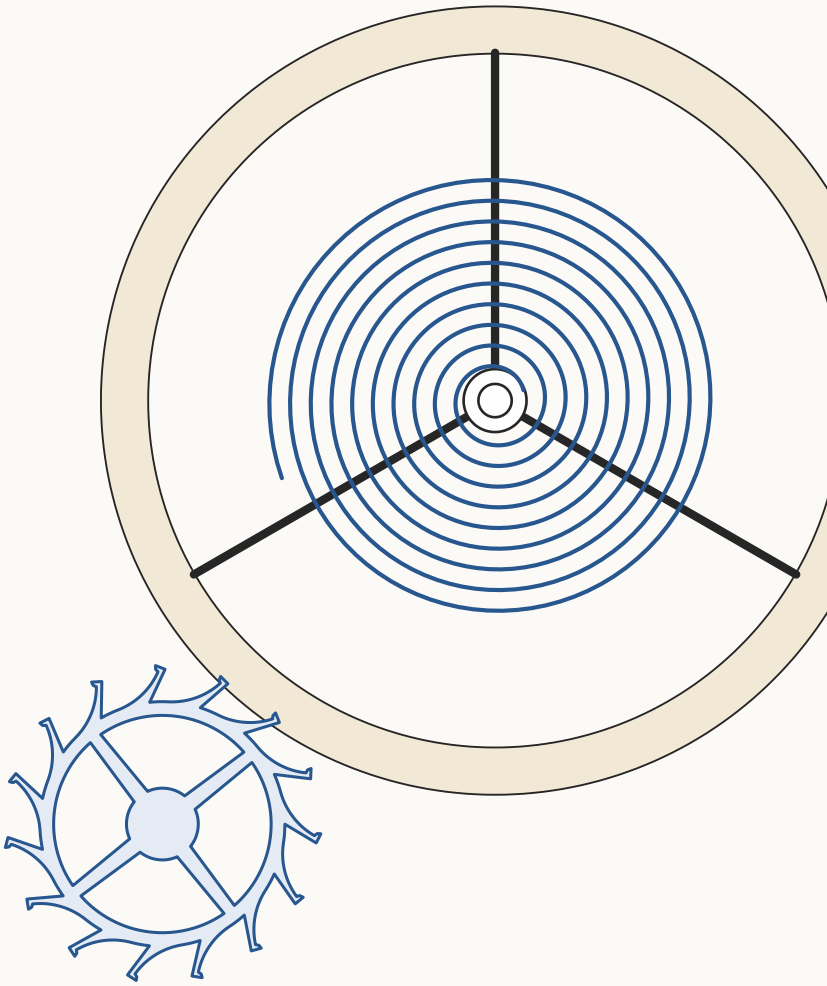
TRAINING · MASTER'S EXAMINATION · WORKSHOP

Watchmaking Formulary

Formulas, sketches and worked examples for watches and clocks



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Horology Edition 2026

How to read this formulary

Each formula has its own card: at the top its name, number and level, below it the formula, rearranged where that helps, and next to it the table of all quantities with their units. Then follow, in one sentence, the conditions under which the formula is valid, and a worked example with numbers and units. We assume that the reader can rearrange formulas; rearranged forms are given only where they are needed constantly in the workshop.

Levels

JOURNEYMAN — Basic knowledge for training and the journeyman's examination.

MASTER — Material of the master's examination.

ADVANCED — Background and design; not calculated in the examination, but useful for understanding what happens in the watch.

Notation

Fractions are always stacked, never written with a slash. Factors are separated by a multiplication dot, decimals use a decimal point, large numbers have thin spaces: $86\,400$.

Most formulas are **quantity equations**: one inserts number and unit and carries the units through the calculation. Any suitable unit is allowed, as long as it is converted cleanly. A few workshop formulas are **numerical-value equations**; they carry the label “Numerical-value equation” and are valid only with exactly the units given in their table of quantities.

Formulas with “≈” are approximations. How far they reach is stated under “Valid”.

Rate, oscillation and beat

The rate G is the deviation of a timepiece per day in seconds ($\frac{\mathrm{s}}{\mathrm{d}}$). Plus means the timepiece gains; minus means it loses.

An **oscillation** is there and back; its duration is the period T, and the frequency f counts full oscillations per second (Hz). A **beat** is a half-oscillation (vibration). The beat rate n_h counts half-oscillations, for watches per hour (A/h): $28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$ equal $4\ \mathrm{Hz}$.

In this formulary T is a time (only for the fits in Chapter XL 9 is it the tolerance). Temperatures are denoted ϑ, temperature changes $\Delta\vartheta$.

Angles

In trigonometric functions ($\sin$, $\cos$, $\tan$) an angle may be given in degrees; the calculator must then be set to degrees. If, however, an angle appears as a factor or as a power in a formula, it is inserted in radians: degrees times $\frac{\pi}{180}$. The table of quantities states which form is meant in each case.

Same symbol, different meaning

The most important symbols mean the same everywhere. Where a chapter needs a symbol in a different sense, for instance A for an area instead of the amplitude, this is stated in the chapter introduction and in the table of quantities of the formula. The table of quantities always applies.

Horology Edition and XL

This edition contains the formulas of the timepiece itself: power, gear train, oscillator, escapement, rate testing, quartz watch, time and calendar. The general foundations from mathematics, mechanics, electrical engineering, manufacturing, materials and statistics are in the XL edition. A reference marked “XL” leads there and gives the number in the XL edition.

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1 Power Source

The power source supplies the energy consumed by gear train, escapement and oscillator. In watches and in many clocks it is stored by a mainspring in the barrel, in the pendulum clock often by a weight. The spring is compact, but its torque decreases as it runs down. The weight needs a height of fall, but in return it always pulls with the same force.

Calculations use workshop units: dimensions in mm, forces in N, torques in $\mathrm{N} \cdot \mathrm{mm}$. How the torque reaches the escapement through the gear train is given in Chapter 2. Departing from the usual assignment, in this chapter P stands for power and m for the mass of the weight; N counts coils, turns and revolutions.

1.1 Mainspring in the Barrel: Geometry

The mainspring lies as a pack in the barrel: run down against the wall, wound around the barrel arbor. For the calculation we assume the coils lie tightly against each other. The spring then covers the same area in both states, namely length times blade thickness. All formulas in this section follow from this.

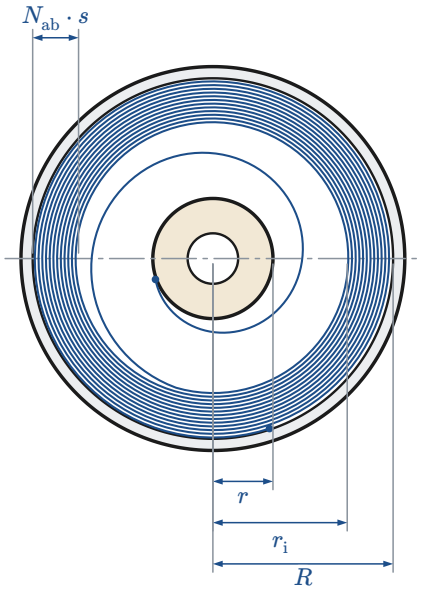


Fig. 1.1 Run down: the pack lies against the wall and reaches inwards to r_{i}

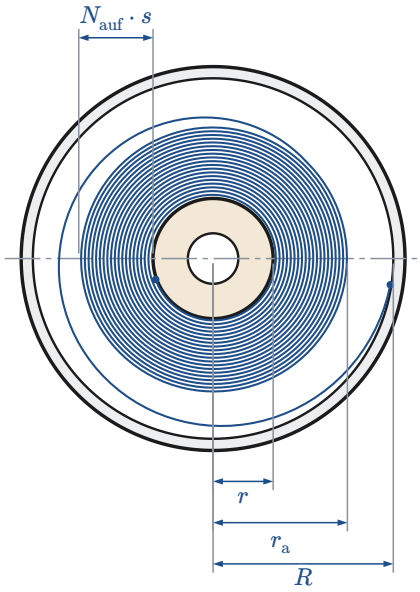


Fig. 1.2 Wound: the pack lies around the arbor and reaches outwards to r_{a}

Radius of the Barrel Arbor (Guide Value)

(1.1) APPROXIMATION JOURNEYMAN

$$r \approx \frac{R}{3}$$

r	radius (of the barrel arbor)	mm
R	inner radius (of the barrel)	mm

The barrel arbor takes up about one third of the inner radius. A thinner arbor would give more turns, but bends the innermost coil too sharply.

Valid: guide value for watches and clocks.

Good to know: Second condition: depending on the spring material, the arbor radius should be at least about 10 to 16 times the blade thickness (guide value). A thinner arbor bends the spring too sharply; it fatigues at the arbor and breaks there.

EXAMPLE A barrel has an inside diameter of 11.4 mm, i.e. a radius of 5.7 mm. What should the radius of the barrel arbor be?

Given: $R = 5.7\,\mathrm{mm}$

$$\begin{aligned} r &\approx \frac{R}{3} \\ &= \frac{5.7\,\mathrm{mm}}{3} \\ &= 1.9\,\mathrm{mm} \end{aligned}$$

The arbor gets a radius of about 1.9 mm, i.e. a diameter of 3.8 mm. With a blade of 0.12 mm this is just under 16 times the blade thickness.

Coils of the Wound Spring

(1.2) MASTER

$$N_{\mathrm{auf}} = \frac{\sqrt{r^2 + \dfrac{l \cdot s}{\pi}} - r}{s}$$

N_{auf}	coils of the wound spring (around the arbor)	1
r	radius (of the barrel arbor)	mm
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm

The square root is the outer radius r_{a} of the wound pack. It follows from equal areas, $l \cdot s = \pi \cdot (r_{\mathrm{a}}^2 - r^2)$. The width of the pack $r_{\mathrm{a}} - r$ divided by the blade thickness gives the coils.

Valid: coils lie tightly together; spring ends, hooks and bridle not included.

EXAMPLE A mainspring 420 mm long with a blade thickness of 0.12 mm is wound around an arbor of 1.9 mm radius. How many coils does the pack have?

Given: $r = 1.9\,\mathrm{mm}$; $l = 420\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$

$$\begin{aligned} N_{\mathrm{auf}} &= \frac{\sqrt{r^2 + \dfrac{l \cdot s}{\pi}} - r}{s} \\ &= \frac{\sqrt{(1.9\,\mathrm{mm})^2 + \dfrac{420\,\mathrm{mm} \cdot 0.12\,\mathrm{mm}}{\pi}} - 1.9\,\mathrm{mm}}{0.12\,\mathrm{mm}} \\ &\approx 21.1 \end{aligned}$$

About 21 coils. The pack extends to a radius of 4.43 mm.

Coils of the Run-Down Spring

(1.3) MASTER

$$N_{\text{ab}} = \frac{R - \sqrt{R^2 - \dfrac{l \cdot s}{\pi}}}{s}$$

Here the square root is the inner radius r_{i} of the pack against the wall, again from equal areas: $l \cdot s = \pi \cdot (R^2 - r_{\text{i}}^2)$.

Valid: coils lie tightly against the wall. The spring must fit into the barrel: $l \cdot s$ smaller than $\pi \cdot (R^2 - r^2)$.

N_{ab}	coils of the run-down spring (against the wall)	1
R	inner radius (of the barrel)	mm
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm

EXAMPLE The same spring (420 mm long, 0.12 mm thick) lies run down against the wall of a barrel with an inner radius of 5.7 mm.

Given: $R = 5.7\,\text{mm}$; $l = 420\,\text{mm}$; $s = 0.12\,\text{mm}$

$$\begin{aligned} N_{\text{ab}} &= \frac{R - \sqrt{R^2 - \dfrac{l \cdot s}{\pi}}}{s} \\ &= \frac{5.7\,\text{mm} - \sqrt{(5.7\,\text{mm})^2 - \dfrac{420\,\text{mm} \cdot 0.12\,\text{mm}}{\pi}}}{0.12\,\text{mm}} \\ &\approx 13.7 \end{aligned}$$

Just under 14 coils. On the inside the pack ends at a radius of 4.06 mm.

Turns of the Barrel

(1.4) JOURNEYMAN

$$N = N_{\text{auf}} - N_{\text{ab}}$$

As the spring runs down, the coils move from the arbor to the wall. The barrel turns through as many turns as the wound spring has more coils than the run-down one.

Valid: coils counted in both states inside the barrel. The value is the upper limit: the spring ends do not take part, and in practice up to one turn is lost.

N	turns (of the barrel)	1
N_{auf}	coils of the wound spring (around the arbor)	1
N_{ab}	coils of the run-down spring (against the wall)	1

EXAMPLE The spring from the two previous examples has 21.1 coils wound and 13.7 coils run down.

Given: $N_{\text{auf}} = 21.1$; $N_{\text{ab}} = 13.7$

$$\begin{aligned} N &= N_{\text{auf}} - N_{\text{ab}} \\ &= 21.1 - 13.7 \\ &= 7.4 \end{aligned}$$

By calculation the barrel makes 7.4 turns.

Optimum Spring Length

(1.5) MASTER

$$l = \frac{\pi \cdot \left(R^2 - r^2\right)}{2 \cdot s}$$

l	length (of the mainspring)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm
s	blade thickness (thickness of the spring)	mm

The most turns are given by the spring that covers exactly half of the free area between arbor and wall. A shorter spring has too few coils, a longer one leaves them no room to unwind.

Valid: coils lying tightly; allowance for attaching the ends not included.

Good to know: The maximum is flat (Fig. 1.3). In the example, a spring 20 % shorter or longer costs only about 3 % of the turns. The length therefore need not be exact to the millimetre, whereas the blade thickness matters a great deal (Section 1.2).

EXAMPLE Which length is optimum for a blade of 0.12 mm in a barrel with an inner radius of 5.7 mm and an arbor radius of 1.9 mm?

Given: $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$

$$\begin{aligned} l &= \frac{\pi \cdot \left(R^2 - r^2\right)}{2 \cdot s} \\ &= \frac{\pi \cdot \left((5.7\,\mathrm{mm})^2 - (1.9\,\mathrm{mm})^2\right)}{2 \cdot 0.12\,\mathrm{mm}} \\ &\approx 378\,\mathrm{mm} \end{aligned}$$

About 378 mm. The 420 mm spring from the previous examples is therefore somewhat too long, but hardly loses any turns as a result.

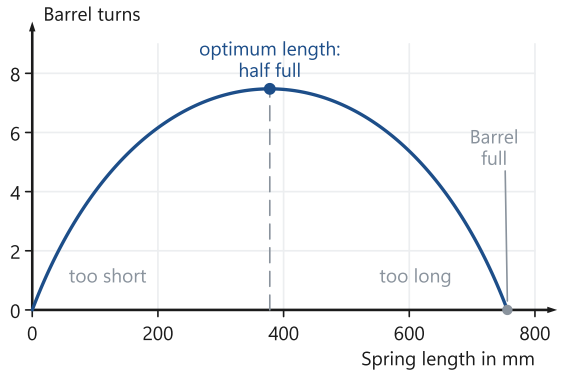


Fig. 1.3 Turns as a function of spring length, calculated for $R = 5.7\,\mathrm{mm}$, $r = 1.9\,\mathrm{mm}$, $s = 0.12\,\mathrm{mm}$

Filling Ratio of the Barrel

(1.6) MASTER

$$k_\mathrm{F} = \frac{l \cdot s}{\pi \cdot (R^2 - r^2)}$$

The filling ratio shows whether an existing or ordered spring suits the barrel. The optimum is 0.5.

Valid: coils lying tightly.

k_F	filling ratio: share of the free area covered by the spring	1
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm

EXAMPLE How much does the spring 420 mm long with a blade thickness of 0.12 mm fill the barrel (inner radius 5.7 mm, arbor radius 1.9 mm)?

Given: $l = 420\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$; $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$

$$\begin{aligned} k_\mathrm{F} &= \frac{l \cdot s}{\pi \cdot (R^2 - r^2)} \\ &= \frac{420\,\mathrm{mm} \cdot 0.12\,\mathrm{mm}}{\pi \cdot \left((5.7\,\mathrm{mm})^2 - (1.9\,\mathrm{mm})^2\right)} \\ &\approx 0.56 \end{aligned}$$

The spring covers 56 % of the free area, slightly more than the optimum half.

Dividing Circle at Optimum Length

(1.7) MASTER

$$\rho = \sqrt{\frac{R^2 + r^2}{2}}$$

At optimum length, the wound pack extends exactly as far outwards as the run-down pack extends inwards. This dividing circle splits the free area into two equal halves.

Valid: spring of optimum length, filling ratio 0.5.

ρ	radius (of the dividing circle)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm

Good to know: Check on the open barrel: if the run-down spring reaches further in than the dividing circle, it is too long. If it stays further out, it is too short.

EXAMPLE Where is the dividing circle in the barrel with an inner radius of 5.7 mm and an arbor radius of 1.9 mm?

Given: $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$

$$\begin{aligned} \rho &= \sqrt{\frac{R^2 + r^2}{2}} \\ &= \sqrt{\frac{(5.7\,\mathrm{mm})^2 + (1.9\,\mathrm{mm})^2}{2}} \\ &\approx 4.25\,\mathrm{mm} \end{aligned}$$

At a radius of 4.25 mm. The run-down pack is then 1.45 mm wide, the wound pack 2.35 mm.

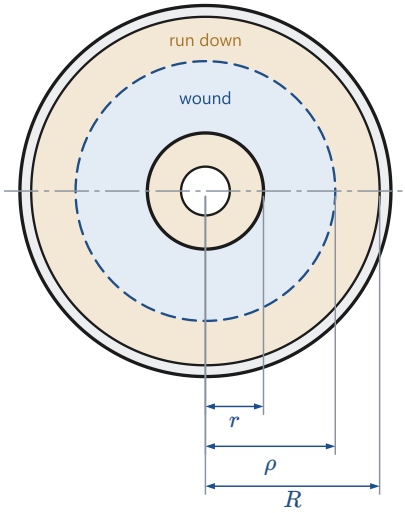


Fig. 1.4 The dividing circle ρ halves the free area: the wound spring lies inside, the run-down spring outside

Maximum Number of Turns

(1.8) MASTER

$$N_{\max} = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{s}$$

REARRANGED $$s = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{N_{\max}}$$

This many turns are possible with the optimum spring length. Rearranged, the formula gives the blade thickness for a required number of turns.

Valid: spring of optimum length (filling ratio 0.5), coils lying tightly. Theoretical upper limit as for formula (1.4).

$N_{\max}$	maximum possible turns of the barrel	1
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm
s	blade thickness (thickness of the spring)	mm

EXAMPLE What is the maximum number of turns the barrel (inner radius 5.7 mm, arbor radius 1.9 mm) can achieve with a blade of 0.12 mm?

Given: $R = 5.7\,\text{mm}$; $r = 1.9\,\text{mm}$; $s = 0.12\,\text{mm}$

$$N_{\max} = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{s}$$

$$= \frac{\sqrt{2 \cdot \left((5.7\,\text{mm})^2 + (1.9\,\text{mm})^2\right)} - 5.7\,\text{mm} - 1.9\,\text{mm}}{0.12\,\text{mm}}$$

$$\approx 7.5$$

About 7.5 turns. With a blade of 0.10 mm it would be 9, with 0.14 mm only 6.4.

Guide Values for Arbor Radius $r = \frac{1}{3}\,R$ and Optimum Spring Length

Quantity	Value
radius of the dividing circle	$\rho = 0.745 \cdot R$
width of the run-down pack	$0.255 \cdot R$, about one quarter of the inner radius
width of the wound pack	$0.412 \cdot R$
optimum spring length	$l = 1.40 \cdot \frac{R^2}{s}$
maximum number of turns	$N_{\max} = 0.157 \cdot \frac{R}{s}$

Calculated from formulas (1.5), (1.7) and (1.8). The values apply only if the arbor measures one third of the inner radius.

Spring Length from the Coils of the Run-Down Spring

(1.9) JOURNEYMAN

$$l = \pi \cdot (R + r_\mathrm{i}) \cdot N_\mathrm{ab}$$

The length is the mean circumference of the pack times the number of coils. This allows the spring length to be determined without removing the spring. The blade thickness follows from the width of the pack: $s = (R - r_\mathrm{i}) : N_\mathrm{ab}$.

Valid: coils lie tightly against the wall. The piece from the pack to the arbor is not included; the actual spring is longer by about one inner coil.

l	length (of the mainspring)	mm
R	inner radius (of the barrel)	mm
r_i	inner radius of the run-down spring pack	mm
N_ab	coils of the run-down spring (against the wall)	1

EXAMPLE In a barrel with an inner radius of 5.7 mm, the run-down spring lies against the wall with 13.75 coils. The inner edge of the pack has a radius of 4.05 mm.

Given: $R = 5.7\,\mathrm{mm}$; $r_\mathrm{i} = 4.05\,\mathrm{mm}$; $N_\mathrm{ab} = 13.75$

$$\begin{aligned} l &= \pi \cdot (R + r_\mathrm{i}) \cdot N_\mathrm{ab} \\ &= \pi \cdot (5.7\,\mathrm{mm} + 4.05\,\mathrm{mm}) \cdot 13.75 \\ &\approx 421\,\mathrm{mm} \end{aligned}$$

The pack contains about 421 mm of spring. The blade thickness is 1.65 mm divided by 13.75, i.e. 0.12 mm.

Key point: The spring should fill half of the free area of the barrel. More turns are then only possible with a thinner blade.

1.2 Torque and Energy of the Mainspring

The wound spring tends to bend back and in doing so turns the barrel. Its torque depends mainly on the blade thickness, the stored energy on the volume of the spring and on the material.

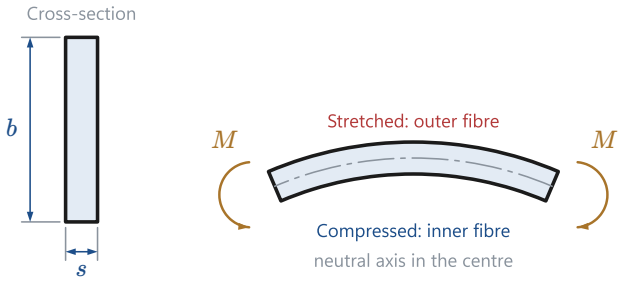


Fig. 1.5 Blade cross-section with height b and thickness s (thickness exaggerated); in bending the outside is stretched, the inside compressed

Torque of the Mainspring

(1.10) MASTER

$$M = \frac{E \cdot b \cdot s^3 \cdot \varphi}{12 \cdot l}$$

The torque increases with the angle of twist, with the blade height and with the third power of the blade thickness. A longer spring is softer. One revolution corresponds to $\varphi = 2\pi$.

Valid: elastic range, rectangular cross-section, coils do not touch or rub against each other. The angle is counted from the relaxed shape of the spring outside the barrel.

M	torque (of the spring)	$\mathrm{N \cdot mm}$
E	modulus of elasticity (of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm
φ	angle of twist (relative to the relaxed shape, in radians)	rad
l	length (of the mainspring)	mm

Caution: In the barrel, coils lie against each other at the arbor and at the wall. They do not work and they rub. The actual characteristic is therefore not a straight line (Fig. 1.6). The formula shows what the torque depends on; the torque of a particular spring is measured.

EXAMPLE A mainspring ($E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}}$, height 1.3 mm, thickness 0.12 mm, length 420 mm) is tensioned by 12 revolutions relative to its relaxed shape, i.e. 75.4 rad.

Given: $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}}$; $b = 1.3\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$; $\varphi = 75.4\,\mathrm{rad}$; $l = 420\,\mathrm{mm}$

$$M = \frac{E \cdot b \cdot s^3 \cdot \varphi}{12 \cdot l}$$
$$= \frac{200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}} \cdot 1.3\,\mathrm{mm} \cdot (0.12\,\mathrm{mm})^3 \cdot 75.4\,\mathrm{rad}}{12 \cdot 420\,\mathrm{mm}}$$
$$\approx 6.72\,\mathrm{N \cdot mm}$$

About 6.7 $\mathrm{N \cdot mm}$, a usual order of magnitude for the barrel of a man's wristwatch.

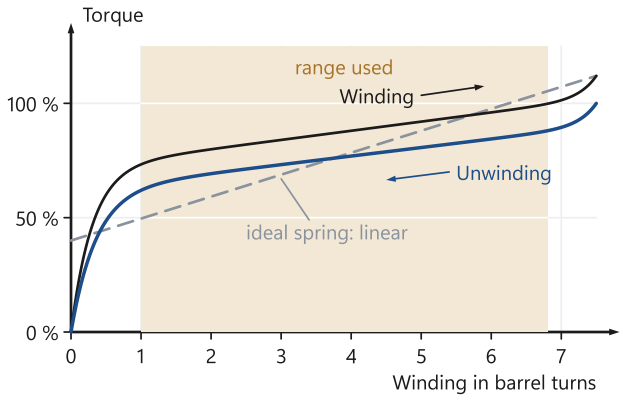


Fig. 1.6 Torque in the barrel during winding and running down (schematic, not a measured curve); 100 % is the torque at which the run-down begins from full wind

Fig. 1.6 shows the curve in the barrel. At the very top and the very bottom the torque changes sharply; in between it falls only slowly. The flat part is used. During winding the curve lies higher than during running down; the difference is the friction between the coils. Reverse-curve (S-shaped) springs (Fig. 1.8) are bent against their later winding direction in the relaxed state. They are stressed more evenly over their whole length and deliver a more even torque than springs with a simple spiral shape.

Maximum Torque from the Permissible Bending Stress

(1.11) MASTER

$$M_{\max} = \frac{\sigma_{\text{zul}} \cdot b \cdot s^2}{6}$$

$M_{\max}$	maximum torque transmissible elastically	$\mathrm{N{\cdot}mm}$
σ_{zul}	permissible bending stress (elastic limit of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm

The blade cannot withstand more torque elastically: the outer fibre reaches the permissible stress. The fraction $\frac{b \cdot s^2}{6}$ is the section modulus of the rectangle (Chapter XL 2).

Valid: rectangular cross-section, pure bending. Beyond the elastic limit the spring deforms permanently; it sets.

EXAMPLE What torque does a blade of 1.3 mm height and 0.12 mm thickness transmit at a permissible bending stress of $2500\ \frac{\mathrm{N}}{\mathrm{mm^2}}$?

Given: $\sigma_{\text{zul}} = 2500\ \frac{\mathrm{N}}{\mathrm{mm^2}}$; $b = 1.3\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$

$$\begin{aligned} M_{\max} &= \frac{\sigma_{\text{zul}} \cdot b \cdot s^2}{6} \\ &= \frac{2500\ \frac{\mathrm{N}}{\mathrm{mm^2}} \cdot 1.3\,\mathrm{mm} \cdot (0.12\,\mathrm{mm})^2}{6} \\ &= 7.8\,\mathrm{N{\cdot}mm} \end{aligned}$$

At most $7.8\ \mathrm{N \cdot mm}$. The previous example, at $6.72\ \mathrm{N \cdot mm}$, lies below this.

Spring Materials for Mainsprings (Guide Values)

Material	E in $\frac{\mathrm{N}}{\mathrm{mm^2}}$	σ_{zul} in $\frac{\mathrm{N}}{\mathrm{mm^2}}$
spring steel (carbon steel, hardened and tempered)	206 000 … 210 000	1 500 … 2 000
cobalt-nickel-chromium alloy (for example Nivaflex)	200 000 … 225 000	2 200 … 2 800

Rough guide values for estimates. The permissible stress depends strongly on alloy, degree of rolling and heat treatment. The spring manufacturer's data are authoritative. Modulus of elasticity and tensile strength of further materials are given in Section XL 2.7.

Influence of Blade Thickness

(1.12) JOURNEYMAN

$$M_2 = M_1 \cdot \left(\frac{s_2}{s_1}\right)^3$$

M_2	torque of the replacement spring	$\mathrm{N{\cdot}mm}$
M_1	torque of the original spring	$\mathrm{N{\cdot}mm}$
s_2	blade thickness of the replacement spring	mm
s_1	blade thickness of the original spring	mm

The torque changes with the third power of the blade thickness. Even one hundredth of a millimetre makes a clear difference.

Valid: same blade height, same length, same material, same angle of twist.

Caution: A thicker blade fills the barrel more and achieves fewer turns. With a different blade thickness, therefore also recalculate length and turns according to Section 1.1.

EXAMPLE Instead of a spring with a blade thickness of 0.12 mm and $6.0\ \mathrm{N \cdot mm}$, one with 0.125 mm is fitted. What will the torque be?

Given: $M_1 = 6.0\,\mathrm{N{\cdot}mm}$; $s_2 = 0.125\,\mathrm{mm}$; $s_1 = 0.12\,\mathrm{mm}$

$$\begin{aligned} M_2 &= M_1 \cdot \left(\frac{s_2}{s_1}\right)^3 \\ &= 6.0\,\mathrm{N{\cdot}mm} \cdot \left(\frac{0.125\,\mathrm{mm}}{0.12\,\mathrm{mm}}\right)^3 \\ &\approx 6.8\,\mathrm{N{\cdot}mm} \end{aligned}$$

About $6.8\ \mathrm{N \cdot mm}$. The blade, only 0.005 mm thicker, gives 13 % more torque.

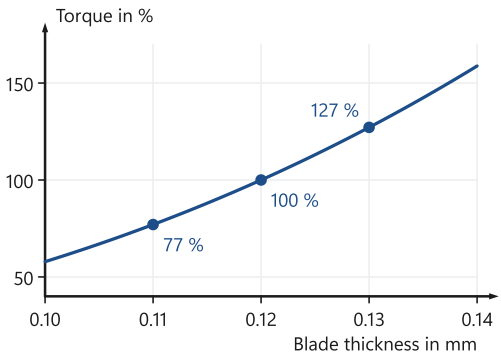


Fig. 1.7 The torque increases with the third power of the blade thickness (100 % at $s = 0.12\,\mathrm{mm}$)

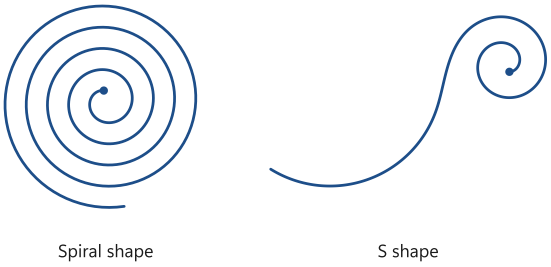


Fig. 1.8 Relaxed mainsprings: simple spiral shape and reverse-curve (S) shape

Replacement Spring with the Same Effect

(1.13) JOURNEYMAN

$$b_1 \cdot s_1^3 = b_2 \cdot s_2^3$$

REARRANGED $s_2 = s_1 \cdot \sqrt[3]{\dfrac{b_1}{b_2}}$

If the original spring is not available, a slightly thicker blade compensates for a lower height and vice versa. The torque remains the same if the product of height and the third power of thickness remains the same.

Valid: same length, same material. The replacement spring must not be higher than the barrel allows.

b_1	blade height of the original spring	mm
s_1	blade thickness of the original spring	mm
b_2	blade height of the replacement spring	mm
s_2	blade thickness of the replacement spring	mm

EXAMPLE The original spring measures 1.50 mm × 0.12 mm. Only a spring with a height of 1.40 mm is available. Which blade thickness gives the same torque?

Given: $s_1 = 0.12\,\mathrm{mm}$; $b_1 = 1.50\,\mathrm{mm}$; $b_2 = 1.40\,\mathrm{mm}$

$$\begin{aligned} s_2 &= s_1 \cdot \sqrt[3]{\frac{b_1}{b_2}} \\ &= 0.12\,\mathrm{mm} \cdot \sqrt[3]{\frac{1.50\,\mathrm{mm}}{1.40\,\mathrm{mm}}} \\ &\approx 0.1228\,\mathrm{mm} \end{aligned}$$

By calculation just under 0.123 mm. The next thickness on the list is chosen, here 0.125 mm, accepting slightly more torque.

Stored Energy of the Ideal Spring

(1.14) MASTER

$$W = \frac{M \cdot \varphi}{2}$$

Because the torque rises evenly from the relaxed state, on average half the final torque counts. $1\,\mathrm{N \cdot mm} = 1\,\mathrm{mJ}$.

Valid: ideal spring with a linear characteristic according to formula (1.10).

W	work (energy stored in the spring, tensioning work)	mJ
M	torque (of the spring)	$\mathrm{N{\cdot}mm}$
φ	angle of twist (relative to the relaxed shape, in radians)	rad

EXAMPLE The spring from the example for formula (1.10) has a torque of 6.72 $\mathrm{N \cdot mm}$ at 75.4 rad.

Given: $M = 6.72\,\mathrm{N{\cdot}mm}$; $\varphi = 75.4\,\mathrm{rad}$

$$\begin{aligned} W &= \frac{M \cdot \varphi}{2} \\ &= \frac{6.72\,\mathrm{N{\cdot}mm} \cdot 75.4\,\mathrm{rad}}{2} \\ &\approx 253\,\mathrm{mJ} \end{aligned}$$

The spring contains about 253 mJ, i.e. 0.25 J. Only the part delivered by the barrel while running down is usable (formula (1.16)).

Maximum Storable Energy

(1.15) ADVANCED

$$W_{\max} = \frac{\sigma_{\mathrm{zul}}^2 \cdot b \cdot s \cdot l}{6 \cdot E}$$

The product $b \cdot s \cdot l$ is the volume of the spring. The storable energy therefore increases with the spring volume and with $\sigma_{\mathrm{zul}}^2 : E$. This is why the barrel is given as much space as possible in the movement, and why materials with a high elastic limit are worthwhile.

Valid: ideal spring, bent to the permissible stress over its whole length. Upper limit; real springs do not reach it.

$W_{\max}$	maximum elastically storable energy	mJ
σ_{zul}	permissible bending stress	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm
l	length (of the mainspring)	mm
E	modulus of elasticity (of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$

EXAMPLE Spring 1.3 mm × 0.12 mm × 420 mm, $\sigma_{\mathrm{zul}} = 2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}$, $E = 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}$.

Given: $\sigma_{\mathrm{zul}} = 2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}$; $b = 1.3\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$; $l = 420\,\mathrm{mm}$; $E = 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}$

$$\begin{aligned} W_{\max} &= \frac{\sigma_{\mathrm{zul}}^2 \cdot b \cdot s \cdot l}{6 \cdot E} \\ &= \frac{\left(2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}\right)^2 \cdot 1.3\,\mathrm{mm} \cdot 0.12\,\mathrm{mm} \cdot 420\,\mathrm{mm}}{6 \cdot 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}} \\ &\approx 341\,\mathrm{mJ} \end{aligned}$$

At most about 0.34 J. This is the energy needed to lift 35 g by one metre.

Work Delivered while Running Down

(1.16) JOURNEYMAN

$$W = 2 \cdot \pi \cdot N \cdot M_{\mathrm{m}}$$

Work is torque times angle of rotation; in each turn the barrel rotates through 2π. The mean torque is read from the measured let-down curve.

Valid: torque averaged over the run-down.

W	work (delivered by the barrel while running down)	mJ
N	turns (of the barrel)	1
M_{m}	mean torque over these turns	$\mathrm{N{\cdot}mm}$

EXAMPLE A barrel delivers 6.0 $\mathrm{N \cdot mm}$ on average over 6.5 turns.

Given: $N = 6.5$; $M_{\mathrm{m}} = 6.0\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} W &= 2 \cdot \pi \cdot N \cdot M_{\mathrm{m}} \\ &= 2 \cdot \pi \cdot 6.5 \cdot 6.0\,\mathrm{N{\cdot}mm} \\ &\approx 245\,\mathrm{mJ} \end{aligned}$$

The movement receives about 245 mJ, just under a quarter of a joule for almost two days of running.

Efficiency of the Mainspring

(1.17) MASTER

$$\eta_{\mathrm{F}} = \frac{W_{\mathrm{ab}}}{W_{\mathrm{auf}}}$$

Friction between the coils and at the wall consumes part of the winding work. Clean blades and the right grease in the barrel keep the loss small.

Valid: both quantities of work measured over the same turns.

η_{F}	efficiency of the spring in the barrel	1
W_{ab}	work delivered while running down (area under the let-down curve)	mJ
W_{auf}	work expended in winding (area under the winding curve)	mJ

EXAMPLE The barrel from the previous example delivers 245 mJ while running down. 288 mJ were expended in winding over the same turns.

Given: $W_{\mathrm{ab}} = 245\,\mathrm{mJ}$; $W_{\mathrm{auf}} = 288\,\mathrm{mJ}$

$$\begin{aligned} \eta_{\mathrm{F}} &= \frac{W_{\mathrm{ab}}}{W_{\mathrm{auf}}} \\ &= \frac{245\,\mathrm{mJ}}{288\,\mathrm{mJ}} \\ &= 0.85 \end{aligned}$$

The spring returns 85 % of the winding work; 15 % is lost as friction.

Key point: Double the blade height, double the torque. Double the blade thickness, eight times the torque.

1.3 Running Time

The running time is the time for which the movement runs on one full wind. It follows from the turns of the barrel and the time taken by one turn. The power reserve is the part of the running time that still lies ahead of the movement. As in Chapter 2, the number of teeth of a pinion carries a prime and the index of its arbor.

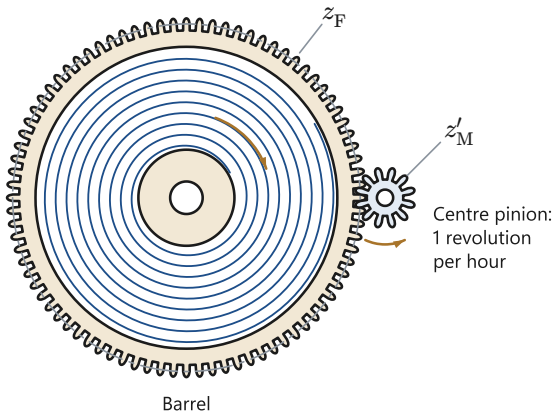


Fig. 1.9 The barrel with z_F teeth drives the centre pinion with z'_M teeth

Rotation Period of the Barrel

(1.18) JOURNEYMAN

$$t_F = \frac{z_F}{z'_M} \cdot t_M$$

The barrel meshes with the pinion of the centre wheel, which turns once per hour. If the barrel has seven times as many teeth as the pinion, one turn takes seven hours.

Valid: barrel meshes directly with the pinion of a wheel that turns once per hour. If the driven wheel turns differently, insert its rotation period.

t_F	rotation period of the barrel (time for one turn)	h
z_F	number of teeth of the barrel	1
z'_M	number of teeth of the centre pinion	1
t_M	rotation period of the centre wheel (1 h)	h

EXAMPLE A barrel with 84 teeth drives a centre pinion with 12 teeth.

Given: $z_F = 84$; $z'_M = 12$; $t_M = 1\,\mathrm{h}$

$$\begin{aligned} t_F &= \frac{z_F}{z'_M} \cdot t_M \\ &= \frac{84}{12} \cdot 1\,\mathrm{h} \\ &= 7\,\mathrm{h} \end{aligned}$$

The barrel takes 7 hours for one turn.

Running Time from the Turns of the Barrel

(1.19) JOURNEYMAN

$$t_G = N \cdot t_F$$

REARRANGED $N = \frac{t_G}{t_F}$

t_G	running time	h
N	turns (of the barrel)	1
t_F	rotation period of the barrel (time for one turn)	h

The turns that the barrel actually delivers give the running time. Rearranged, the formula gives the turns required for a specified running time.

Valid: evenly running movement; turns from full wind to standstill. For the usable running time count only the turns in which the torque is sufficient for an adequate amplitude.

Good to know: The power reserve is the running time remaining after a certain period of running: remaining turns times rotation period of the barrel.

EXAMPLE The barrel from the previous example (7 h per turn) delivers 6.5 usable turns.

Given: $N = 6.5$; $t_F = 7\,\text{h}$

$$\begin{aligned} t_G &= N \cdot t_F \\ &= 6.5 \cdot 7\,\text{h} \\ &= 45.5\,\text{h} \end{aligned}$$

The watch runs for 45.5 hours. After 24 hours, 21.5 hours of power reserve remain.

Rotation Period of the Barrel with Intermediate Wheel

(1.20) JOURNEYMAN

$$t_F = \frac{z_F \cdot z_B}{z'_B \cdot z'_M} \cdot t_M$$

t_F	rotation period of the barrel (time for one turn)	h
z_F	number of teeth of the barrel	1
z_B	number of teeth of the intermediate wheel	1
z'_B	number of teeth of the intermediate pinion	1
z'_M	number of teeth of the centre pinion	1
t_M	rotation period of the centre wheel (1 h)	h

For long running times an intermediate wheel sits between barrel and centre wheel. The numerator contains the driving wheels, the denominator the driven pinions.

Valid: barrel drives the intermediate pinion, the intermediate wheel drives the centre pinion; centre wheel once per hour.

EXAMPLE Eight-day movement: barrel 84 teeth, intermediate pinion 12, intermediate wheel 72, centre pinion 12.

Given: $z_F = 84$; $z_B = 72$; $z'_B = 12$; $z'_M = 12$; $t_M = 1\,\text{h}$

$$\begin{aligned} t_F &= \frac{z_F \cdot z_B}{z'_B \cdot z'_M} \cdot t_M \\ &= \frac{84 \cdot 72}{12 \cdot 12} \cdot 1\,\text{h} \\ &= 42\,\text{h} \end{aligned}$$

One turn takes 42 hours. For eight days, i.e. 192 hours, the movement needs about 4.6 turns according to formula (1.19).

In an automatic movement the outer end of the spring is not hooked in. It carries a slipping bridle that presses outwards against the barrel wall and slips as soon as the spring is fully wound. Its slipping torque determines the maximum torque the spring delivers to the movement. If the bridle slips too early, the spring is never fully wound and power reserve is lost. If it holds too firmly, the torque becomes too high and the balance knocks.

1.4 Weight Drive

A weight pulls on the barrel via a cable, gut line or chain. Its torque remains the same over the entire running time; this makes the weight drive the most uniform power source. The price is the height of fall.

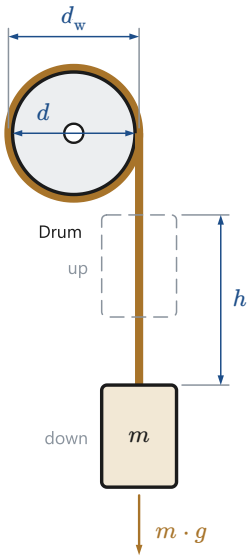


Fig. 1.10 Weight hanging directly on the line ($k = 1$): effective diameter d_w to the centre of the line, height of fall h

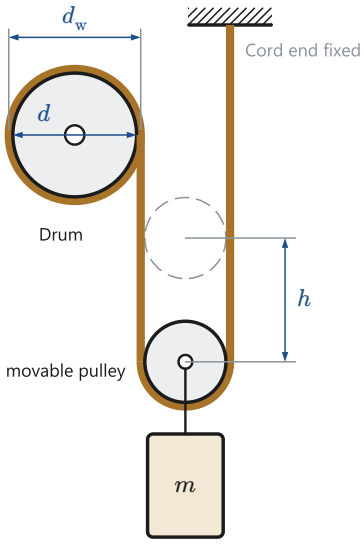


Fig. 1.11 Weight on a loose pulley ($k = 2$): the line runs off twice as far as the weight falls

Effective Barrel Diameter

(1.21) JOURNEYMAN

$$d_w = d + d_S$$

The force of the line acts at the centre of the line. The lever arm is therefore half the barrel diameter plus half the thickness of the line.

Valid: smooth barrel, line in one layer. For grooved barrels measure the diameter to the centre of the line.

d_w	effective barrel diameter (to the centre of the line)	mm
d	diameter (of the barrel)	mm
d_S	thickness of the cable or gut line	mm

EXAMPLE A line 1.5 mm thick lies on a barrel of 36 mm diameter.

Given: $d = 36\,\mathrm{mm}$; $d_S = 1.5\,\mathrm{mm}$

$$\begin{aligned} d_w &= d + d_S \\ &= 36\,\mathrm{mm} + 1.5\,\mathrm{mm} \\ &= 37.5\,\mathrm{mm} \end{aligned}$$

The calculation uses 37.5 mm.

Torque at the Weight Barrel

(1.22) JOURNEYMAN

$$M = \frac{m \cdot g \cdot d_{\mathrm{w}}}{2 \cdot k}$$

The weight force acts at half the effective diameter. If the weight hangs on a loose pulley, each of the two strands carries half the load, so only half the weight force pulls on the barrel.

Valid: friction of the pulley and weight of line and pulley neglected.

M	torque (at the barrel)	$\mathrm{N{\cdot}mm}$
m	mass (of the weight)	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
d_{w}	effective barrel diameter (to the centre of the line)	mm
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1

EXAMPLE A longcase clock has a weight of 4 kg on a loose pulley; effective barrel diameter 37.5 mm.

Given: $m = 4\,\mathrm{kg}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $d_{\mathrm{w}} = 37.5\,\mathrm{mm}$; $k = 2$

$$\begin{aligned} M &= \frac{m \cdot g \cdot d_{\mathrm{w}}}{2 \cdot k} \\ &= \frac{4\,\mathrm{kg} \cdot 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot 37.5\,\mathrm{mm}}{2 \cdot 2} \\ &\approx 368\,\mathrm{N{\cdot}mm} \end{aligned}$$

About $368\,\mathrm{N \cdot mm}$, i.e. $0.368\,\mathrm{N \cdot m}$, about fifty times that of a wristwatch. Without a pulley the torque would be twice as large.

Height of Fall of the Weight

(1.23) JOURNEYMAN

$$h = \frac{\pi \cdot d_{\mathrm{w}} \cdot N_{\mathrm{W}}}{k}$$

In each revolution the barrel releases one circumference of line. With a loose pulley the line is divided between two strands, so the weight falls only half as far.

Valid: line in one layer on the barrel.

h	height (height of fall of the weight)	mm
d_{w}	effective barrel diameter (to the centre of the line)	mm
N_{W}	revolutions of the barrel	1
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1

Good to know: The space required in the case must also include the height of the weight and, with a loose pulley, the pulley with its hook.

EXAMPLE The barrel of the longcase clock (effective diameter 37.5 mm, loose pulley) makes 16 revolutions in eight days.

Given: $d_{\mathrm{w}} = 37.5\,\mathrm{mm}$; $N_{\mathrm{W}} = 16$; $k = 2$

$$\begin{aligned} h &= \frac{\pi \cdot d_{\mathrm{w}} \cdot N_{\mathrm{W}}}{k} \\ &= \frac{\pi \cdot 37.5\,\mathrm{mm} \cdot 16}{2} \\ &\approx 942\,\mathrm{mm} \end{aligned}$$

The weight falls about 94 cm in eight days. Without a pulley it would be 1.88 m, more than a usual case provides.

Running Time from the Height of Fall

(1.24) JOURNEYMAN

$$t_{\mathrm{G}} = \frac{k \cdot h}{\pi \cdot d_{\mathrm{w}}} \cdot t_{\mathrm{W}}$$

The fraction is the number of barrel revolutions that fit into the height of fall. As with the mainspring barrel, the rotation period of the weight barrel follows from the numbers of teeth (formula (1.18)).

Valid: line in one layer. Only the distance the weight can actually fall counts as height of fall, excluding the height of weight and pulley.

t_{G}	running time	h
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1
h	height (height of fall of the weight)	mm
d_{w}	effective barrel diameter (to the centre of the line)	mm
t_{W}	rotation period of the barrel	h

EXAMPLE In a wall regulator the weight can fall 1100 mm. The barrel (effective diameter 31 mm) turns once in 12 hours. How long does the clock run with a loose pulley?

Given: $k = 2$; $h = 1100\,\mathrm{mm}$; $d_{\mathrm{w}} = 31\,\mathrm{mm}$; $t_{\mathrm{W}} = 12\,\mathrm{h}$

$$\begin{aligned} t_{\mathrm{G}} &= \frac{k \cdot h}{\pi \cdot d_{\mathrm{w}}} \cdot t_{\mathrm{W}} \\ &= \frac{2 \cdot 1100\,\mathrm{mm}}{\pi \cdot 31\,\mathrm{mm}} \cdot 12\,\mathrm{h} \\ &\approx 271\,\mathrm{h} \end{aligned}$$

About 271 hours, i.e. a little over 11 days. Without a pulley it would be only 5.6 days; for an eight-day movement this case therefore needs the loose pulley.

Energy of the Weight

(1.25) JOURNEYMAN

$$W = m \cdot g \cdot h$$

The energy depends only on the mass and height of fall of the weight. A loose pulley changes nothing: half the force at the barrel, but twice the length of line.

Valid: friction of the pulley neglected.

W	work (delivered by the weight, potential energy)	J
m	mass (of the weight)	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	height (height of fall of the weight)	m

EXAMPLE The weight of the longcase clock (4 kg) falls 0.942 m in eight days.

Given: $m = 4\,\mathrm{kg}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 0.942\,\mathrm{m}$

$$\begin{aligned} W &= m \cdot g \cdot h \\ &= 4\,\mathrm{kg} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 0.942\,\mathrm{m} \\ &\approx 37.0\,\mathrm{J} \end{aligned}$$

The longcase clock converts 37 J in eight days. The wristwatch from Section 1.2 manages on 0.245 J in just under two days.

Key point: The loose pulley halves height of fall and torque; the running time remains the same. For the same torque it needs twice the weight.

1.5 Fusee and Chain

The fusee compensates for the decreasing torque of the mainspring. With the spring fully wound, the chain pulls at the small radius of the fusee, with the spring almost run down at the large radius. The torque at the fusee wheel thus remains almost constant.

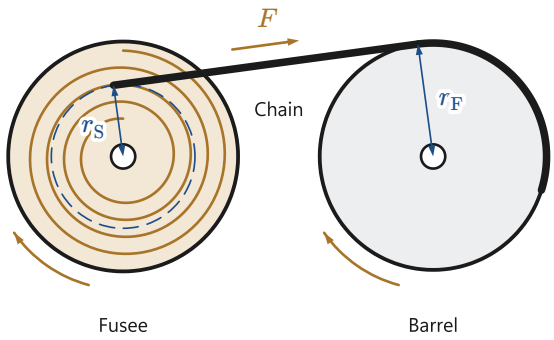


Fig. 1.12 Fusee and barrel seen from above: the chain pulls with the force F at the effective radius r_S

Torque at the Fusee

(1.26) ADVANCED

$$M_S = \frac{M_F \cdot r_S}{r_F}$$

REARRANGED $r_S = \dfrac{M_S \cdot r_F}{M_F}$

The chain transmits the force $F = M_F : r_F$ from the barrel to the fusee; there it acts at the radius r_S. Rearranged, the formula gives for every state of winding the radius at which the fusee delivers the required torque.

Valid: both radii measured to the centre of the chain; friction of chain and pivots neglected.

M_S	torque at the fusee	$\mathrm{N{\cdot}mm}$
M_F	torque of the barrel at the same moment	$\mathrm{N{\cdot}mm}$
r_S	effective radius of the fusee at the point where the chain runs off	mm
r_F	radius of the barrel (to the centre of the chain)	mm

EXAMPLE The barrel of a fusee pocket watch has a radius of 9 mm. The fusee is to deliver a constant $32\ \mathrm{N \cdot mm}$. Fully wound, the spring delivers $48\ \mathrm{N \cdot mm}$. What radius does the fusee need at this point?

Given: $M_S = 32\,\mathrm{N{\cdot}mm}$; $r_F = 9\,\mathrm{mm}$; $M_F = 48\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} r_S &= \frac{M_S \cdot r_F}{M_F} \\ &= \frac{32\,\mathrm{N{\cdot}mm} \cdot 9\,\mathrm{mm}}{48\,\mathrm{N{\cdot}mm}} \\ &= 6.0\,\mathrm{mm} \end{aligned}$$

6 mm at the thin end of the fusee. When the spring has dropped to $32\ \mathrm{N \cdot mm}$, the radius must be 9 mm.

1.6 Power of the Drive

Power is energy per unit time. A mechanical watch needs very little of it: at the barrel of a wristwatch it is one to a few microwatts, at the weight barrel of a longcase clock some tens of microwatts.

Mean Power

(1.27) JOURNEYMAN

$$P = \frac{W}{t_{\mathrm{G}}}$$

For substitution, convert the running time to seconds: $1\,\mathrm{h} = 3600\,\mathrm{s}$. Then $1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{W}$.

Valid: mean value over the entire running time.

P	power (of the drive)	$\mu\mathrm{W}$
W	work (delivered during the running time)	mJ
t_{G}	running time	h

EXAMPLE The barrel from Section 1.2 delivers 245 mJ in 45.5 hours.

Given: $W = 245\,\mathrm{mJ}$; $t_{\mathrm{G}} = 45.5\,\mathrm{h}$

$$\begin{aligned} P &= \frac{W}{t_{\mathrm{G}}} \\ &= \frac{245\,\mathrm{mJ}}{45.5\,\mathrm{h}} \\ &= \frac{0{,}245\,\mathrm{J}}{163\,800\,\mathrm{s}} \\ &\approx 1.5\,\mu\mathrm{W} \end{aligned}$$

About 1.5 microwatts. A 1 W LED needs about 700 000 times as much.

Power from Torque and Rotation Period

(1.28) MASTER

$$P = \frac{2 \cdot \pi \cdot M}{t_{\mathrm{U}}}$$

Power is torque times angular velocity. The driving arbor turns once through 2π in the rotation period.

Valid: torque constant over the period considered; insert the rotation period in seconds.

P	power (of the drive)	$\mu\mathrm{W}$
M	torque (at the driving arbor: mainspring barrel or weight barrel)	$\mathrm{N{\cdot}m}$
t_{U}	rotation period of the driving arbor	h

EXAMPLE $0.368\,\mathrm{N \cdot m}$ act at the weight barrel of the longcase clock; it turns once in 12 hours.

Given: $M = 0.368\,\mathrm{N{\cdot}m}$; $t_{\mathrm{U}} = 12\,\mathrm{h}$

$$\begin{aligned} P &= \frac{2 \cdot \pi \cdot M}{t_{\mathrm{U}}} \\ &= \frac{2 \cdot \pi \cdot 0.368\,\mathrm{N{\cdot}m}}{12\,\mathrm{h}} \\ &= \frac{2 \cdot \pi \cdot 0{,}368\,\mathrm{N{\cdot}m}}{43\,200\,\mathrm{s}} \\ &\approx 53.5\,\mu\mathrm{W} \end{aligned}$$

About 54 microwatts, roughly 36 times as much as for the wristwatch. The same calculation with $6\,\mathrm{N \cdot mm}$ and 7 hours again gives 1.5 microwatts for the wristwatch.

Power at the Oscillator

(1.29) MASTER

$$P_{\mathrm{U}} = \eta \cdot P$$

On the way through gear train and escapement, the greater part of the drive power is lost. The overall efficiency is the product of the individual efficiencies (Chapters XL 2 and 2).

Valid: steady-state running with constant amplitude. The efficiency of 0.3 in the example is a rough guide value for watches with lever escapement.

P_{U}	power reaching the balance or pendulum	$\mu\mathrm{W}$
η	efficiency (of gear train and escapement together)	1
P	power (of the drive)	$\mu\mathrm{W}$

EXAMPLE 1.5 microwatts come from the barrel. Gear train and escapement together have an efficiency of 0.3.

Given: $\eta = 0.3$; $P = 1.5\,\mu\mathrm{W}$

$$\begin{aligned} P_{\mathrm{U}} &= \eta \cdot P \\ &= 0.3 \cdot 1.5\,\mu\mathrm{W} \\ &= 0.45\,\mu\mathrm{W} \end{aligned}$$

About half a microwatt arrives at the balance. Depending on calibre and beat rate, the power at the balance lies between a few tenths of a microwatt and about one microwatt.

1.7 Winding and Hand Setting

In manual winding, the crown turns the barrel arbor via winding pinion, crown wheel and ratchet wheel. In an automatic movement this is done by the oscillating weight, the rotor, via a strongly reducing gear train. In the pulled-out position the crown drives the motion work via the sliding pinion. In winding, small wheels drive large ones; the gearing is to slow speed. The calculation therefore uses, as with the motion work (Section 2.3), the reduction ratio: numbers of teeth of the driven wheels divided by those of the driving ones. This is the reciprocal of the gear ratio from Section 2.1.

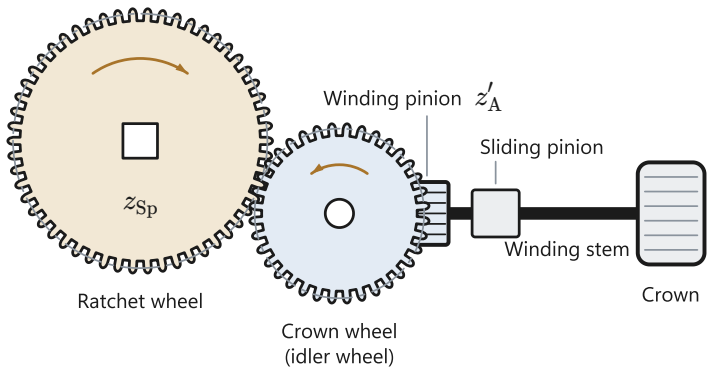


Fig. 1.13 Crown winding seen from above (schematic): the winding pinion with z'_{A} teeth drives, via the crown wheel, the ratchet wheel with z_{Sp} teeth on the barrel arbor

Crown Revolutions for Winding

(1.30) JOURNEYMAN

$$N_{\mathrm{K}} = \frac{N \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{A}}}$$

REARRANGED $$N = \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{A}}}{z_{\mathrm{Sp}}}$$

N_{K}	revolutions of the crown	1
N	turns (by which the barrel arbor tensions the spring)	1
z_{Sp}	number of teeth of the ratchet wheel	1
z'_{A}	number of teeth of the winding pinion (on the winding stem)	1

The crown wheel sits between winding pinion and ratchet wheel and is only an idler: its number of teeth drops out of the calculation. The reduction ratio of the winding is the number of teeth of the ratchet wheel divided by that of the winding pinion.

Valid: winding pinion, crown wheel and ratchet wheel in one chain; the crown wheel meshes with both using the same toothing. If the arbor of the crown wheel carries two different wheels, this stage counts as in formula (2.3).

Good to know: Each revolution of the crown adds as much running time as the rotation period of the barrel divided by the reduction ratio. In the example, at 7 hours per turn, this is just under 2 hours per revolution of the crown.

EXAMPLE The winding pinion has 15 teeth, the crown wheel 36, the ratchet wheel 54. The spring can be tensioned by 6.5 turns. How often must the crown be turned for full winding?

Given: $N = 6.5$; $z_{\mathrm{Sp}} = 54$; $z'_{\mathrm{A}} = 15$

$$\begin{aligned} N_{\mathrm{K}} &= \frac{N \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{A}}} \\ &= \frac{6.5 \cdot 54}{15} \\ &= 23.4 \end{aligned}$$

23.4 full revolutions. By hand one usually turns half revolutions, i.e. about 47 strokes. The 36 teeth of the crown wheel play no part in the calculation.

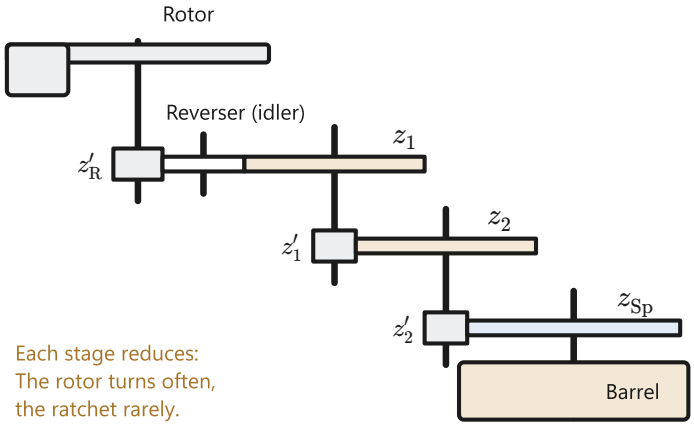


Fig. 1.14 Automatic winding, developed in side view (schematic): rotor pinion, reverser, two reduction stages and ratchet wheel

Reduction Ratio of Automatic Winding

(1.31) MASTER

$$i_{\mathrm{R}} = \frac{z_1 \cdot z_2 \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{R}} \cdot z'_1 \cdot z'_2}$$

The rotor has little torque but plenty of travel. The reduction gear train therefore reduces strongly: the numerator contains the driven wheels, the denominator the driving pinions. Reverser wheels that only rectify the direction of rotation and mesh with both neighbours using the same toothing are idlers and drop out.

Valid: two reduction stages between rotor pinion and ratchet wheel; with a different number of stages, correspondingly more or fewer factors. Guide value: the reduction ratio usually lies between 100 and 180.

i_{R}	reduction ratio of automatic winding (rotor revolutions per turn of the ratchet wheel)	1
z_1	number of teeth of the first reduction wheel	1
z_2	number of teeth of the second reduction wheel	1
z_{Sp}	number of teeth of the ratchet wheel	1
z'_{R}	number of teeth of the rotor pinion	1
z'_1	number of teeth of the pinion on the first reduction wheel	1
z'_2	number of teeth of the pinion on the second reduction wheel	1

EXAMPLE The rotor pinion has 24 teeth. The first reduction wheel has 66 teeth and a pinion with 8, the second 64 teeth and a pinion with 10, the ratchet wheel 70 teeth.

Given: $z_1 = 66$; $z_2 = 64$; $z_{\mathrm{Sp}} = 70$; $z'_{\mathrm{R}} = 24$; $z'_1 = 8$; $z'_2 = 10$

$$\begin{aligned} i_{\mathrm{R}} &= \frac{z_1 \cdot z_2 \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{R}} \cdot z'_1 \cdot z'_2} \\ &= \frac{66 \cdot 64 \cdot 70}{24 \cdot 8 \cdot 10} \\ &= 154 \end{aligned}$$

The rotor turns 154 times until the ratchet wheel has made one revolution and tensioned the spring by one turn.

Rotor Revolutions for a Power Reserve

(1.32) MASTER

$$N_{\mathrm{R}} = \frac{i_{\mathrm{R}} \cdot t_{\mathrm{R}}}{t_{\mathrm{F}}}$$

N_{R}	revolutions of the rotor (in the winding direction)	1
i_{R}	reduction ratio of automatic winding (rotor revolutions per turn of the ratchet wheel)	1
t_{R}	power reserve (to be built up)	h
t_{F}	rotation period of the barrel (time for one turn)	h

One turn of the ratchet wheel costs i_{R} rotor revolutions and gives as many hours of running as the barrel needs for one turn. With 24 hours the formula gives the daily requirement: this is how often the rotor must turn per day for the movement to maintain its state of wind.

Valid: minimum value without losses: the idle travel of the reverser on change of direction is not included, the slipping bridle does not slip. With unidirectional winding only the revolutions in the winding direction count.

Good to know: When worn, more revolutions usually accumulate than necessary; the slipping bridle lets the excess slip (Section 1.3). If an automatic watch stops despite being worn, either the movement is insufficient for the daily requirement, or the winding loses travel, for example through gummed reverser wheels.

EXAMPLE The movement from the previous example has a barrel that turns once in 7 hours. How many rotor revolutions does it need for 24 hours of running?

Given: $i_{\mathrm{R}} = 154$; $t_{\mathrm{R}} = 24\,\mathrm{h}$; $t_{\mathrm{F}} = 7\,\mathrm{h}$

$$\begin{aligned} N_{\mathrm{R}} &= \frac{i_{\mathrm{R}} \cdot t_{\mathrm{R}}}{t_{\mathrm{F}}} \\ &= \frac{154 \cdot 24\,\mathrm{h}}{7\,\mathrm{h}} \\ &= 528 \end{aligned}$$

528 revolutions per day, i.e. 22 per hour of running. For 42 hours of power reserve from standstill it is 924 revolutions.

Hand Setting: Travel of the Minute Hand

(1.33) JOURNEYMAN

$$N_{\mathrm{M}} = \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{K}}}{z_{\mathrm{V}}}$$

N_{M}	revolutions of the minute hand	1
N_{K}	revolutions of the crown	1
z'_{K}	number of teeth of the sliding pinion (toothing for hand setting)	1
z_{V}	number of teeth of the cannon pinion	1

In the hand-setting position the sliding pinion meshes with the setting wheel. This drives the cannon pinion with the minute hand via the minute wheel. Setting wheel and minute wheel are idlers here. One revolution of the minute hand is 60 minutes of hand travel.

Valid: setting wheel, further idlers and minute wheel each mesh with their two neighbours using the same toothing. The hour hand follows via the motion work in the ratio 12 to 1 (Section 2.3).

EXAMPLE The sliding pinion has 9 teeth for hand setting, the cannon pinion 12. How far does the minute hand advance for one revolution of the crown?

Given: $N_{\mathrm{K}} = 1$; $z'_{\mathrm{K}} = 9$; $z_{\mathrm{V}} = 12$

$$\begin{aligned} N_{\mathrm{M}} &= \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{K}}}{z_{\mathrm{V}}} \\ &= \frac{1 \cdot 9}{12} \\ &= 0.75 \end{aligned}$$

By three quarters of a revolution, i.e. 45 minutes of hand travel per revolution of the crown.

Key point: Crown wheel, setting wheel and minute wheel are only idlers here. What counts are the first driving and the last driven wheel.

2 Gear Train and Toothing

The gear train transmits the force of the drive to the escapement and in doing so counts the oscillations of the oscillator: its ratios determine how fast each arbor turns and which beat rate belongs to a movement. This is why calculating an unknown movement starts with counting teeth.

Sections 2.1 to 2.3 calculate with numbers of teeth, Section 2.4 with the dimensions of the toothing. Striking work, calendar and moon phase follow the same rules; friction in the gear train concludes the chapter. Depending on the formula, the symbol k stands for the addendum coefficient or for a number; the table of quantities of the formula is authoritative.

2.1 Gear Ratio

A wheel drives a pinion: this is the basic element of every gear train. Pinions are small gear wheels with usually 6 to 16 leaves; they are fixed on the arbor of the next wheel.

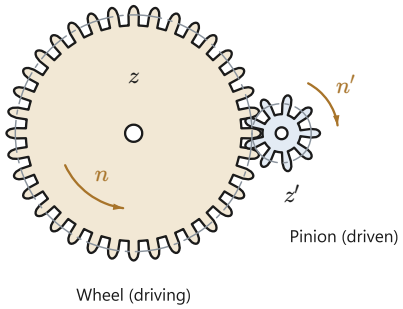


Fig. 2.1 Wheel drives pinion: the pinion turns faster by the gear ratio and in the opposite direction

Gear Ratio of One Mesh

(2.1) JOURNEYMAN

$$i = \frac{z}{z'}$$

The gear ratio states how often the pinion turns while the wheel makes one revolution. In this chapter z is the number of teeth of a wheel and z' that of a pinion; an index names the arbor.

Valid: wheel drives pinion, as in the going train and the striking work: the speed is stepped up. Where a pinion drives a wheel (motion work, winding), the speed is stepped down; the calculation then uses the reduction ratio, the number of teeth of the driven wheel divided by that of the driving pinion, formula (2.11).

i	gear ratio (greater than 1 means: stepping up in speed)	1
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The centre wheel has 80 teeth and drives the third pinion with 10 leaves.

Given: $z = 80$; $z' = 10$

$$\begin{aligned} i &= \frac{z}{z'} \\ &= \frac{80}{10} \\ &= 8 \end{aligned}$$

The third pinion turns eight times while the centre wheel makes one revolution.

Rotational Speed of the Pinion

(2.2) JOURNEYMAN

$$n' = \frac{n \cdot z}{z'}$$

In the mesh, the same number of teeth pass on both sides: $z \cdot n = z' \cdot n'$. The pinion therefore turns faster than the wheel by the gear ratio.

n'	rotational speed of the driven pinion	$\frac{1}{\mathrm{h}}$
n	rotational speed (of the driving wheel)	$\frac{1}{\mathrm{h}}$
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The centre wheel (80 teeth) turns once per hour. How fast does the third pinion with 10 leaves turn?

Given: $n = 1\,\frac{1}{\mathrm{h}}$; $z = 80$; $z' = 10$

$$\begin{aligned} n' &= \frac{n \cdot z}{z'} \\ &= \frac{1\,\frac{1}{\mathrm{h}} \cdot 80}{10} \\ &= 8\,\frac{1}{\mathrm{h}} \end{aligned}$$

Pinion and third wheel turn eight times per hour; one revolution takes 7.5 min.

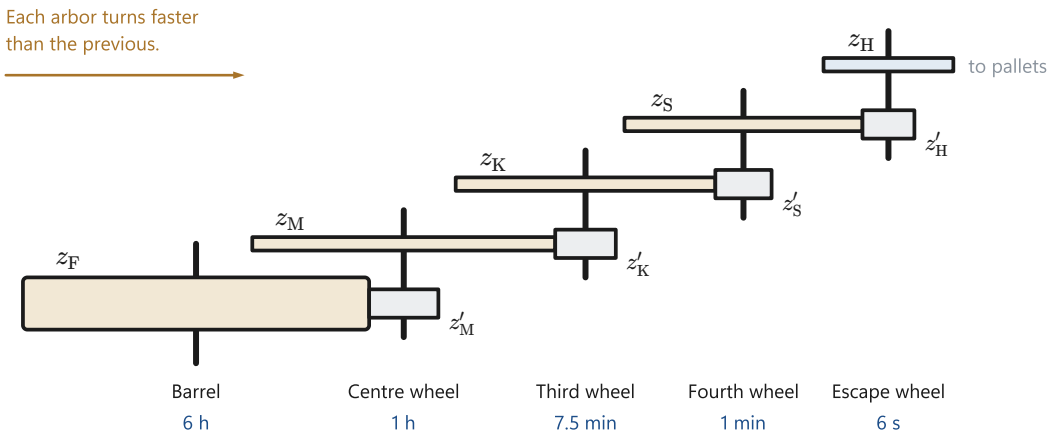


Fig. 2.2 Gear train of a watch, developed in side view. Below, the rotation periods for the tooth counts 72 : 12, 80 : 10, 75 : 10 and 80 : 8

Overall Gear Ratio

(2.3) JOURNEYMAN

$$i_{\text{ges}} = \frac{z_1 \cdot z_2 \cdot z_3}{z'_1 \cdot z'_2 \cdot z'_3}$$

If several meshes follow each other, the individual gear ratios are multiplied: product of the numbers of teeth of all driving wheels divided by the product of all driven pinions. With more or fewer meshes the fraction becomes correspondingly longer or shorter.

i_{ges}	overall gear ratio (from the first to the last arbor)	1
z_1	number of teeth of the first driving wheel	1
z_2	number of teeth of the second driving wheel	1
z_3	number of teeth of the third driving wheel	1
z'_1	number of leaves of the first driven pinion	1
z'_2	number of leaves of the second driven pinion	1
z'_3	number of leaves of the third driven pinion	1

Good to know: An idler that is driven and drives on with the same teeth appears once in the numerator and once in the denominator. It does not change the gear ratio, only the direction of rotation.

EXAMPLE A pocket watch movement has centre wheel 80, third wheel 75 and fourth wheel 80 teeth; third and fourth pinions have 10 leaves each, the escape pinion 8 leaves.

Given: $z_1 = 80;\quad z_2 = 75;\quad z_3 = 80;\quad z'_1 = 10;\quad z'_2 = 10;\quad z'_3 = 8$

$$\begin{aligned} i_{\text{ges}} &= \frac{z_1 \cdot z_2 \cdot z_3}{z'_1 \cdot z'_2 \cdot z'_3} \\ &= \frac{80 \cdot 75 \cdot 80}{10 \cdot 10 \cdot 8} \\ &= 600 \end{aligned}$$

The escape wheel turns 600 times while the centre wheel makes one revolution. The individual gear ratios are 8, 7.5 and 10.

Rotation Period

(2.4) JOURNEYMAN

$$t'_{\text{U}} = \frac{t_{\text{U}} \cdot z'}{z}$$

The rotation period is the reciprocal of the rotational speed. From arbor to arbor it becomes shorter by the gear ratio. In Fig. 2.2 the rotation periods are shown below the arbors.

t'_{U}	rotation period of the pinion (time for one revolution)	s
t_{U}	rotation period of the driving wheel	s
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The fourth wheel (80 teeth) turns once in 60 s and drives the escape pinion with 8 leaves.

Given: $t_{\text{U}} = 60\,\text{s};\quad z' = 8;\quad z = 80$

$$\begin{aligned} t'_{\text{U}} &= \frac{t_{\text{U}} \cdot z'}{z} \\ &= \frac{60\,\text{s} \cdot 8}{80} \\ &= 6\,\text{s} \end{aligned}$$

The escape wheel needs 6 s for one revolution.

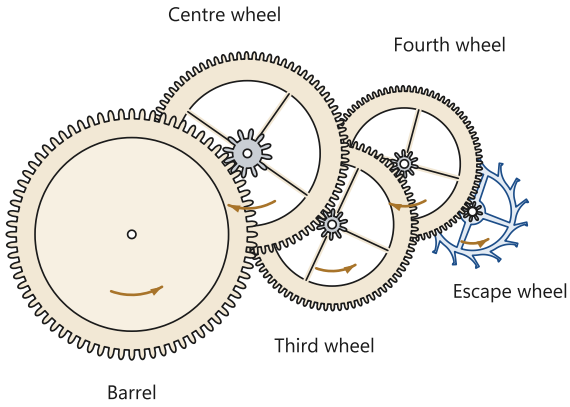


Fig. 2.3 The same gear train in plan view: each arbor turns opposite to the previous one

Key point: Each mesh reverses the direction of rotation. After an even number of meshes the last arbor turns like the first, after an odd number in the opposite direction.

Torque at the Pinion

(2.5) JOURNEYMAN

$$M' = \frac{M \cdot \eta \cdot z'}{z}$$

What the gear train gains in rotational speed it loses in torque. Friction at the tooth flanks and in the bearings takes away an additional part.

Valid: uniform rotation. The torque fluctuates somewhat during the engagement of one tooth; the formula gives the mean value.

M'	torque at the arbor of the driven pinion	$\mathrm{N{\cdot}mm}$
M	torque (at the arbor of the driving wheel)	$\mathrm{N{\cdot}mm}$
η	efficiency (per mesh; guide value 0.90 to 0.95; without friction 1)	1
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE A barrel with 72 teeth delivers 8 N·mm and drives the centre pinion with 12 leaves. The mesh has an efficiency of 0.95.

Given: $M = 8\,\mathrm{N{\cdot}mm}$; $\eta = 0.95$; $z' = 12$; $z = 72$

$$\begin{aligned} M' &= \frac{M \cdot \eta \cdot z'}{z} \\ &= \frac{8\,\mathrm{N{\cdot}mm} \cdot 0.95 \cdot 12}{72} \\ &\approx 1.27\,\mathrm{N{\cdot}mm} \end{aligned}$$

1.27 N·mm arrive at the centre arbor. Without friction it would be 1.33 N·mm.

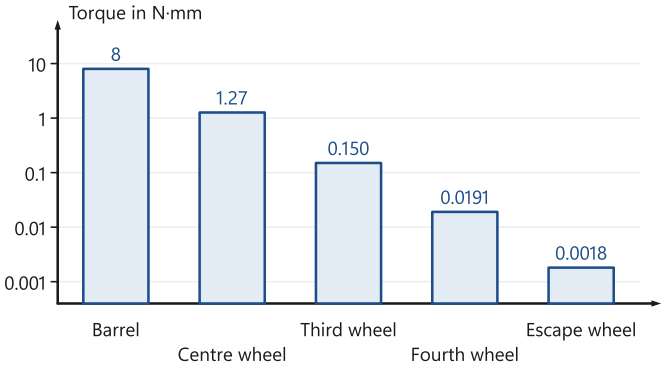


Fig. 2.4 Torque at the arbors of the gear train from Fig. 2.2 with 8 N·mm at the barrel and $\eta = 0.95$ per mesh

2.2 Calculating the Going Train

The going train connects the centre wheel, which turns once per hour, with the escape wheel. Its gear ratio and the number of teeth of the escape wheel determine how many vibrations per hour the oscillator must make for the hands to run correctly.

These formulas are used to determine the beat rate of an unknown movement, the length of a missing pendulum (Chapter 4) and the number of teeth of a missing wheel or pinion.

Beat Rate from the Gear Train

(2.6) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot z_\mathrm{H} \cdot i \cdot n_\mathrm{M}$$

Each tooth of the escape wheel makes the oscillator perform two vibrations: one at the entry, one at the exit. Per revolution of the escape wheel these are $2 \cdot z_\mathrm{H}$ beats.

Valid: all common escapements; i according to formula (2.3). The centre wheel turns once per hour; if the calculation starts from another arbor, its rotational speed is inserted.

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_H	number of teeth of the escape wheel	1
i	gear ratio (from the centre wheel to the escape wheel)	1
n_M	rotational speed of the centre wheel (one revolution per hour)	$\frac{1}{\mathrm{h}}$

EXAMPLE A regulator has a centre wheel with 80 and an intermediate wheel with 75 teeth, two pinions with 10 leaves each and an escape wheel with 30 teeth. The gear ratio from the centre wheel to the escape wheel is $8 \cdot 7.5 = 60$.

Given: $z_\mathrm{H} = 30; \quad i = 60; \quad n_\mathrm{M} = 1\ \frac{1}{\mathrm{h}}$

$$\begin{aligned} n_\mathrm{h} &= 2 \cdot z_\mathrm{H} \cdot i \cdot n_\mathrm{M} \\ &= 2 \cdot 30 \cdot 60 \cdot 1\ \tfrac{1}{\mathrm{h}} \\ &= 3600\ \tfrac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

3600 beats per hour are 60 per minute. This movement requires a seconds pendulum 994 mm long (Chapter 4).

Beat Rate of a Watch from the Numbers of Teeth

(2.7) JOURNEYMAN

$$n_{\mathrm{h}} = \frac{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{M}} \cdot z_{\mathrm{M}} \cdot z_{\mathrm{K}} \cdot z_{\mathrm{S}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}} \cdot z'_{\mathrm{H}}}$$

The numerator contains the driving wheels, the denominator the driven pinions. Each pinion carries the index of the arbor on which it sits (Fig. 2.2).

Valid: gear train with three meshes between centre wheel and escape wheel. For a different design, adapt the fraction according to formula (2.3).

n_{h}	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_{H}	number of teeth of the escape wheel	1
n_{M}	rotational speed of the centre wheel (one revolution per hour)	$\frac{1}{\mathrm{h}}$
z_{M}	number of teeth of the centre wheel	1
z_{K}	number of teeth of the third wheel	1
z_{S}	number of teeth of the fourth wheel	1
z'_{K}	number of leaves of the third pinion	1
z'_{S}	number of leaves of the fourth pinion	1
z'_{H}	number of leaves of the escape pinion	1

Good to know: If a wheel or pinion is missing, the formula is rearranged for its number of teeth. The result must be a whole number; otherwise a number of teeth has been miscounted or the beat rate wrongly assumed.

EXAMPLE Counted were: centre wheel 80, third wheel 75, fourth wheel 84, escape wheel 20 teeth; third pinion 10, fourth pinion 10, escape pinion 7 leaves.

Given: $z_{\mathrm{H}} = 20$; $n_{\mathrm{M}} = 1\,\frac{1}{\mathrm{h}}$; $z_{\mathrm{M}} = 80$; $z_{\mathrm{K}} = 75$; $z_{\mathrm{S}} = 84$; $z'_{\mathrm{K}} = 10$; $z'_{\mathrm{S}} = 10$; $z'_{\mathrm{H}} = 7$

$$\begin{aligned} n_{\mathrm{h}} &= \frac{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{M}} \cdot z_{\mathrm{M}} \cdot z_{\mathrm{K}} \cdot z_{\mathrm{S}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}} \cdot z'_{\mathrm{H}}} \\ &= \frac{2 \cdot 20 \cdot 1\,\frac{1}{\mathrm{h}} \cdot 80 \cdot 75 \cdot 84}{10 \cdot 10 \cdot 7} \\ &= 28\,800\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

The movement beats 28 800 vibrations per hour; the balance oscillates at 4 Hz.

Rotational Speed of the Escape Wheel

(2.8) JOURNEYMAN

$$n_{\mathrm{H}} = \frac{n_{\mathrm{h}}}{2 \cdot z_{\mathrm{H}}}$$

This gives the rotational speed that the gear train must deliver at the escape wheel. More teeth on the escape wheel means the gear train can run more slowly.

n_{H}	rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$
n_{h}	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_{H}	number of teeth of the escape wheel	1

EXAMPLE How fast does an escape wheel with 20 teeth turn at 28 800 A/h?

Given: $n_{\mathrm{h}} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$; $z_{\mathrm{H}} = 20$

$$\begin{aligned} n_{\mathrm{H}} &= \frac{n_{\mathrm{h}}}{2 \cdot z_{\mathrm{H}}} \\ &= \frac{28\,800\ \frac{\mathrm{A}}{\mathrm{h}}}{2 \cdot 20} \\ &= 12\ \frac{1}{\mathrm{min}} \end{aligned}$$

Twelve revolutions per minute; one revolution takes 5 s. An escape wheel with 15 teeth would need 16 revolutions per minute at the same beat rate.

Watches up to 21 600 A/h almost always have an escape wheel with 15 teeth, movements with 28 800 A/h and more often one with 20 teeth. For clocks the range extends from about 15 to 40 teeth. If the seconds hand sits on the arbor of the escape wheel and the pendulum beats seconds, the escape wheel has 30 teeth.

Condition for the Fourth Wheel

(2.9) JOURNEYMAN

$$\frac{z_{\mathrm{M}} \cdot z_{\mathrm{K}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}} = 60$$

REARRANGED $z_{\mathrm{K}} = \dfrac{60 \cdot z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}}{z_{\mathrm{M}}}$

z_{M}	number of teeth of the centre wheel	1
z_{K}	number of teeth of the third wheel	1
z'_{K}	number of leaves of the third pinion	1
z'_{S}	number of leaves of the fourth pinion	1

If the arbor of the fourth wheel carries the seconds hand, it must turn 60 times as fast as the centre wheel. The usual division is $8 \cdot 7.5$.

Valid: movements whose fourth wheel turns once per minute (small seconds or central seconds wheel).

EXAMPLE In a movement the third wheel is missing. The centre wheel has 80 teeth, the third and fourth pinions have 10 leaves each.

Given: $z'_{\mathrm{K}} = 10$; $z'_{\mathrm{S}} = 10$; $z_{\mathrm{M}} = 80$

$$\begin{aligned} z_{\mathrm{K}} &= \frac{60 \cdot z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}}{z_{\mathrm{M}}} \\ &= \frac{60 \cdot 10 \cdot 10}{80} \\ &= 75 \end{aligned}$$

The missing third wheel has 75 teeth. The module is given by the centre distance, formula (2.16).

Beat Rate from the Last Mesh

(2.10) JOURNEYMAN

$$n_{\mathrm{h}} = \frac{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{S}} \cdot z_{\mathrm{S}}}{z'_{\mathrm{H}}}$$

REARRANGED $$z_{\mathrm{S}} = \frac{n_{\mathrm{h}} \cdot z'_{\mathrm{H}}}{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{S}}}$$

If the fourth wheel turns once per minute, the last mesh is sufficient to check the beat rate or to calculate a missing fourth wheel.

Valid: fourth wheel turns once per minute.

n_{h}	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_{H}	number of teeth of the escape wheel	1
n_{S}	rotational speed of the fourth wheel (one revolution per minute)	$\frac{1}{\mathrm{min}}$
z_{S}	number of teeth of the fourth wheel	1
z'_{H}	number of leaves of the escape pinion	1

EXAMPLE A movement with 28 800 A/h has an escape wheel with 20 teeth and an escape pinion with 7 leaves. The fourth wheel is missing.

Given: $n_{\mathrm{h}} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$; $z'_{\mathrm{H}} = 7$; $z_{\mathrm{H}} = 20$; $n_{\mathrm{S}} = 1\ \frac{1}{\mathrm{min}}$

$$\begin{aligned} z_{\mathrm{S}} &= \frac{n_{\mathrm{h}} \cdot z'_{\mathrm{H}}}{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{S}}} \\ &= \frac{28\,800\ \frac{\mathrm{A}}{\mathrm{h}} \cdot 7}{2 \cdot 20 \cdot 1\ \frac{1}{\mathrm{min}}} \\ &= 84 \end{aligned}$$

The fourth wheel must have 84 teeth.

Tooth-Count Sets for Watches (Calculated Examples, Fourth Wheel Turns Once per Minute)

Beat rate in A/h	Centre wheel : third pinion	Third wheel : fourth pinion	Fourth wheel : escape pinion	Escape wheel	Gear ratio
18 000	80 : 10	75 : 10	80 : 8	15	600
18 000	64 : 8	60 : 8	70 : 7	15	600
21 600	80 : 10	75 : 10	72 : 6	15	720
21 600	64 : 8	60 : 8	84 : 7	15	720
28 800	80 : 10	75 : 10	84 : 7	20	720
36 000	80 : 10	75 : 10	90 : 6	20	900

The sets are calculated with formula (2.7) and show how the beat rate arises from the numbers of teeth. Actual calibres differ from them; the counted teeth are always authoritative.

Key point: Beat rate equals twice the escape-wheel teeth times the gear ratio from centre wheel to escape wheel. Driving wheels in the numerator, driven pinions in the denominator.

2.3 Motion Work

The motion work sits under the dial and derives the rotation of the hour hand from that of the minute hand. The cannon pinion sits with friction on the arbor of the centre wheel so that the hands can be set.

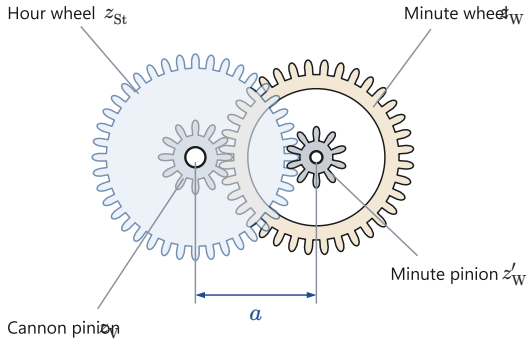


Fig. 2.5 Motion work in plan view: both meshes have the same centre distance a

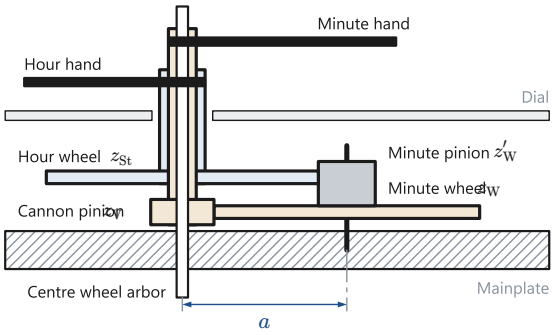


Fig. 2.6 Motion work in section (schematic): the hour wheel turns loosely on the cannon pinion

Reduction Ratio of the Motion Work

(2.11) JOURNEYMAN

$$\frac{z_W \cdot z_{St}}{z_V \cdot z'_W} = 12$$

REARRANGED $$z_{St} = \frac{12 \cdot z_V \cdot z'_W}{z_W}$$

The cannon pinion carries the minute hand and turns once per hour, the hour wheel once in 12 hours. Because pinions drive here, the driven wheels are in the numerator: the motion work reduces 12 to 1.

Valid: display with 12 hours per revolution of the hour hand. After two meshes the hour wheel turns in the same direction as the cannon pinion.

z_W	number of teeth of the minute wheel	1
z_{St}	number of teeth of the hour wheel	1
z_V	number of leaves of the cannon pinion (drives the minute wheel)	1
z'_W	number of leaves of the minute pinion (drives the hour wheel)	1

EXAMPLE The hour wheel of a motion work is missing. The cannon pinion has 14 leaves, the minute wheel 42 teeth, the minute pinion 12 leaves.

Given: $z_V = 14$; $z'_W = 12$; $z_W = 42$

$$z_{St} = \frac{12 \cdot z_V \cdot z'_W}{z_W} = \frac{12 \cdot 14 \cdot 12}{42} = 48$$

The hour wheel has 48 teeth. The reduction is divided into 3 at the first and 4 at the second mesh.

24-Hour Display

(2.12) MASTER

$$z_{24} = 2 \cdot z_{A}$$

The 24-hour hand turns half as fast as the hour hand. An idler does not change the reduction ratio but makes the hand run clockwise. The same 2 to 1 reduction drives the date driving wheel, which advances once a day (Section 2.6).

Valid: drive from the hour wheel. Calculated from the cannon pinion, the reduction ratio is 24 to 1.

z_{24}	number of teeth of the 24-hour wheel	1
z_{A}	number of teeth of the driving wheel on the hour-wheel pipe	1

EXAMPLE A wheel with 24 teeth sits on the hour-wheel pipe and drives the 24-hour wheel via an idler.

Given: $z_{A} = 24$

$$\begin{aligned} z_{24} &= 2 \cdot z_{A} \\ &= 2 \cdot 24 \\ &= 48 \end{aligned}$$

The 24-hour wheel needs 48 teeth. The number of teeth of the idler can be chosen freely.

Equal Centre Distance in the Motion Work

(2.13) MASTER

$$m_1 \cdot (z_{V} + z_{W}) = m_2 \cdot (z'_{W} + z_{St})$$

REARRANGED $$m_2 = \frac{m_1 \cdot (z_{V} + z_{W})}{z'_{W} + z_{St}}$$

Cannon pinion and hour wheel turn about the same axis, and so do minute wheel and minute pinion. Both meshes therefore have the same centre distance according to formula (2.16). If the sums of the numbers of teeth differ, the modules must differ too.

m_1	module of the cannon pinion and minute wheel mesh	mm
z_{V}	number of leaves of the cannon pinion (drives the minute wheel)	1
z_{W}	number of teeth of the minute wheel	1
m_2	module of the minute pinion and hour wheel mesh	mm
z'_{W}	number of leaves of the minute pinion (drives the hour wheel)	1
z_{St}	number of teeth of the hour wheel	1

EXAMPLE A motion work has cannon pinion 12, minute wheel 36, minute pinion 10 and hour wheel 40 teeth. The first mesh has a module of 0.25 mm.

Given: $m_1 = 0.25\,\mathrm{mm}$; $z_{V} = 12$; $z_{W} = 36$; $z'_{W} = 10$; $z_{St} = 40$

$$\begin{aligned} m_2 &= \frac{m_1 \cdot (z_{V} + z_{W})}{z'_{W} + z_{St}} \\ &= \frac{0.25\,\mathrm{mm} \cdot (12 + 36)}{10 + 40} \\ &= 0.24\,\mathrm{mm} \end{aligned}$$

The second mesh needs a module of 0.24 mm. With the tooth counts 10 and 30 as well as 8 and 32, both meshes could use the same module.

2.4 Tooth Geometry

All dimensions of a toothing depend on the module. Module and number of teeth fix pitch circle, centre distance and tooth size; the tooth form then determines tip and root circles.

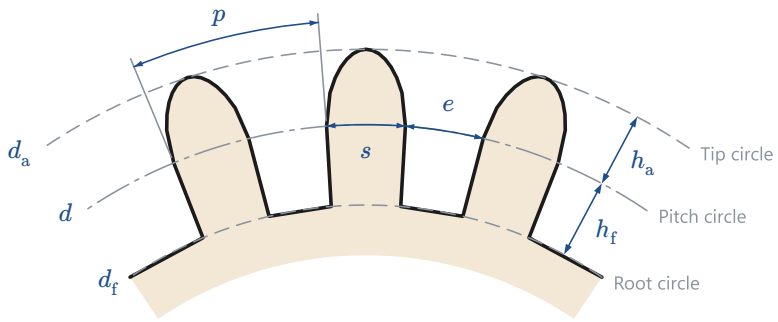


Fig. 2.7 Dimensions on the gear wheel: tip circle d_a, pitch circle d, root circle d_f, pitch p, tooth thickness s, space e, addendum h_a and dedendum h_f

Pitch and Module

(2.14) JOURNEYMAN

$$p = \pi \cdot m$$

REARRANGED $m = \frac{p}{\pi}$

The module is the pitch divided by π. It is the measure of tooth size: only wheel and pinion with the same module fit together.

p	circular pitch (distance from tooth to tooth, as an arc on the pitch circle)	mm
m	module	mm

EXAMPLE What is the pitch for module 0.2 mm?

Given: $m = 0.2\,\mathrm{mm}$

$$\begin{aligned} p &= \pi \cdot m \\ &= \pi \cdot 0.2\,\mathrm{mm} \\ &\approx 0.628\,\mathrm{mm} \end{aligned}$$

From tooth to tooth it is 0.628 mm on the pitch circle; tooth and space are therefore each about 0.3 mm wide.

Pitch Circle Diameter

(2.15) JOURNEYMAN

$$d = m \cdot z$$

REARRANGED $m = \frac{d}{z}$

The pitch circle is the imaginary circle on which wheel and pinion roll on each other like two smooth friction wheels.

d	pitch circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE A wheel has 80 teeth and a module of 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}$; $z = 80$

$$\begin{aligned} d &= m \cdot z \\ &= 0.2\,\mathrm{mm} \cdot 80 \\ &= 16\,\mathrm{mm} \end{aligned}$$

The pitch circle measures 16 mm. It cannot be seen on the finished wheel; only the tip circle can be measured.

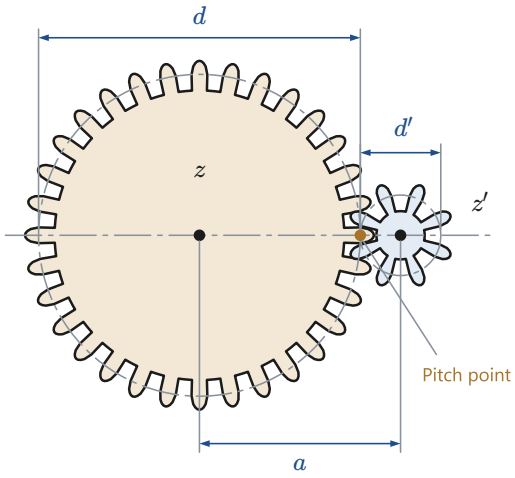


Fig. 2.8 Mesh: the pitch circles d and d' touch at the pitch point

Centre Distance

(2.16) JOURNEYMAN

$$a = \frac{m \cdot (z + z')}{2}$$

REARRANGED $m = \dfrac{2 \cdot a}{z + z'}$

a	centre distance	mm
m	module	mm
z	number of teeth (of the wheel)	1
z'	number of leaves of the pinion	1

The pitch circles touch at the pitch point. The centre distance is the sum of the two pitch circle radii. Rearranged, the formula gives the module from the measured centre distance.

Valid: external toothing with correctly set depth of engagement.

EXAMPLE A centre distance of 8.5 mm is measured between two bearings. The wheel has 75 teeth, the pinion 10.

Given: $a = 8.5\,\mathrm{mm}$; $z = 75$; $z' = 10$

$$\begin{aligned} m &= \frac{2 \cdot a}{z + z'} \\ &= \frac{2 \cdot 8.5\,\mathrm{mm}}{75 + 10} \\ &= 0.2\,\mathrm{mm} \end{aligned}$$

The mesh has a module of 0.2 mm.

Pitch Circle from the Centre Distance

(2.17) MASTER

$$d = \frac{2 \cdot a \cdot z}{z + z'}$$

The centre distance is divided between wheel and pinion in the ratio of the numbers of teeth. For the pinion, z' is in the numerator.

d	pitch circle diameter (of the wheel)	mm
a	centre distance	mm
z	number of teeth (of the wheel)	1
z'	number of leaves of the pinion	1

EXAMPLE Centre distance 8.5 mm, wheel 75 teeth, pinion 10 leaves.

Given: $a = 8.5\,\text{mm}; \quad z = 75; \quad z' = 10$

$$\begin{aligned} d &= \frac{2 \cdot a \cdot z}{z + z'} \\ &= \frac{2 \cdot 8.5\,\text{mm} \cdot 75}{75 + 10} \\ &= 15\,\text{mm} \end{aligned}$$

The pitch circle of the wheel measures 15 mm, leaving 2 mm for the pinion.

Horological toothing. In the going train the wheel drives the pinion, with the most uniform force possible and little friction. For this the cycloidal toothing has become established, today usually executed as circular-arc toothing according to the standards of the watch industry: above the pitch circle the wheel tooth carries an ogival addendum, the pinion leaf has straight radial flanks and a round or ogival tip. The addenda depend on the numbers of teeth. The following formulas combine them in an allowance added to the number of teeth.

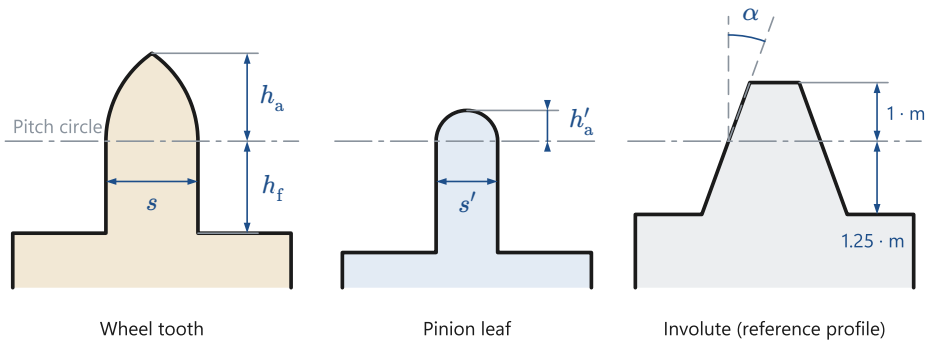


Fig. 2.9 Tooth forms drawn on a straight pitch line: wheel tooth with ogival addendum, pinion leaf with semicircular tip, basic rack profile of involute toothing with $\alpha = 20°$

Tip Circle of the Wheel

(2.18) APPROXIMATION JOURNEYMAN

$$d_\mathrm{a} \approx m \cdot (z + k)$$

The tip circle is the outside diameter of the wheel and the only dimension that is easy to measure. The addendum is $h_\mathrm{a} = \frac{k}{2} \cdot m$.

Valid: horological toothing, wheel drives pinion. k from the table; without a table use $k \approx 3$, in course exercises also $k \approx \pi$.

d_a	tip circle diameter (of the wheel)	mm
m	module	mm
z	number of teeth (of the wheel)	1
k	addendum coefficient (of the wheel: twice the addendum, measured in modules)	1

EXAMPLE A wheel with 80 teeth and module 0.2 mm meshes with a pinion with 10 leaves; according to the table $k = 3.19$.

Given: $m = 0.2\,\mathrm{mm}$; $z = 80$; $k = 3.19$

$$\begin{aligned} d_\mathrm{a} &\approx m \cdot (z + k) \\ &= 0.2\,\mathrm{mm} \cdot (80 + 3.19) \\ &\approx 16.64\,\mathrm{mm} \end{aligned}$$

The wheel is turned to a tip circle of 16.64 mm; the pitch circle lies 0.32 mm below the tooth tips.

Addendum Coefficient k of the Wheel at a Glance (Rounded, for Gear Ratios from 6 to 10)

Number of leaves of the pinion	6	7	8	10	12
Addendum coefficient k	2.6	2.8	2.9	3.2	3.4

The more leaves the pinion has, the higher the ogival addendum of the wheel tooth. The exact values also depend on the number of teeth of the wheel; they are given in the following table.

Addendum Coefficient k of the Wheel by Numbers of Teeth of Wheel and Pinion (Table Values for Horological Cycloidal Toothing)

Number of teeth z of the wheel	Pinion $z' = 6$	$z' = 7$	$z' = 8$	$z' = 9$	$z' = 10$	$z' = 12$	$z' = 15$
20	2.53	2.65	2.76	2.86	2.95	3.10	3.24
24	2.56	2.68	2.79	2.90	2.99	3.14	3.31
25	2.57	2.69	2.80	2.91	3.00	3.15	3.33
28	2.57	2.71	2.82	2.93	3.03	3.18	3.38
30	2.58	2.71	2.83	2.94	3.04	3.20	3.40
32	2.59	2.72	2.84	2.95	3.06	3.22	3.43
35	2.60	2.73	2.85	2.96	3.07	3.25	3.46
36	2.60	2.73	2.85	2.96	3.08	3.26	3.47
40	2.60	2.74	2.86	2.98	3.10	3.29	3.51
42	2.61	2.74	2.87	2.98	3.10	3.30	3.53
45	2.61	2.74	2.88	3.00	3.12	3.32	3.55
48	2.62	2.75	2.88	3.00	3.12	3.33	3.56
50	2.62	2.75	2.89	3.01	3.12	3.33	3.57
54	2.62	2.75	2.89	3.01	3.13	3.35	3.59
56	2.62	2.76	2.90	3.02	3.13	3.35	3.60
60	2.63	2.76	2.90	3.02	3.14	3.36	3.61
64	2.63	2.77	2.91	3.03	3.15	3.37	3.62
70	2.63	2.77	2.92	3.05	3.17	3.39	3.64
72	2.64	2.78	2.92	3.05	3.17	3.39	3.64
75	2.64	2.78	2.93	3.06	3.18	3.40	3.65
80	2.64	2.79	2.93	3.07	3.19	3.41	3.66
84	2.64	2.79	2.93	3.07	3.20	3.42	3.67
90	2.65	2.80	2.94	3.08	3.21	3.44	3.69
96	2.65	2.80	2.94	3.09	3.22	3.45	3.70
100	2.66	2.81	2.95	3.10	3.23	3.46	3.71
110	2.66	2.81	2.96	3.11	3.24	3.47	3.73
120	2.67	2.82	2.97	3.12	3.25	3.49	3.75
140	2.68	2.84	2.99	3.13	3.27	3.51	3.78
150	2.69	2.84	3.00	3.14	3.28	3.52	3.80
160	2.69	2.85	3.01	3.15	3.29	3.52	3.81
180	2.70	2.86	3.01	3.15	3.30	3.53	3.83
200	2.71	2.87	3.02	3.16	3.31	3.55	3.85

Course and examination exercises calculate with these values. For numbers of teeth between the rows, take the nearest row; from row to row k changes by only a few hundredths. The addendum of the wheel tooth is $h_{\mathrm{a}} = \frac{k}{2} \cdot m$. The values belong to the ogival tooth of cycloidal toothing: its flank is generated when a circle of half the diameter of the pinion pitch circle rolls on the outside of the wheel's pitch circle (epicycloid). Recalculated, the table values lie at about 95 % of the height at which the two flanks meet at the tip when the tooth is half a pitch thick on the pitch circle. For manufacture, the toothing standards of the watch industry (NHS, NIHS) apply. Depending on standard and profile, they give somewhat different values from this table, for wheel toothing by a few hundredths up to a few tenths; anyone making a wheel therefore calculates with the standard sheet.

Module from the Measured Tip Circle

(2.19) APPROXIMATION JOURNEYMAN

$$m \approx \frac{d_a}{z + k}$$

The usual route to a missing wheel or pinion: count the teeth on the existing mating part, measure the tip circle and calculate the module. Check the result against the centre distance according to formula (2.16).

Valid: as for formula (2.18). For an existing pinion, formula (2.20) applies accordingly.

m	module	mm
d_a	tip circle diameter (of the wheel)	mm
z	number of teeth (of the wheel)	1
k	addendum coefficient (of the wheel: twice the addendum, measured in modules)	1

EXAMPLE The pinion of a wheel is missing. The wheel has 64 teeth and a tip circle of 10.06 mm; it is to drive a pinion with 8 leaves, i.e. $k = 2.9$.

Given: $d_a = 10.06\,\text{mm}$; $z = 64$; $k = 2.9$

$$\begin{aligned} m &\approx \frac{d_a}{z + k} \\ &= \frac{10.06\,\text{mm}}{64 + 2.9} \\ &\approx 0.150\,\text{mm} \end{aligned}$$

The module is 0.15 mm. The missing pinion therefore has a pitch circle of 1.2 mm, the centre distance is 5.4 mm.

Tip Circle of the Pinion

(2.20) APPROXIMATION MASTER

$$d'_a \approx m \cdot (z' + k')$$

In the going train, the pinion tip works only briefly before the line of centres. It is therefore low; with a semicircular tip the addendum equals half the leaf thickness. In general the addendum is $h'_a = \frac{k'}{2} \cdot m$.

Valid: horological toothing, pinion is driven. k' from the two tables.

d'_a	tip circle diameter of the pinion	mm
m	module	mm
z'	number of leaves of the pinion	1
k'	addendum coefficient of the pinion (table)	1

EXAMPLE A pinion with 8 leaves and module 0.15 mm is given semicircular tips; from the leaf thickness $k' = 1.05$.

Given: $m = 0.15\,\text{mm}$; $z' = 8$; $k' = 1.05$

$$\begin{aligned} d'_a &\approx m \cdot (z' + k') \\ &= 0.15\,\text{mm} \cdot (8 + 1.05) \\ &\approx 1.36\,\text{mm} \end{aligned}$$

The pinion has an outside diameter of 1.36 mm with a pitch circle of 1.2 mm. With the rounded table value $k' = 1.0$ the result is 1.35 mm.

Pinion Leaf of Horological Toothing: Leaf Thickness and Resulting Addendum Coefficient (Guide Values)

Number of leaves of the pinion	Leaf thickness s'	Addendum coefficient k' for semicircular tip
6 to 10	about one third of the pitch, $1.05 \cdot m$	1.05
from 11	about two fifths of the pitch, $1.25 \cdot m$	1.25

With a semicircular tip the addendum is half the leaf thickness; these values follow from this. The boundary between 10 and 11 leaves follows the toothing standard of the watch industry.

Addendum Coefficient k' of the Pinion by Tip Form (Rounded Table Values of Course and Examination Exercises)

Tip form of the pinion leaf	Pinion with 6 to 10 leaves	Pinion from 11 leaves
semicircular (rounded)	1.0	1.2
ogival (pointed)	1.5	1.8

The values for the semicircular tip are the rounded values 1.05 and 1.25 from the table above. The difference changes the tip circle by only $0.05 \cdot m$, i.e. by 0.01 mm for module 0.2 mm. As in the toothing standard of the watch industry, the boundary lies between 10 and 11 leaves: up to 10 leaves the pinion leaf must stay thin so that the wheel tooth has room in the space. Some course materials already count the pinion with 10 leaves in the second group; in the examination the value given by the exercise or its table applies. Ogival tips are higher; depending on the profile they reach up to about $k' = 2$.

Root Circle of the Wheel

(2.21) APPROXIMATION MASTER

$$d_\mathrm{f} \approx m \cdot (z - \pi)$$

d_f	root circle diameter (of the wheel)	mm
m	module	mm
z	number of teeth (of the wheel)	1

The dedendum of the wheel tooth is about half the pitch, $h_\mathrm{f} \approx \frac{\pi}{2} \cdot m$. This lets the pinion tip pass freely through the space.

Valid: guide value for horological toothing. The root depth of the wheel is not critical as long as the pinion tip does not bottom.

EXAMPLE Wheel with 80 teeth, module 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}; \quad z = 80$

$$\begin{aligned} d_\mathrm{f} &\approx m \cdot (z - \pi) \\ &= 0.2\,\mathrm{mm} \cdot (80 - \pi) \\ &\approx 15.37\,\mathrm{mm} \end{aligned}$$

The tooth root lies on a diameter of 15.37 mm. The whole tooth depth is therefore about 0.63 mm.

Root Circle of the Pinion (Guide Value)

(2.22) APPROXIMATION JOURNEYMAN

$$d'_\mathrm{f} \approx m \cdot (z' - 4)$$

d'_f	root circle diameter of the pinion (core diameter)	mm
m	module	mm
z'	number of leaves of the pinion	1

The dedendum of the pinion leaf is about $2 \cdot m$. This is sufficient for every wheel addendum in the table: the highest measures $1.93 \cdot m$, the usual ones about $1.3 \cdot m$ to $1.7 \cdot m$. The remainder is tip clearance.

Valid: guide value of horological toothing for course and examination exercises. The following formula calculates more precisely with the addendum coefficient of the mating wheel.

EXAMPLE A pinion with 12 leaves has a module of 0.31 mm. What is its root circle?

Given: $m = 0.31\,\mathrm{mm}; \quad z' = 12$

$$\begin{aligned} d'_\mathrm{f} &\approx m \cdot (z' - 4) \\ &= 0.31\,\mathrm{mm} \cdot (12 - 4) \\ &= 2.48\,\mathrm{mm} \end{aligned}$$

2.48 mm with a pitch circle of 3.72 mm.

Root Circle of the Pinion

(2.23) ADVANCED

$$d'_{\mathrm{f}} = m \cdot (z' - k) - 2 \cdot c$$

The space of the pinion must take the tip of the wheel tooth and still leave clearance below it: dedendum of the pinion equals addendum of the wheel plus tip clearance.

d'_{f}	root circle diameter of the pinion (core diameter)	mm
m	module	mm
z'	number of leaves of the pinion	1
k	addendum coefficient (of the mating wheel)	1
c	tip clearance between wheel tip and pinion root (guide value $0.4 \cdot m$)	mm

EXAMPLE Pinion with 10 leaves, module 0.2 mm, mating wheel with $k = 3.2$, tip clearance 0.08 mm.

Given: $m = 0.2\,\mathrm{mm}$; $z' = 10$; $k = 3.2$; $c = 0.08\,\mathrm{mm}$

$$\begin{aligned} d'_{\mathrm{f}} &= m \cdot (z' - k) - 2 \cdot c \\ &= 0.2\,\mathrm{mm} \cdot (10 - 3.2) - 2 \cdot 0.08\,\mathrm{mm} \\ &= 1.20\,\mathrm{mm} \end{aligned}$$

The core of the pinion measures 1.2 mm with a pitch circle of 2 mm. Without tip clearance it would be 1.36 mm; at this dimension the wheel tooth would already bottom.

Tooth Thickness of the Wheel

(2.24) JOURNEYMAN

$$s = \frac{\pi \cdot m}{2}$$

Tooth and space share the pitch: $p = s + e$. On the wheel both are equal, so the tooth thickness is half the pitch.

Valid: wheel of horological toothing (guide value). Pinion leaves are thinner, see table at formula (2.20).

s	tooth thickness (of the wheel on the pitch circle)	mm
m	module	mm

EXAMPLE Module 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}$

$$\begin{aligned} s &= \frac{\pi \cdot m}{2} \\ &= \frac{\pi \cdot 0.2\,\mathrm{mm}}{2} \\ &\approx 0.314\,\mathrm{mm} \end{aligned}$$

Tooth and space of the wheel are each 0.314 mm wide on the pitch circle.

Backlash

(2.25) MASTER

$$j = p - s - s'$$

The pinion leaf is narrower than the space of the wheel. The difference is the backlash. It leaves room for oil, dirt, pitch errors and runout.

Valid: measurement on the pitch circle at the correct centre distance.

j	backlash on the pitch circle	mm
p	circular pitch (distance from tooth to tooth, as an arc on the pitch circle)	mm
s	tooth thickness (of the wheel on the pitch circle)	mm
s'	leaf thickness of the pinion on the pitch circle	mm

EXAMPLE Module 0.2 mm: pitch 0.628 mm, tooth thickness of the wheel 0.314 mm, of the pinion with 8 leaves 0.209 mm.

Given: $p = 0.628\,\mathrm{mm}; \quad s = 0.314\,\mathrm{mm}; \quad s' = 0.209\,\mathrm{mm}$

$$\begin{aligned} j &= p - s - s' \\ &= 0.628\,\mathrm{mm} - 0.314\,\mathrm{mm} - 0.209\,\mathrm{mm} \\ &= 0.105\,\mathrm{mm} \end{aligned}$$

About 0.1 mm of clearance remains between the flanks, one sixth of the pitch.

Depthing. If the arbors are too close, the pitch circles intersect: the depthing is too deep, the teeth jam against each other, the movement runs heavily and in the worst case locks. If they are too far apart, the depthing is too shallow: the tooth tips lead, the force is transmitted unevenly and the mesh drops audibly. Checking is done with the depthing tool or by feeling the play between the teeth; correction is made via the centre distance or with a suitable pinion.

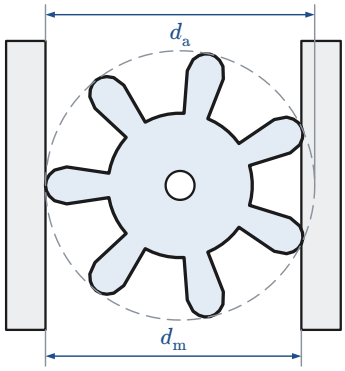


Fig. 2.10 Odd number of teeth: the measured dimension d_m is smaller than the tip circle d_a

Measuring the Tip Circle with an Odd Number of Teeth

(2.26) MASTER

$$d_a = \frac{d_m}{\cos^2\left(\frac{90^\circ}{z}\right)}$$

d_a	tip circle diameter	mm
d_m	measured dimension between the measuring faces	mm
z	number of teeth (odd)	1

With an odd number of teeth, each tooth is opposite a space. One measuring face rests on one tooth tip, the other on two. The measured dimension is therefore smaller than the tip circle.

Valid: odd number of teeth; measuring faces flat, parallel and wider than one pitch. With an even number of teeth the tip circle is measured directly.

EXAMPLE A pinion with 7 leaves measures 1.62 mm between the jaws of the calliper.

Given: $d_m = 1.62\,\text{mm}; \quad z = 7$

$$d_a = \frac{d_m}{\cos^2\left(\frac{90^\circ}{z}\right)} = \frac{1.62\,\text{mm}}{\cos^2\left(\frac{90^\circ}{7}\right)} \approx 1.70\,\text{mm}$$

The tip circle is 0.08 mm larger than the measured dimension. With 15 teeth the difference still amounts to about 1 %, from about 60 teeth less than 0.1 %.

Involute toothing. The toothing of mechanical engineering is found in the watch where the speed is stepped down or the direction of force changes: in winding, in hand setting, in automatic gearing and in calendar mechanisms. Its flanks are involutes, i.e. curves traced by a thread unwound from the base circle. The toothing tolerates small errors in the centre distance: the rotation is still transmitted uniformly. Pitch, pitch circle and centre distance are calculated as above.

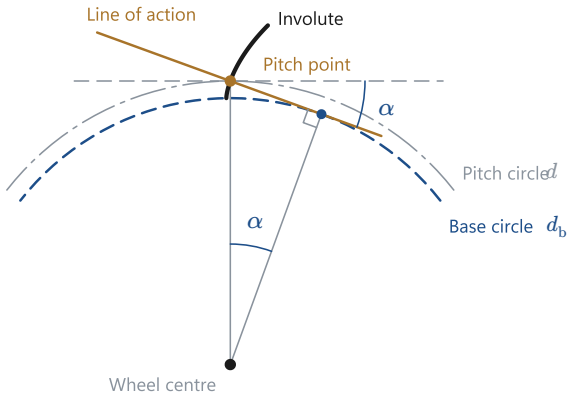


Fig. 2.11 Involute toothing: the line of action touches the base circle and is inclined by the pressure angle α

Tip Circle of Involute Toothing

(2.27) MASTER

$$d_\mathrm{a} = m \cdot (z + 2)$$

REARRANGED $m = \dfrac{d_\mathrm{a}}{z + 2}$

The addendum equals the module. Rearranged, the formula gives the module of an involute wheel from tip circle and number of teeth.

Valid: standard toothing without profile shift, pressure angle 20°, addendum $1 \cdot m$.

d_a	tip circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE A wheel with 24 teeth and module 0.5 mm.

Given: $m = 0.5\,\mathrm{mm}$; $z = 24$

$$\begin{aligned} d_\mathrm{a} &= m \cdot (z + 2) \\ &= 0.5\,\mathrm{mm} \cdot (24 + 2) \\ &= 13\,\mathrm{mm} \end{aligned}$$

Tip circle 13 mm with a pitch circle of 12 mm.

Root Circle of Involute Toothing

(2.28) MASTER

$$d_\mathrm{f} = m \cdot (z - 2.5)$$

The dedendum is 1.25 times the module: addendum of the mating wheel plus $0.25 \cdot m$ tip clearance.

Valid: standard toothing without profile shift. In precision engineering larger dedenda are also common; then twice the dedendum factor replaces 2.5.

d_f	root circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE The same wheel: 24 teeth, module 0.5 mm.

Given: $m = 0.5\,\mathrm{mm}$; $z = 24$

$$\begin{aligned} d_\mathrm{f} &= m \cdot (z - 2.5) \\ &= 0.5\,\mathrm{mm} \cdot (24 - 2.5) \\ &= 10.75\,\mathrm{mm} \end{aligned}$$

Root circle 10.75 mm; the tooth depth is $2.25 \cdot m$, i.e. 1.125 mm.

Base Circle

(2.29) ADVANCED

$$d_\mathrm{b} = d \cdot \cos\alpha$$

The involute is unwound from the base circle. The teeth always touch on the line of action, which is tangent to both base circles and inclined by the pressure angle to the common tangent of the pitch circles.

d_b	base circle diameter	mm
d	pitch circle diameter	mm
α	pressure angle (standardized 20°)	°

EXAMPLE Pitch circle 12 mm, pressure angle 20°.

Given: $d = 12\,\mathrm{mm}$; $\alpha = 20°$

$$\begin{aligned} d_\mathrm{b} &= d \cdot \cos\alpha \\ &= 12\,\mathrm{mm} \cdot \cos(20°) \\ &\approx 11.28\,\mathrm{mm} \end{aligned}$$

The base circle lies 0.36 mm below the pitch circle. There is no involute below the base circle.

2.5 Striking Work

The striking work is a separate train with its own drive, gear train and fly. The going train releases it; a counting mechanism determines the number of blows: the count wheel or the rack.

Sum of the Hour Blows

(2.30) MASTER

$$N = \frac{k \cdot (k+1)}{2}$$

The hour blows 1 + 2 + 3 and so on add up according to the sum formula.

N	number (of blows for the hours 1 to k)	1
k	last hour struck	1

EXAMPLE How many blows are struck from 1 o'clock to 12 o'clock?

Given: $k = 12$

$$\begin{aligned} N &= \frac{k \cdot (k+1)}{2} \\ &= \frac{12 \cdot (12+1)}{2} \\ &= 78 \end{aligned}$$

78 blows in 12 hours, 156 per day.

Blows per Day

(2.31) MASTER

$$N_{\mathrm{d}} = 2 \cdot (N_{12} + 12 \cdot b)$$

The half-hour blow adds 12 blows in 12 hours.

Valid: hour striking with at most one blow at the half hour. Quarter-striking work has its own train and is calculated separately.

N_{d}	blows in 24 hours	1
N_{12}	hour blows in 12 hours (78)	1
b	blows at each half hour (0 or 1)	1

EXAMPLE A striking work strikes the hours and one blow at each half hour.

Given: $N_{12} = 78; \quad b = 1$

$$\begin{aligned} N_{\mathrm{d}} &= 2 \cdot (N_{12} + 12 \cdot b) \\ &= 2 \cdot (78 + 12 \cdot 1) \\ &= 180 \end{aligned}$$

180 blows per day: 90 in 12 hours instead of 78.

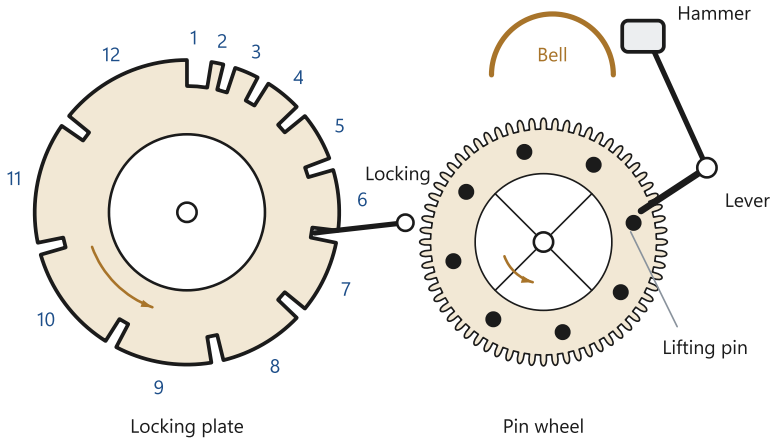


Fig. 2.12 Count wheel with 78 divisions (each hour occupies as many divisions as it has blows) and pin wheel with hammer

Count Wheel

(2.32) MASTER

$$N_{\mathrm{Sch}} = \frac{z_{\mathrm{N}} \cdot z_{\mathrm{Sch}}}{z'_{\mathrm{Sch}}}$$

The count wheel counts the blows. It turns once in 12 hours; its circumference is divided into 78 or 90 parts, and after each sequence of blows the locking detent drops into a notch and stops the train.

Valid: count wheel driven from the pin-wheel arbor and turning once in 12 hours.

N_{Sch}	blows per revolution of the count wheel (78, with half-hour striking 90)	1
z_{N}	number of lifting pins	1
z_{Sch}	number of teeth of the count-wheel drive wheel	1
z'_{Sch}	number of leaves of the pinion on the pin-wheel arbor that drives the count wheel	1

EXAMPLE A pin wheel carries 10 lifting pins. On its arbor sits a pinion with 8 leaves that drives a count-wheel drive wheel with 72 teeth.

Given: $z_{\mathrm{N}} = 10$; $z_{\mathrm{Sch}} = 72$; $z'_{\mathrm{Sch}} = 8$

$$\begin{aligned} N_{\mathrm{Sch}} &= \frac{z_{\mathrm{N}} \cdot z_{\mathrm{Sch}}}{z'_{\mathrm{Sch}}} \\ &= \frac{10 \cdot 72}{8} \\ &= 90 \end{aligned}$$

90 blows per revolution: a striking work with half-hour striking. For 78 blows, 13 lifting pins with a reduction ratio of 6 are suitable.

In rack striking, the hour snail, a stepped cam with 12 steps on the hour wheel, determines how far the rack falls. With each blow the gathering pallet gathers the rack back by one tooth until the train is locked again. Unlike the count wheel, rack striking cannot get out of step with the hands.

Blows per Revolution of the Driving Wheel

(2.33) MASTER

$$N_{\mathrm{A}} = \frac{z_{\mathrm{N}} \cdot z_{\mathrm{A}}}{z'_{\mathrm{N}}}$$

Each lifting pin raises the hammer once. The pin wheel turns faster than the driving wheel by the gear ratio.

Valid: driving wheel meshes directly with the pinion of the pin wheel. If an intermediate wheel lies between them, its gear ratio is added as a factor.

N_{A}	blows per revolution of the driving wheel (great wheel or barrel)	1
z_{N}	number of lifting pins	1
z_{A}	number of teeth of the driving wheel	1
z'_{N}	number of leaves of the pinion on the pin-wheel arbor	1

EXAMPLE A great wheel with 96 teeth drives the pinion (8 leaves) of a pin wheel with 8 lifting pins.

Given: $z_{\mathrm{N}} = 8$; $z_{\mathrm{A}} = 96$; $z'_{\mathrm{N}} = 8$

$$\begin{aligned} N_{\mathrm{A}} &= \frac{z_{\mathrm{N}} \cdot z_{\mathrm{A}}}{z'_{\mathrm{N}}} \\ &= \frac{8 \cdot 96}{8} \\ &= 96 \end{aligned}$$

96 blows are struck per revolution of the barrel.

Revolutions per Winding

(2.34) MASTER

$$U = \frac{t_{\mathrm{G}} \cdot n_{\mathrm{d}}}{N_{\mathrm{A}}}$$

The number of blows per day is given by formula (2.31). Going train and striking work are designed so that both run equally long on one winding.

U	required revolutions of the driving wheel	1
t_{G}	running time	d
n_{d}	blows per day	$\frac{1}{\mathrm{d}}$
N_{A}	blows per revolution of the driving wheel	1

EXAMPLE The striking work from the previous example is to run for 8 days and strikes 180 times a day.

Given: $t_{\mathrm{G}} = 8\,\mathrm{d}$; $\quad n_{\mathrm{d}} = 180\,\frac{1}{\mathrm{d}}$; $\quad N_{\mathrm{A}} = 96$

$$\begin{aligned} U &= \frac{t_{\mathrm{G}} \cdot n_{\mathrm{d}}}{N_{\mathrm{A}}} \\ &= \frac{8\,\mathrm{d} \cdot 180\,\frac{1}{\mathrm{d}}}{96} \\ &= 15 \end{aligned}$$

1440 blows are struck in 8 days; for this the barrel turns 15 times. Line thickness and barrel diameter then give the height of fall of the weight (Chapter 1).

Revolutions of the Fly per Blow

(2.35) MASTER

$$U_{\mathrm{W}} = \frac{i_{\mathrm{W}}}{z_{\mathrm{N}}}$$

The fly brakes the train by the air resistance of its vanes. Because the resistance increases sharply with rotational speed, a uniform speed is established and the blows follow at equal intervals. Larger or more steeply set vanes make the striking slower.

U_{W}	revolutions of the fly per blow	1
i_{W}	gear ratio from pin wheel to fly	1
z_{N}	number of lifting pins	1

EXAMPLE From the pin wheel (64 teeth, 8 lifting pins) to the fly there are three meshes: 64 to 8, 56 to 7 and 49 to 7. The gear ratio is $8 \cdot 8 \cdot 7 = 448$.

Given: $i_{\mathrm{W}} = 448$; $\quad z_{\mathrm{N}} = 8$

$$\begin{aligned} U_{\mathrm{W}} &= \frac{i_{\mathrm{W}}}{z_{\mathrm{N}}} \\ &= \frac{448}{8} \\ &= 56 \end{aligned}$$

The fly turns 56 times per blow.

2.6 Calendar and Moon Phase

Calendar mechanisms count days. Their wheels are not driven continuously but advanced by one tooth once a day; the number of teeth is then the number of days per revolution.

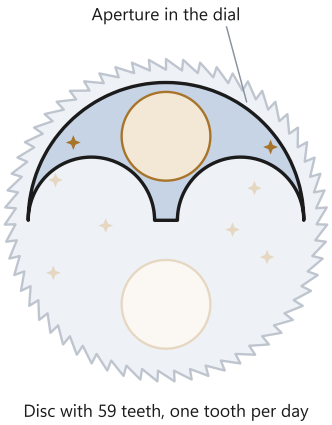


Fig. 2.13 Moon-phase disc behind the aperture in the dial

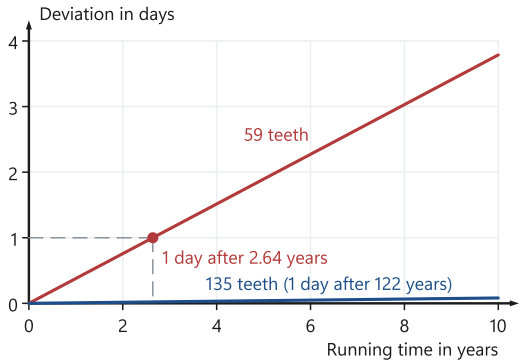


Fig. 2.14 Deviation of the moon-phase display from the mean moon

Moon-Phase Disc

(2.36) MASTER

$$t_{\mathrm{L}} = \frac{z \cdot t_{\mathrm{s}}}{k}$$

From new moon to new moon, an average of 29.530 59 days passes, the synodic month. Because a wheel cannot have half teeth, the disc carries two moons and 59 teeth.

t_{L}	displayed duration of one lunation	d
z	number of teeth (of the moon disc)	1
t_{s}	time between two advances (one tooth per day)	d
k	number (of moons on the disc)	1

EXAMPLE A moon disc with 59 teeth and two moons is advanced by one tooth daily.

Given: $z = 59; \quad t_{\mathrm{s}} = 1\,\mathrm{d}; \quad k = 2$

$$\begin{aligned} t_{\mathrm{L}} &= \frac{z \cdot t_{\mathrm{s}}}{k} \\ &= \frac{59 \cdot 1\,\mathrm{d}}{2} \\ &= 29.5\,\mathrm{d} \end{aligned}$$

The display passes through all phases in 29.5 days. The moon needs 29.530 59 days on average, so the display runs slightly fast.

Running Time until the Moon Phase Deviates

(2.37) MASTER

$$t_{\mathrm{F}} = \frac{\Delta t \cdot t_{\mathrm{L}}}{|T_{\mathrm{syn}} - t_{\mathrm{L}}|}$$

With each lunation the display deviates by the difference between the displayed and the true lunation. The deviations add up.

Valid: mean lunar motion. The true moon deviates from it by up to about half a day in the course of a month.

t_{F}	running time until the display deviates by Δt	d
Δt	deviation of the display from the true age of the moon	d
t_{L}	displayed duration of one lunation	d
T_{syn}	synodic month (mean value 29.530 59 d, Section 8.1)	d

EXAMPLE After what time does the display with 59 teeth deviate by one day?

Given: $\Delta t = 1\,\mathrm{d}$; $t_{\mathrm{L}} = 29.5\,\mathrm{d}$; $T_{\mathrm{syn}} = 29.530\,59\,\mathrm{d}$

$$\begin{aligned} t_{\mathrm{F}} &= \frac{\Delta t \cdot t_{\mathrm{L}}}{|T_{\mathrm{syn}} - t_{\mathrm{L}}|} \\ &= \frac{1\,\mathrm{d} \cdot 29.5\,\mathrm{d}}{|29.530\,59\,\mathrm{d} - 29.5\,\mathrm{d}|} \\ &\approx 964\,\mathrm{d} \end{aligned}$$

After 964 days, i.e. about 2.6 years, the display is one day ahead and must be corrected.

Precise Moon Phase with Gearing

(2.38) ADVANCED

$$t_{\mathrm{L}} = \frac{t_{\mathrm{A}} \cdot z}{z' \cdot k}$$

A gear ratio between the advancing wheel and the moon disc matches the synodic month much more precisely than the simple disc with 59 teeth.

Valid: moon disc driven via wheels. The disc with 135 teeth is widespread; the drive in the example is one possible tooth-count set, others are in use.

t_{L}	displayed duration of one lunation	d
t_{A}	rotation period of the driving arbor (star wheel with 7 teeth, advanced daily: 7 d)	d
z	number of teeth (of the moon disc)	1
z'	number of leaves of the pinion (drives the moon disc)	1
k	number (of moons on the disc)	1

EXAMPLE A star wheel with 7 teeth is advanced daily. On its arbor sits a pinion with 16 leaves that drives, via an idler, a moon disc with 135 teeth and two moons.

Given: $t_{\mathrm{A}} = 7\,\mathrm{d}$; $z = 135$; $z' = 16$; $k = 2$

$$\begin{aligned} t_{\mathrm{L}} &= \frac{t_{\mathrm{A}} \cdot z}{z' \cdot k} \\ &= \frac{7\,\mathrm{d} \cdot 135}{16 \cdot 2} \\ &= 29.531\,25\,\mathrm{d} \end{aligned}$$

The display is now only 57 s too slow per lunation. According to formula (2.37) with $T_{\mathrm{syn}} = 29.530\,59\,\mathrm{d}$, it takes 44 744 days, a good 122 years, until it lags by one day.

Counting Wheels of Calendar Displays

Display	Wheel	Advances	One revolution in
date	date ring or date wheel with 31 teeth	daily	31 days
day of the week	star wheel with 7 teeth	daily	7 days
month	star wheel with 12 teeth	monthly	1 year
moon phase	disc with 59 teeth and two moons	daily	59 days
perpetual calendar	programme wheel with 48 month steps	monthly	4 years (1461 days)

The simple calendar is advanced by hand five times a year, namely after every month with fewer than 31 days. The annual calendar needs one correction at the end of February. The perpetual calendar also knows the leap years; it need not be corrected until the year 2100, which is not a leap year.

The date driving wheel turns once in 24 hours and is driven by the hour wheel with a reduction ratio of 2 to 1, formula (2.12). Leap years, the length of the year and the equation of time are given in Chapter 8.

2.7 Efficiency of the Gear Train

Every mesh and every bearing consumes part of the driving force through friction. The efficiency states what proportion of the torque remains.

Overall Efficiency of the Gear Train

(2.39) MASTER

$$\eta_{\text{ges}} = \eta^{k}$$

The efficiencies of meshes in series are multiplied. If they differ, the product of the individual values replaces the power. The general form is given in formula (XL 2.30).

Valid: guide values for clean, oiled movements. Pinions with few leaves, dry bearings and depthings that are too deep or too shallow reduce the efficiency considerably.

η_{ges}	overall efficiency of the gear train	1
η	efficiency (per mesh; including bearing friction, guide value 0.90 to 0.95)	1
k	number (of meshes)	1

EXAMPLE From the barrel to the escape wheel there are four meshes, each with an efficiency of 0.95.

Given: $\eta = 0.95; \quad k = 4$

$$\begin{aligned}\eta_{\text{ges}} &= \eta^{k} \\ &= 0.95^{4} \\ &\approx 0.81\end{aligned}$$

About one fifth of the torque is lost in the gear train. At 0.90 per mesh it would already be one third.

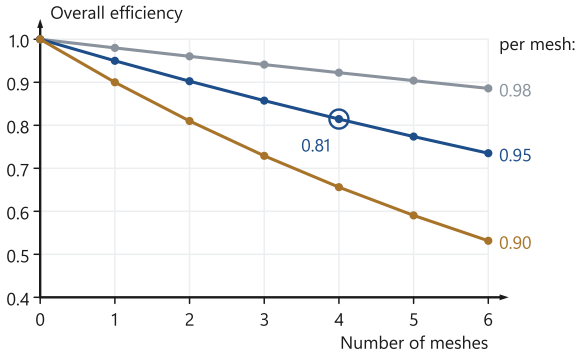


Fig. 2.15 Overall efficiency by number of meshes

Torque at the Escape Wheel

(2.40) MASTER

$$M_{\mathrm{H}} = \frac{M_{\mathrm{F}} \cdot \eta_{\mathrm{ges}}}{i_{\mathrm{ges}}}$$

From arbor to arbor the torque falls by the gear ratio and by the friction losses (Fig. 2.4).

M_{H}	torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
M_{F}	torque at the barrel	$\mathrm{N{\cdot}mm}$
η_{ges}	overall efficiency of the gear train	1
i_{ges}	overall gear ratio (from barrel to escape wheel)	1

EXAMPLE A barrel delivers 8 N·mm. The gear ratio to the escape wheel is $6 \cdot 600 = 3600$, the overall efficiency 0.81.

Given: $M_{\mathrm{F}} = 8\,\mathrm{N{\cdot}mm}$; $\eta_{\mathrm{ges}} = 0.81$; $i_{\mathrm{ges}} = 3600$

$$\begin{aligned} M_{\mathrm{H}} &= \frac{M_{\mathrm{F}} \cdot \eta_{\mathrm{ges}}}{i_{\mathrm{ges}}} \\ &= \frac{8\,\mathrm{N{\cdot}mm} \cdot 0.81}{3600} \\ &= 1.8\,\mu\mathrm{N{\cdot}m} \end{aligned}$$

1.8 µN·m remain at the escape wheel, a little over one five-thousandth of the spring torque. This is why even a little dirt on the fast arbors brings a movement to a stop.

Efficiency from Measured Torques

(2.41) ADVANCED

$$\eta_{\mathrm{ges}} = \frac{M_{\mathrm{H}} \cdot i_{\mathrm{ges}}}{M_{\mathrm{F}}}$$

Without friction, the spring torque divided by the gear ratio would arrive at the escape wheel. The efficiency is the ratio of the measured value to this ideal value.

η_{ges}	overall efficiency of the gear train	1
M_{H}	torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
i_{ges}	overall gear ratio (from barrel to escape wheel)	1
M_{F}	torque at the barrel	$\mathrm{N{\cdot}mm}$

EXAMPLE On a movement, 8 N·mm are measured at the barrel and 1.7 µN·m at the escape wheel; the gear ratio is 3600.

Given: $M_{\mathrm{H}} = 1.7\,\mu\mathrm{N{\cdot}m}$; $i_{\mathrm{ges}} = 3600$; $M_{\mathrm{F}} = 8\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} \eta_{\mathrm{ges}} &= \frac{M_{\mathrm{H}} \cdot i_{\mathrm{ges}}}{M_{\mathrm{F}}} \\ &= \frac{1.7\,\mu\mathrm{N{\cdot}m} \cdot 3600}{8\,\mathrm{N{\cdot}mm}} \\ &= 0.765 \end{aligned}$$

The gear train transmits a little over three quarters of the torque. This corresponds to 0.935 per mesh with four meshes.

3 Oscillator: Balance and Balance Spring

Balance and balance spring form the oscillator of the watch. The balance is the inert mass, the balance spring the spring that returns it to its rest position. How fast the two oscillate together is determined by only two quantities: the moment of inertia of the balance and the restoring torque coefficient of the balance spring.

Almost all regulating formulas in this chapter follow from one rule: the period increases with the square root of the moment of inertia and decreases with the square root of the restoring torque coefficient. One percent more moment of inertia or one percent less restoring torque coefficient makes the watch half a percent slower, i.e. 432 s per day. In this chapter N stands for a number (coils of the balance spring, identical screws on the rim), W for energy and P for power.

3.1 Oscillation of the Balance

The balance oscillates back and forth about its rest position. One oscillation is there and back; the watch ticks twice in the process, because each beat is a vibration (half-oscillation).

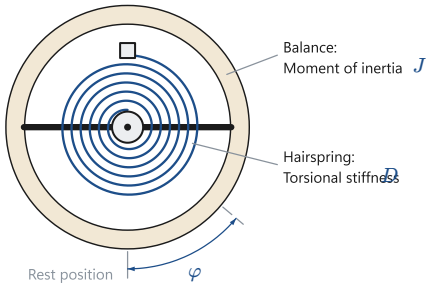


Fig. 3.1 Balance with balance spring: the balance provides the moment of inertia J, the balance spring the restoring torque coefficient D

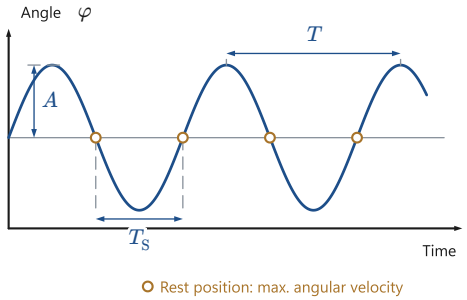


Fig. 3.2 Angle of the balance over time: amplitude A, period T, beat duration $T_S = \frac{1}{2} T$

Period of the Balance

(3.1) JOURNEYMAN

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J}{D}}$$

REARRANGED $D = \dfrac{4 \cdot \pi^2 \cdot J}{T^2}$

A more inert balance oscillates more slowly, a stronger balance spring faster. Both quantities are under the square root: four times the moment of inertia doubles the period.

Valid: restoring torque of the balance spring proportional to the angle, without friction and without influence of the escapement. J includes rim, arms, staff, double roller and collet.

T	period (full oscillation, there and back)	s
J	moment of inertia (of the balance; with everything that oscillates with it)	$\mathrm{mg{\cdot}cm^2}$
D	restoring torque coefficient (of the balance spring; torque per angle)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

Good to know: The amplitude does not appear in the formula. An ideal oscillator oscillates equally fast at small and large amplitude; this property is called isochronism (Section 3.7).

EXAMPLE A balance has a moment of inertia of $10\,\mathrm{mg{\cdot}cm^2}$, its balance spring a restoring torque coefficient of $0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. How long does one oscillation last?

Given: $J = 10\,\mathrm{mg{\cdot}cm^2}$; $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{J}{D}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{10\,\mathrm{mg{\cdot}cm^2}}{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}} \\ &\approx 0.250\,\mathrm{s} \end{aligned}$$

One oscillation lasts a quarter of a second. The balance therefore makes 4 oscillations and 8 beats per second.

Frequency of the Balance

(3.2) JOURNEYMAN

$$f = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{D}{J}}$$

The frequency is the reciprocal of the period: $f = \frac{1}{T}$.

Valid: as for formula (3.1).

f	frequency (full oscillations per second)	Hz
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE What is the frequency of the oscillator from the previous example?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned} f &= \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{D}{J}} \\ &= \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}{10\,\mathrm{mg{\cdot}cm^2}}} \\ &\approx 4.00\,\mathrm{Hz} \end{aligned}$$

4 Hz, the usual frequency of today's wristwatches.

Beat Rate from the Frequency

(3.3) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot f$$

REARRANGED $f = \frac{n_\mathrm{h}}{2}$

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
f	frequency (full oscillations per second)	Hz

The watch beats twice per oscillation. For watches the beat rate is stated in vibrations per hour (A/h, A for alternation). The formula with the beat duration is already given in Section XL 1.1.

EXAMPLE A balance oscillates at 3 Hz. What beat rate is this?

Given: $f = 3\,\mathrm{Hz}$

$$\begin{aligned} n_\mathrm{h} &= 2 \cdot f \\ &= 2 \cdot 3\,\mathrm{Hz} \\ &= 6\,\frac{1}{\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &\approx 21\,600\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

21 600 vibrations per hour. The beat rate is given in the data sheet of the calibre, and the timing machine calculates with it.

Common Beat Rates

Beat rate n_h in $\frac{\mathrm{A}}{\mathrm{h}}$	Beats per second	Frequency f in Hz	Period T in s	Beat duration T_S in s
18 000	5	2.5	0.4	0.2
19 800	5.5	2.75	0.3636	0.1818
21 600	6	3	0.3333	0.1667
25 200	7	3.5	0.2857	0.1429
28 800	8	4	0.25	0.125
36 000	10	5	0.2	0.1

The beat duration T_S is the duration of one vibration, i.e. half the period.

Angular Frequency

(3.4) MASTER

$$\omega = 2 \cdot \pi \cdot f$$

ω	angular frequency	$\frac{\mathrm{rad}}{\mathrm{s}}$
f	frequency (full oscillations per second)	Hz

The angular frequency is the frequency in radians: 2π per oscillation. For the balance also $\omega = \sqrt{\frac{D}{J}}$.

EXAMPLE What is the angular frequency at 4 Hz?

Given: $f = 4\,\mathrm{Hz}$

$$\begin{aligned} \omega &= 2 \cdot \pi \cdot f \\ &= 2 \cdot \pi \cdot 4\,\mathrm{Hz} \\ &\approx 25.1\,\frac{\mathrm{rad}}{\mathrm{s}} \end{aligned}$$

About 25.1 rad per second. The following formulas calculate with this number.

Angle of the Balance over Time

(3.5) MASTER

$$\varphi = A \cdot \sin(\omega \cdot t)$$

The balance oscillates sinusoidally: it is fastest at the rest position and stands still for an instant at the turning points.

Valid: free, undamped oscillation. The product $\omega \cdot t$ is an angle in radians; the calculator must be set to rad for the sine.

φ	angle (of the balance from the rest position)	$^\circ$
A	amplitude (greatest angle from the rest position)	$^\circ$
ω	angular frequency	$\frac{\text{rad}}{\text{s}}$
t	time (since passing through the rest position)	s

EXAMPLE A balance oscillates at 4 Hz ($\omega = 25.13\ \frac{\text{rad}}{\text{s}}$) with 270° amplitude. Where is it 0.02 s after passing through the rest position?

Given: $A = 270^\circ; \quad \omega = 25.13\ \frac{\text{rad}}{\text{s}}; \quad t = 0.02\,\text{s}$

$$\begin{aligned}\varphi &= A \cdot \sin(\omega \cdot t)\\ &= 270^\circ \cdot \sin\left(25.13\ \tfrac{\text{rad}}{\text{s}} \cdot 0.02\,\text{s}\right)\\ &\approx 130^\circ\end{aligned}$$

After 0.02 s the balance has already covered about 130°, almost half the amplitude. It needs 0.0625 s in total to reach the turning point, a quarter oscillation.

Maximum Angular Velocity

(3.6) MASTER

$$\omega_{\max} = A \cdot \omega$$

At the rest position the entire energy of the oscillation is kinetic. This is where the escapement acts, because a disturbance at this point changes the rate least.

Valid: insert the amplitude in radians: degrees times $\frac{\pi}{180}$.

$\omega_{\max}$	angular velocity of the balance when passing through the rest position	$\frac{\text{rad}}{\text{s}}$
A	amplitude (in radians)	rad
ω	angular frequency	$\frac{\text{rad}}{\text{s}}$

EXAMPLE How fast does the balance at 4 Hz and 270° amplitude (4.71 rad) turn at the rest position?

Given: $A = 4.71\,\text{rad}; \quad \omega = 25.13\ \frac{\text{rad}}{\text{s}}$

$$\begin{aligned}\omega_{\max} &= A \cdot \omega\\ &= 4.71\,\text{rad} \cdot 25.13\ \tfrac{\text{rad}}{\text{s}}\\ &\approx 118\ \tfrac{\text{rad}}{\text{s}}\end{aligned}$$

About 118 rad per second; turning on uniformly, this would be almost 19 revolutions per second. A point on the rim at 5 mm radius moves at about 0.6 m per second.

Key point: 28 800 vibrations per hour are 8 beats per second and 4 Hz.

3.2 Balance Spring

The balance spring is a bending spring. Its restoring torque coefficient states how much torque it produces per angle. It depends on the material, the cross-section of the blade and the effective length.

Restoring Torque of the Balance Spring

(3.7) JOURNEYMAN

$$M = D \cdot \varphi$$

The restoring torque increases uniformly with the angle and always points towards the rest position. Only because of this does the balance oscillate equally fast at every amplitude.

Valid: balance spring in the elastic range; coils do not touch.

M	torque (restoring torque of the balance spring)	$\mu\mathrm{N{\cdot}m}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
φ	angle (of the balance from the rest position; radians)	rad

EXAMPLE With what torque does a balance spring with $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ pull the balance back at the turning point at 270° (4.71 rad)?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $\varphi = 4.71\,\mathrm{rad}$

$$\begin{aligned} M &= D \cdot \varphi \\ &= 0.632\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \cdot 4.71\,\mathrm{rad} \\ &\approx 2.98\,\mu\mathrm{N{\cdot}m} \end{aligned}$$

Just under 3 µN·m. This is the order of magnitude of the torque that arrives at the escape wheel.

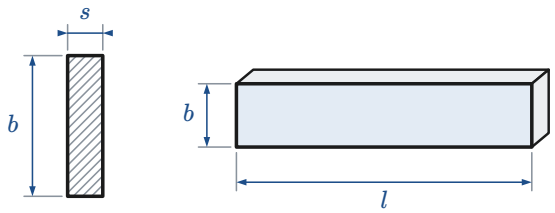


Fig. 3.3 Blade of the balance spring: blade height b, blade thickness s, effective length l

Restoring Torque Coefficient from the Dimensions of the Balance Spring

(3.8) MASTER

$$D = \frac{E \cdot b \cdot s^3}{12 \cdot l}$$

Height and modulus of elasticity enter linearly, the thickness to the third power. A longer balance spring is softer.

Valid: rectangular cross-section, uniform blade, pure bending. With N and mm, D is obtained in N·mm per rad; $1\,\mathrm{N{\cdot}mm} = 1000\,\mu\mathrm{N{\cdot}m}$.

D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
E	modulus of elasticity (of the balance-spring material)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
b	blade height (in the direction of the balance staff)	mm
s	blade thickness (in the radial direction)	mm
l	length (of the balance spring, effective)	mm

EXAMPLE A balance spring has $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2}$, a blade height of 0.14 mm, a blade thickness of 0.032 mm and an effective length of 121 mm.

Given: $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2}$; $b = 0.14\,\mathrm{mm}$; $s = 0.032\,\mathrm{mm}$; $l = 121\,\mathrm{mm}$

$$\begin{aligned} D &= \frac{E \cdot b \cdot s^3}{12 \cdot l} \\ &= \frac{200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2} \cdot 0.14\,\mathrm{mm} \cdot (0.032\,\mathrm{mm})^3}{12 \cdot 121\,\mathrm{mm}} \\ &\approx 0.632\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \end{aligned}$$

This balance spring matches the balance with $10\,\mathrm{mg{\cdot}cm}^2$ from Section 3.1 and gives 4 Hz with it.

Rate Change due to Blade Thickness

(3.9) APPROXIMATION MASTER

$$\Delta G \approx \frac{3 \cdot \Delta s}{2 \cdot s} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δs	change in blade thickness (+ means: thicker)	$\mu\mathrm{m}$
s	blade thickness	$\mu\mathrm{m}$

Because the restoring torque coefficient increases with s^3, 1 % more thickness changes the restoring torque coefficient by 3 % and the rate by 1.5 %.

Valid: small changes; height, length and balance unchanged. For the blade height, correspondingly $\Delta G \approx \frac{\Delta b}{2 \cdot b} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$.

EXAMPLE A balance spring is 0.1 µm thicker than 32 µm.

Given: $\Delta s = 0.1\,\mu\mathrm{m}; \quad s = 32\,\mu\mathrm{m}$

$$\begin{aligned}\Delta G &\approx \frac{3 \cdot \Delta s}{2 \cdot s} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{3 \cdot 0.1\,\mu\mathrm{m}}{2 \cdot 32\,\mu\mathrm{m}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 405\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

One tenth of a micrometre makes almost 7 minutes per day. No blade can be measured this precisely; this is why balance springs are not sorted by dimension but by their measured restoring torque coefficient and matched to the balance.

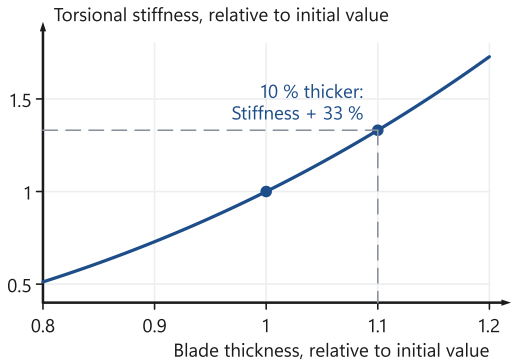


Fig. 3.4 The restoring torque coefficient increases with the third power of the blade thickness

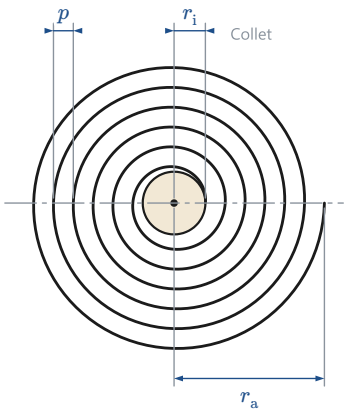


Fig. 3.5 Flat balance spring: radii r_i and r_a, coil pitch p

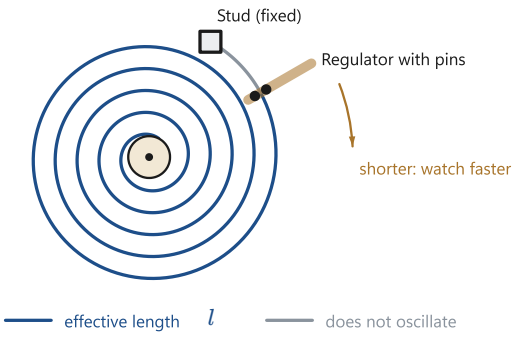


Fig. 3.6 Regulator: the effective length l extends from the collet to the regulator pins

Length of the Flat Balance Spring

(3.10) APPROXIMATION MASTER

$$l \approx \pi \cdot N \cdot (r_\mathrm{a} + r_\mathrm{i})$$

Every coil is almost a circle. The length is the number of coils times the circumference of the mean coil.

Valid: uniformly wound flat balance spring (Archimedean spiral) with at least about 5 coils; terminal curve and attachment ends not included.

l	length (of the balance spring)	mm
N	number (of coils)	1
r_a	radius of the outermost coil	mm
r_i	radius of the innermost coil (at the collet)	mm

EXAMPLE A balance spring has 12 coils between 0.6 mm and 2.6 mm radius.

Given: $N = 12;\quad r_\mathrm{a} = 2.6\,\mathrm{mm};\quad r_\mathrm{i} = 0.6\,\mathrm{mm}$

$$\begin{aligned} l &\approx \pi \cdot N \cdot (r_\mathrm{a} + r_\mathrm{i}) \\ &= \pi \cdot 12 \cdot (2.6\,\mathrm{mm} + 0.6\,\mathrm{mm}) \\ &\approx 121\,\mathrm{mm} \end{aligned}$$

The blade is about 121 mm long, as long as the balance spring in the examples on the restoring torque coefficient and the regulator. A wristwatch thus contains a good 12 cm of spring strip on the area of a shirt button.

Coil Pitch

(3.11) MASTER

$$p = \frac{r_\mathrm{a} - r_\mathrm{i}}{N}$$

The pitch determines how much clearance the coils have when the balance spring breathes. At large amplitude they must not touch.

Valid: uniformly wound flat balance spring.

p	coil pitch (centre of blade to centre of the next blade)	mm
r_a	radius of the outermost coil	mm
r_i	radius of the innermost coil	mm
N	number (of coils)	1

EXAMPLE How far apart are the coils of the balance spring from the previous example?

Given: $r_\mathrm{a} = 2.6\,\mathrm{mm};\quad r_\mathrm{i} = 0.6\,\mathrm{mm};\quad N = 12$

$$\begin{aligned} p &= \frac{r_\mathrm{a} - r_\mathrm{i}}{N} \\ &= \frac{2.6\,\mathrm{mm} - 0.6\,\mathrm{mm}}{12} \\ &\approx 0.167\,\mathrm{mm} \end{aligned}$$

About 0.17 mm from blade to blade. With a blade thickness of 0.032 mm, about 0.13 mm of clearance remains.

CGS Number of the Balance Spring

(3.12) MASTER

$$K = D \cdot \left(d_\mathrm{a}^2 - d_\mathrm{i}^2\right)$$

The CGS value is the restoring torque coefficient times the difference of the squared diameters. It depends only on material, blade and coil pitch, not on the point at which the balance spring is pinned: pinned further out, the restoring torque coefficient decreases to the same extent as the difference of the squares increases. This is why unpinned balance springs are graded and ordered by this value.

Valid: uniformly wound flat balance spring; d_a at the point where the balance spring is pinned, d_i at the collet. The CGS number is the same value in the old unit $\mathrm{dyn \cdot cm^3}$ per rad: numerical value in $\mathrm{N \cdot mm^3}$ times 100.

K	CGS value of the balance spring (times 100 gives the CGS number)	$\frac{\mathrm{N \cdot mm^3}}{\mathrm{rad}}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$
d_a	diameter of the outermost coil ($2 \cdot r_\mathrm{a}$)	mm
d_i	diameter of the innermost coil ($2 \cdot r_\mathrm{i}$)	mm

Good to know: With the length and the coil pitch from the two previous formulas this becomes $K \approx \frac{E \cdot b \cdot s^3 \cdot p}{3 \cdot \pi}$. Neither the length nor the diameters appear in it.

EXAMPLE For 4 Hz, the balance with $10\,\mathrm{mg \cdot cm^2}$ from Section 3.1 requires a restoring torque coefficient of $0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$. The balance spring is to be pinned at a diameter of 5.0 mm, its innermost coil measures 1.2 mm. Which balance spring is ordered?

Given: $D = 0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$; $\quad d_\mathrm{a} = 5.0\,\mathrm{mm}$; $\quad d_\mathrm{i} = 1.2\,\mathrm{mm}$

$$\begin{aligned} K &= D \cdot \left(d_\mathrm{a}^2 - d_\mathrm{i}^2\right) \\ &= 0.632\ \tfrac{\mu\mathrm{N \cdot m}}{\mathrm{rad}} \cdot \left((5.0\,\mathrm{mm})^2 - (1.2\,\mathrm{mm})^2\right) \\ &\approx 0.0149\ \tfrac{\mathrm{N \cdot mm^3}}{\mathrm{rad}} \end{aligned}$$

The CGS value is 0.0149, so the CGS number is 1.49. The next standard number is ordered, here 1.50.

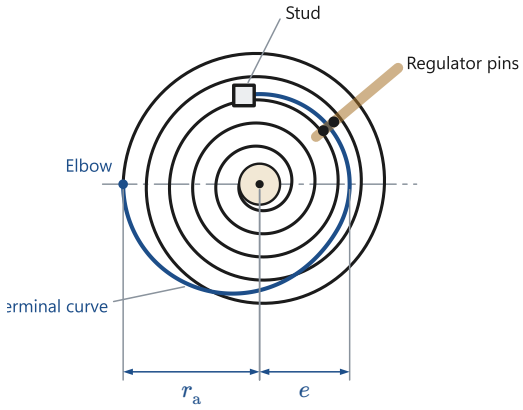


Fig. 3.7 Breguet balance spring from above: the raised terminal curve leads the blade from the outer radius r_a to the terminal radius e, on which the regulator pins move

Terminal Curve Number

(3.13) JOURNEYMAN

$$N_{\mathrm{E}} = \frac{100 \cdot e}{r_{\mathrm{a}}}$$

REARRANGED $e = \dfrac{N_{\mathrm{E}} \cdot r_{\mathrm{a}}}{100}$

N_{E}	terminal curve number	1
e	terminal radius (balance axis to midway between the regulator pins)	mm
r_{a}	radius of the outermost coil	mm

With the Breguet balance spring, the outer end is raised and led inwards in a terminal curve. Its last part is a circular arc about the balance axis, on which the regulator pins move. The terminal curve number gives the radius of this arc as a percentage of the outer radius. The shape of the curve is chosen from the curve template according to this number.

Valid: balance spring with raised overcoil. The curve templates are graded; the nearest number is chosen. Numbers between about 40 and 100 are usual.

EXAMPLE A Breguet balance spring has an outside diameter of 6.4 mm, i.e. an outer radius of 3.2 mm. The regulator pins are 2.4 mm from the balance axis. Which terminal curve is required?

Given: $e = 2.4\,\mathrm{mm}$; $r_{\mathrm{a}} = 3.2\,\mathrm{mm}$

$$\begin{aligned} N_{\mathrm{E}} &= \frac{100 \cdot e}{r_{\mathrm{a}}} \\ &= \frac{100 \cdot 2.4\,\mathrm{mm}}{3.2\,\mathrm{mm}} \\ &= 75 \end{aligned}$$

Terminal curve number 75: its arc lies at three quarters of the outer radius.

Vibrating: Oscillations in the Counting Time

(3.14) JOURNEYMAN

$$N_{\mathrm{S}} = \frac{n_{\mathrm{h}} \cdot t}{2}$$

N_{S}	number of full oscillations (in the counting time)	1
n_{h}	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$
t	time (counting time)	s

In vibrating, one looks for the point of the balance spring at which it gives the nominal beat rate with its balance. The balance spring is held there on trial, the balance is allowed to oscillate freely and the full oscillations are counted.

Valid: n_{h} and t in matching units: convert A/h to beats per second (divide by 3600) if t is in seconds.

EXAMPLE A balance for 18 000 A/h is vibrated with its new balance spring. How many full oscillations must it make in 60 s?

Given: $n_{\mathrm{h}} = 18\,000\ \frac{\mathrm{A}}{\mathrm{h}}$; $t = 60\,\mathrm{s}$

$$\begin{aligned} N_{\mathrm{S}} &= \frac{n_{\mathrm{h}} \cdot t}{2} \\ &= \frac{18\,000\ \frac{\mathrm{A}}{\mathrm{h}} \cdot 60\,\mathrm{s}}{2} \\ &= 150 \end{aligned}$$

150 oscillations per minute. If more are counted, the balance spring is held too short at this point; if fewer, too long.

Effective Length for the Nominal Beat Rate

(3.15) JOURNEYMAN

$$l_2 = l_1 \cdot \left(\frac{n_1}{n_2}\right)^2$$

l_2	required effective length	mm
l_1	effective length in the trial	mm
n_1	beat rate in the trial	$\frac{\mathrm{A}}{\mathrm{h}}$
n_2	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$

The beat rate changes with the square root of the reciprocal of the length. Shorter means faster.

Valid: the same balance, uniform blade.

EXAMPLE With an effective length of 130 mm a balance makes 28 000 A/h. It is to make 28 800 A/h.

Given: $l_1 = 130\,\mathrm{mm}$; $n_1 = 28\,000\,\frac{\mathrm{A}}{\mathrm{h}}$; $n_2 = 28\,800\,\frac{\mathrm{A}}{\mathrm{h}}$

$$l_2 = l_1 \cdot \left(\frac{n_1}{n_2}\right)^2$$

$$= 130\,\mathrm{mm} \cdot \left(\frac{28\,000\,\frac{\mathrm{A}}{\mathrm{h}}}{28\,800\,\frac{\mathrm{A}}{\mathrm{h}}}\right)^2$$

$$\approx 122.9\,\mathrm{mm}$$

The balance spring must be pinned about 7 mm shorter.

Rate Change with the Regulator

(3.16) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{\Delta l}{2 \cdot l} \cdot 86\,400\,\frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta l \approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δl	change in length (of the effective length; + means: longer)	mm
l	length (of the balance spring, effective: collet to regulator pins)	mm

The regulator moves the regulator pins along the outermost coil. Shorter effective length means a larger restoring torque coefficient, and the watch becomes faster; hence the minus sign.

Valid: small changes; the balance spring lies between the regulator pins without play. With play the regulator acts more weakly and the rate depends on the amplitude (Section 3.7).

EXAMPLE A watch with an effective balance-spring length of 121 mm loses 20 s per day. It is therefore to become faster by 20 s per day.

Given: $\Delta G = 20\,\frac{\mathrm{s}}{\mathrm{d}}$; $l = 121\,\mathrm{mm}$

$$\Delta l \approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$$

$$= -\frac{2 \cdot 121\,\mathrm{mm} \cdot 20\,\frac{\mathrm{s}}{\mathrm{d}}}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$$

$$\approx -0.056\,\mathrm{mm}$$

The regulator must shorten the effective length by about 0.06 mm. This is hardly visible; this is why good movements have a fine adjustment on the regulator.

Key point: Shorter balance spring, faster watch. 1 ‰ less effective length gives about 43 s per day.

3.3 Balance

The moment of inertia states how strongly the balance resists being turned. Not only the mass counts, but above all its distance from the axis: the radius enters squared. This is why almost the entire effect lies in the rim.

Moments of inertia of balances are given in mg·cm²: $1\,\mathrm{mg{\cdot}cm^2} = 10^{-10}\,\mathrm{kg{\cdot}m^2}$. Calculating with milligrams and millimetres gives mg·mm²; 100 mg·mm² are 1 mg·cm².

The general formulas for the moment of inertia of simple bodies and the parallel axis theorem are given in Section XL 2.6.

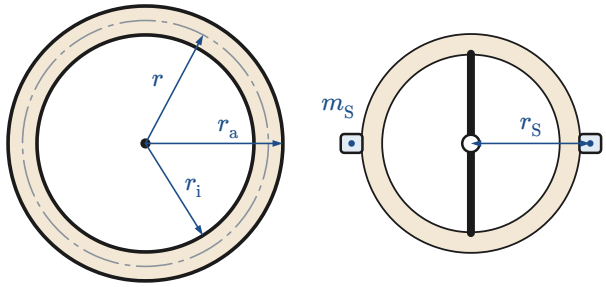


Fig. 3.8 Left, the rim with mean radius r, inner radius r_i and outer radius r_a; right, screws of mass m_S at distance r_S

Moment of Inertia of the Rim (Thin Ring)

(3.17) APPROXIMATION MASTER

$$J \approx m \cdot r^2$$

J	moment of inertia (of the rim)	$\mathrm{mg{\cdot}cm^2}$
m	mass (of the rim)	mg
r	radius (of the rim, mean value)	mm

For an estimate, the entire mass of the rim is assumed to lie on the mean radius. Arms and staff contribute little because their mass lies close to the axis.

Valid: rim narrow compared with the radius; rim only, without arms, staff and screws.

Good to know: Calculated exactly, the rim is a hollow cylinder, formula (XL 2.37). With an outer radius of 4.9 mm and an inner radius of 4.1 mm this gives $8.16\,\mathrm{mg{\cdot}cm^2}$ for the rim from the example; the estimate lies only 0.8 % below.

EXAMPLE A balance rim weighs 40 mg and has a mean radius of 4.5 mm.

Given: $m = 40\,\mathrm{mg}$; $r = 4.5\,\mathrm{mm}$

$$J \approx m \cdot r^2$$
$$= 40\,\mathrm{mg} \cdot (4.5\,\mathrm{mm})^2$$
$$\approx 8.1\,\mathrm{mg{\cdot}cm^2}$$

About $8\,\mathrm{mg{\cdot}cm^2}$ for the rim alone. With arms, staff and screws such a balance reaches about $10\,\mathrm{mg{\cdot}cm^2}$, the value of the other examples. Balances of men's wristwatches usually lie between 5 and 20 mg·cm².

Moment of Inertia of Screws and Masselottes

(3.18) MASTER

$$\Delta J = N \cdot m_\mathrm{S} \cdot r_\mathrm{S}^2$$

ΔJ	change in moment of inertia (increase due to the screws)	$\mathrm{mg{\cdot}cm^2}$
N	number (of identical screws or masselottes)	1
m_S	mass of one screw	mg
r_S	distance of its centre of gravity from the balance axis	mm

Small masses on the rim count as point masses. Screws are always fitted or altered in opposite pairs so that the balance remains in poise.

Valid: screw small compared with its distance from the axis.

EXAMPLE Two opposite screws of 0.5 mg each are fitted in a balance. Their centre of gravity lies 5 mm from the axis.

Given: $N = 2$; $m_\mathrm{S} = 0.5\,\mathrm{mg}$; $r_\mathrm{S} = 5\,\mathrm{mm}$

$$\Delta J = N \cdot m_\mathrm{S} \cdot r_\mathrm{S}^2$$
$$= 2 \cdot 0.5\,\mathrm{mg} \cdot (5\,\mathrm{mm})^2$$
$$= 0.25\,\mathrm{mg{\cdot}cm^2}$$

The two screws add $0.25\,\mathrm{mg{\cdot}cm^2}$.

Rate Change due to the Moment of Inertia

(3.19) APPROXIMATION MASTER

$$\Delta G \approx -\frac{\Delta J}{2 \cdot J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta J \approx -\dfrac{2 \cdot J \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$

A more inert balance oscillates more slowly; hence the minus sign. 1 % more moment of inertia costs 432 s per day.

Valid: small changes (up to a few percent); balance spring unchanged.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
ΔJ	change in moment of inertia (+ means: more inert)	$\mathrm{mg{\cdot}cm^2}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE The two screws from the previous example are fitted to a balance with $10\,\mathrm{mg{\cdot}cm^2}$.

Given: $\Delta J = 0.25\,\mathrm{mg{\cdot}cm^2}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx -\frac{\Delta J}{2 \cdot J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{0.25\,\mathrm{mg{\cdot}cm^2}}{2 \cdot 10\,\mathrm{mg{\cdot}cm^2}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -1080\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The watch becomes 18 minutes per day slower. Conversely, a large losing rate can be made up with two lighter screws.

Fine Regulation with Timing Screws

(3.20) APPROXIMATION MASTER

$$\Delta G \approx -\frac{N \cdot m_\mathrm{S} \cdot r_\mathrm{S} \cdot \Delta r}{J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

Free-sprung balances without a regulator are regulated by moving masses on the rim inwards or outwards: timing screws are turned, eccentric masselottes rotated. Outwards means more inert, and the watch becomes slower.

Valid: small shifts; all N masses adjusted equally. Unequal adjustment creates an out-of-poise error.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	number (of identical screws or masselottes)	1
m_S	mass of one screw	mg
r_S	distance of its centre of gravity from the balance axis	mm
Δr	shift of the centre of gravity (+ means: outwards)	mm
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE Two timing screws of 1.5 mg each sit 4.5 mm from the axis on a balance with $10\,\mathrm{mg{\cdot}cm^2}$. Both are turned out by 0.01 mm.

Given: $N = 2$; $m_\mathrm{S} = 1.5\,\mathrm{mg}$; $r_\mathrm{S} = 4.5\,\mathrm{mm}$; $\Delta r = 0.01\,\mathrm{mm}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx -\frac{N \cdot m_\mathrm{S} \cdot r_\mathrm{S} \cdot \Delta r}{J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{2 \cdot 1.5\,\mathrm{mg} \cdot 4.5\,\mathrm{mm} \cdot 0.01\,\mathrm{mm}}{10\,\mathrm{mg{\cdot}cm^2}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -11.7\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

One hundredth of a millimetre makes about 12 s per day. Turning them in has the same effect in the other direction.

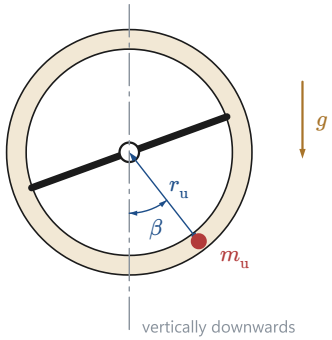


Fig. 3.9 Out-of-poise in a vertical position: excess weight m_u at distance r_u, angle β

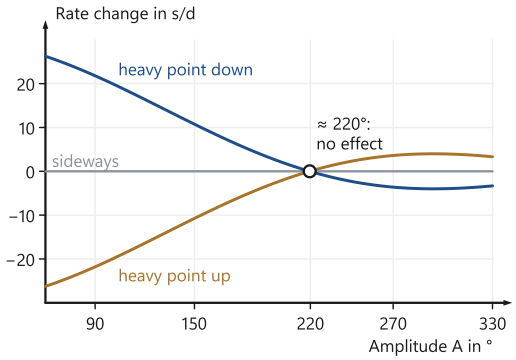


Fig. 3.10 Rate change due to 10 µg excess weight on the rim (example for formula (3.21)), as a function of amplitude

Rate Change due to Out-of-Poise (Centre-of-Gravity Error)

(3.21) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{k_A \cdot m_u \cdot g \cdot r_u \cdot \cos\beta}{2 \cdot D} \cdot 86\,400\ \frac{s}{d}$$

In the vertical positions an excess weight acts like a small pendulum on the balance. Below about 220° amplitude the watch becomes faster when the heavy point is at the bottom, above it the watch becomes slower; when it is at the top, the reverse applies. At about 220° the effect cancels out.

Valid: watch in a vertical position (balance staff horizontal), small excess weight, sinusoidal oscillation; β is measured with the balance at its rest position. In the horizontal positions the out-of-poise has no influence on the rate.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{s}{d}$
k_A	amplitude factor of the out-of-poise (from the table below)	1
m_u	excess weight (mass that shifts the centre of gravity off the axis)	µg
g	gravitational acceleration	$\frac{m}{s^2}$
r_u	distance of the excess weight from the balance axis	mm
β	angle (of the excess weight from the vertical downwards; balance at rest position)	°
D	restoring torque coefficient (of the balance spring)	$\frac{\mu N \cdot m}{rad}$

Good to know: This is how an out-of-poise error is recognized: the rates of opposite vertical positions deviate from the mean in opposite directions, and the difference grows as the amplitude falls. Poising is therefore checked at low amplitude.

EXAMPLE An excess weight of 10 µg sits on a rim 4.5 mm from the axis, exactly at the bottom in the vertical position tested ($\beta = 0°$). The watch runs at 270° amplitude ($k_A = -0.12$), the balance spring has $D = 0.632\ \frac{\mu N \cdot m}{rad}$.

Given: $k_A = -0.12$; $m_u = 10\,\mu g$; $g = 9.81\ \frac{m}{s^2}$; $r_u = 4.5\,mm$; $\beta = 0°$; $D = 0.632\ \frac{\mu N \cdot m}{rad}$

$$\Delta G \approx \frac{k_A \cdot m_u \cdot g \cdot r_u \cdot \cos\beta}{2 \cdot D} \cdot 86\,400\ \frac{s}{d}$$

$$= \frac{-0.12 \cdot 10\,\mu g \cdot 9.81\ \frac{m}{s^2} \cdot 4.5\,mm \cdot \cos(0°)}{2 \cdot 0.632\ \frac{\mu N \cdot m}{rad}} \cdot 86\,400\ \frac{s}{d}$$

$$\approx -3.6\ \frac{s}{d}$$

The out-of-poise makes the watch 3.6 s per day slower in this position, 3.6 s per day faster in the opposite position. If the amplitude falls to 150° ($k_A = 0.36$), this becomes a gain of about 11 s per day.

Amplitude Factor k_A of the Out-of-Poise

Amplitude A	very small	90°	120°	150°	180°	200°	220°	240°	270°	300°	330°
k_A	1	0.72	0.54	0.36	0.18	0.08	0.00	−0.06	−0.12	−0.13	−0.11

Calculated from the sinusoidal oscillation of the balance in the gravitational field; zero at 219.5°.

Key point: Heavy point at the bottom: below 220° amplitude the watch gains, above 220° it loses.

3.4 Regulation from Nominal and Actual Values

Regulating means matching moment of inertia and restoring torque coefficient to each other so that the nominal frequency results. The rate shows how far the frequency is still off.

Rate from the Frequency Deviation

(3.22) JOURNEYMAN

$$G = \frac{f_{\text{ist}} - f_{\text{soll}}}{f_{\text{soll}}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}}$$

G	rate (+ means: the watch gains)	$\frac{\text{s}}{\text{d}}$
f_{ist}	measured frequency	Hz
f_{soll}	nominal frequency	Hz

If the balance oscillates too fast, the motion work counts too many beats: the watch gains. The factor $86\,400\ \frac{\text{s}}{\text{d}}$ converts the relative deviation into seconds per day. The same calculation with beat rates is given in Section 6.1, for quartz in Section 7.2.

Valid: frequency averaged over the measuring time. The beat rates can be inserted instead of the frequencies.

EXAMPLE A balance with a nominal frequency of 4 Hz oscillates at 4.0005 Hz.

Given: $f_{\text{ist}} = 4.0005\,\text{Hz}$; $f_{\text{soll}} = 4\,\text{Hz}$

$$\begin{aligned} G &= \frac{f_{\text{ist}} - f_{\text{soll}}}{f_{\text{soll}}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &= \frac{4.0005\,\text{Hz} - 4\,\text{Hz}}{4\,\text{Hz}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &= 10.8\ \tfrac{\text{s}}{\text{d}} \end{aligned}$$

The watch gains 10.8 s per day. Even one eight-thousandth too much frequency is clearly visible as rate.

Rate Change from Restoring Torque Coefficient and Moment of Inertia (3.23) APPROXIMATION MASTER

$$\Delta G \approx \frac{1}{2} \cdot \left(\frac{\Delta D}{D} - \frac{\Delta J}{J} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

This is the basic formula of regulation. The regulator formula (3.16) and the formulas for screws and masselottes follow from it. To eliminate a rate G, the rate change $\Delta G = -G$ is required.

Valid: small changes (up to a few percent).

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
ΔD	change in restoring torque coefficient (+ means: stronger balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
ΔJ	change in moment of inertia (+ means: more inert balance)	$\mathrm{mg{\cdot}cm^2}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE During a repair the balance spring is replaced; the new one has a restoring torque coefficient $0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ higher than the old one with $0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. At the same time the balance with $10\,\mathrm{mg{\cdot}cm^2}$ becomes more inert by $0.010\,\mathrm{mg{\cdot}cm^2}$ due to washers.

Given: $\Delta D = 0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $\Delta J = 0.010\,\mathrm{mg{\cdot}cm^2}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx \frac{1}{2} \cdot \left(\frac{\Delta D}{D} - \frac{\Delta J}{J} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{1}{2} \cdot \left(\frac{0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}} - \frac{0.010\,\mathrm{mg{\cdot}cm^2}}{10\,\mathrm{mg{\cdot}cm^2}} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 93.5\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

As a result the watch becomes about one and a half minutes per day faster: the stronger balance spring gives 137 s, the more inert balance takes away 43 s.

Matching: Restoring Torque Coefficient for Balance and Beat Rate (3.24) MASTER

$$D = \pi^2 \cdot n_\mathrm{h}^2 \cdot J$$

This is how the balance spring is selected for the balance. Written with the frequency, the formula is $D = 4 \cdot \pi^2 \cdot f^2 \cdot J$.

Valid: insert n_h in beats per second and J in kg·m²; then D is obtained in N·m per rad.

D	restoring torque coefficient (of the balance spring; sought)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
n_h	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE A balance with $12\,\mathrm{mg{\cdot}cm^2}$ is to beat 21 600 A/h, i.e. 6 beats per second.

Given: $n_\mathrm{h} = 21\,600\ \frac{\mathrm{A}}{\mathrm{h}}$; $J = 12\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}D &= \pi^2 \cdot n_\mathrm{h}^2 \cdot J \\ &= \pi^2 \cdot \left(21\,600\ \tfrac{\mathrm{A}}{\mathrm{h}}\right)^2 \cdot 12\,\mathrm{mg{\cdot}cm^2} \\ &\approx 0.426\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}\end{aligned}$$

The balance spring needs about $0.43\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. For 28 800 A/h it would have to be almost twice as strong on the same balance.

Measuring the Moment of Inertia with an Additional Disc

(3.25) ADVANCED

$$J = \frac{J_Z}{\left(\frac{T_2}{T_1}\right)^2 - 1}$$

J	moment of inertia (of the balance; sought)	$\mathrm{mg{\cdot}cm^2}$
J_Z	known moment of inertia of the attached disc	$\mathrm{mg{\cdot}cm^2}$
T_1	period without disc	s
T_2	period with disc	s

An unknown moment of inertia cannot be weighed, but it can be measured via the period: a disc with a known moment of inertia makes the oscillation slower by a calculable amount.

Valid: the same balance spring for both measurements, disc mounted concentrically, free oscillation without escapement.

EXAMPLE A balance oscillates on its balance spring with 0.250 s. With an attached disc of $5\,\mathrm{mg{\cdot}cm^2}$ the oscillation lasts 0.306 s.

Given: $J_Z = 5\,\mathrm{mg{\cdot}cm^2}$; $T_1 = 0.250\,\mathrm{s}$; $T_2 = 0.306\,\mathrm{s}$

$$J = \frac{J_Z}{\left(\frac{T_2}{T_1}\right)^2 - 1} = \frac{5\,\mathrm{mg{\cdot}cm^2}}{\left(\frac{0.306\,\mathrm{s}}{0.250\,\mathrm{s}}\right)^2 - 1} \approx 10.0\,\mathrm{mg{\cdot}cm^2}$$

The balance has about $10\,\mathrm{mg{\cdot}cm^2}$. From J and T_1, formula (3.1) also gives the restoring torque coefficient of the balance spring.

3.5 Temperature

Heat changes the oscillator in two ways. The balance spring becomes softer because its modulus of elasticity decreases, and all parts expand. With a steel balance spring the first effect predominates by far: the watch loses heavily in warmth.

Two solutions have become established. The compensation balance has a cut rim of steel on the inside and brass on the outside; in warmth the free ends bend inwards, the moment of inertia decreases and compensates for the softer balance spring. Today self-compensating balance springs made of nickel-iron alloys such as Nivarox or of oxidized silicon are used, whose modulus of elasticity hardly changes with temperature, together with a monometallic balance.

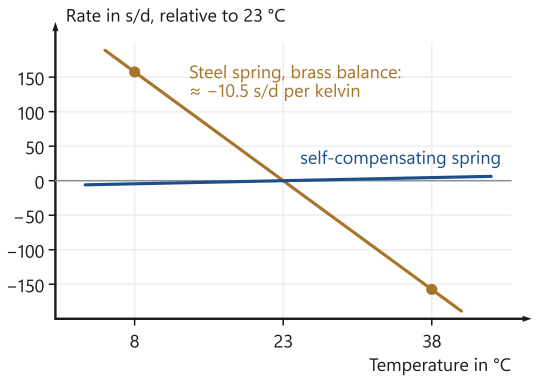


Fig. 3.11 Rate over temperature: steel balance spring with monometallic balance and self-compensating balance spring

Temperature Coefficient from Material Data

(3.26) APPROXIMATION ADVANCED

$$C \approx \frac{1}{2} \cdot (\beta_E + 3 \cdot \alpha_S - 2 \cdot \alpha_U) \cdot 86\,400\ \frac{s}{d}$$

The balance spring becomes softer in warmth (β_E is negative for steel) and at the same time larger in all dimensions; the balance becomes larger and thus more inert. The formula adds up the three contributions.

Valid: monometallic, uncut balance; uniform heating; small changes in temperature. The influence of oil, escapement and regulator pins is not included.

C	temperature coefficient of the rate (+ means: the watch gains in warmth)	$\frac{s}{d \cdot K}$
β_E	temperature coefficient of the modulus of elasticity of the balance spring	$\frac{1}{K}$
α_S	coefficient of linear expansion of the balance spring	$\frac{1}{K}$
α_U	coefficient of linear expansion of the balance	$\frac{1}{K}$

EXAMPLE A steel balance spring ($\beta_E = -240 \cdot 10^{-6}\ \frac{1}{K}$, $\alpha_S = 11.5 \cdot 10^{-6}\ \frac{1}{K}$) works with a monometallic brass balance ($\alpha_U = 18.5 \cdot 10^{-6}\ \frac{1}{K}$).

Given: $\beta_E = -240 \cdot 10^{-6}\ \frac{1}{K}$; $\alpha_S = 11.5 \cdot 10^{-6}\ \frac{1}{K}$; $\alpha_U = 18.5 \cdot 10^{-6}\ \frac{1}{K}$

$$\begin{aligned} C &\approx \frac{1}{2} \cdot (\beta_E + 3 \cdot \alpha_S - 2 \cdot \alpha_U) \cdot 86\,400\ \frac{s}{d} \\ &= \frac{1}{2} \cdot \left(-240 \cdot 10^{-6}\ \frac{1}{K} + 3 \cdot 11.5 \cdot 10^{-6}\ \frac{1}{K} - 2 \cdot 18.5 \cdot 10^{-6}\ \frac{1}{K}\right) \cdot 86\,400\ \frac{s}{d} \\ &\approx -10.5\ \frac{s}{d \cdot K} \end{aligned}$$

The watch loses about 10.5 s per day per kelvin of warming, i.e. more than 5 minutes per day between 8 °C and 38 °C. Almost all of it comes from the modulus of elasticity.

Materials of Balance Spring and Balance (Guide Values at 20 °C)

Material	Use	β_E in $10^{-6}\ \frac{1}{K}$	α in $10^{-6}\ \frac{1}{K}$
carbon steel, hardened	balance spring of older watches	−200 … −270	11 … 13
Nivarox and similar nickel-iron alloys	self-compensating balance spring	about 0, adjustable	um 8
silicon, uncoated	for comparison; not compensated without oxide layer	um −60	2.6
silicon with oxide layer	self-compensating balance spring	about 0, adjustable	um 2.6
brass	monometallic balance		18 … 21
beryllium bronze (Glucydur)	monometallic balance		17

For self-compensating balance springs, the manufacturer sets β_E so that it just compensates for the expansion of balance spring and balance.

Temperature Coefficient from Measurement

(3.27) MASTER

$$C = \frac{G_2 - G_1}{\vartheta_2 - \vartheta_1}$$

REARRANGED $G_2 = G_1 + C \cdot (\vartheta_2 - \vartheta_1)$

Testing is done at two temperatures in the same position and with the same state of wind. Usual values are 8 °C and 38 °C; the denominator is then 30 K. In chronometer testing both temperatures are fixed (Section 6.4).

Valid: mean coefficient between the two test temperatures; the watch must be thoroughly warmed or cooled before each measurement.

C	temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}}$
G_2	rate at the higher temperature	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	rate at the lower temperature	$\frac{\mathrm{s}}{\mathrm{d}}$
ϑ_2	higher test temperature	$^\circ\mathrm{C}$
ϑ_1	lower test temperature	$^\circ\mathrm{C}$

EXAMPLE A watch gains 6.0 s per day at 8 °C and 1.5 s per day at 38 °C, each in the same position.

Given: $G_2 = 1.5\,\frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 6.0\,\frac{\mathrm{s}}{\mathrm{d}}$; $\vartheta_2 = 38^\circ\mathrm{C}$; $\vartheta_1 = 8^\circ\mathrm{C}$

$$\begin{aligned} C &= \frac{G_2 - G_1}{\vartheta_2 - \vartheta_1} \\ &= \frac{1.5\,\frac{\mathrm{s}}{\mathrm{d}} - 6.0\,\frac{\mathrm{s}}{\mathrm{d}}}{38^\circ\mathrm{C} - 8^\circ\mathrm{C}} \\ &= -0.15\,\frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}} \end{aligned}$$

The watch loses 0.15 s per day per kelvin of warming. Chronometer testing according to ISO 3159 permits up to 0.6 s per day per kelvin in either direction for movements over 20 mm in diameter.

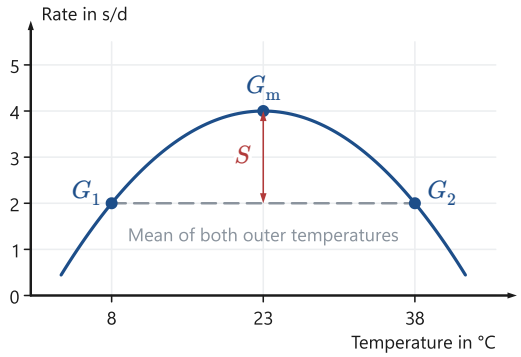


Fig. 3.12 Middle temperature error S: at the middle temperature the watch gains, although the rates G_1 and G_2 at 8 °C and 38 °C are equal

Middle Temperature Error

(3.28) ADVANCED

$$S = G_{\mathrm{m}} - \frac{G_1 + G_2}{2}$$

The compensation balance compensates almost linearly, but the steel balance spring does not soften linearly. If the rate is correct at the two outer temperatures, the watch gains in between. Self-compensating balance springs hardly have this error any more.

Valid: three measurements in the same position; middle temperature exactly between the two outer ones.

S	middle temperature error (+ means: the watch gains at the middle temperature)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{m}	rate at the middle temperature (23 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	rate at the lower temperature (8 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_2	rate at the higher temperature (38 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE A pocket watch with compensation balance and steel balance spring gains 2.0 s per day at both 8 °C and 38 °C, but 4.0 s per day at 23 °C.

Given: $G_{\mathrm{m}} = 4.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_2 = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} S &= G_{\mathrm{m}} - \frac{G_1 + G_2}{2} \\ &= 4.0\ \tfrac{\mathrm{s}}{\mathrm{d}} - \frac{2.0\ \frac{\mathrm{s}}{\mathrm{d}} + 2.0\ \frac{\mathrm{s}}{\mathrm{d}}}{2} \\ &= 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The middle temperature error is 2 s per day; a few seconds per day are typical for this design. The temperature coefficient according to formula (3.27) is zero here and does not reveal the error.

Key point: Steel balance spring with plain balance: about 11 s per day loss per kelvin of warming.

3.6 Energy and Quality Factor

The oscillating balance is an energy store. In each oscillation a small part of the energy is lost through air and bearing friction; the escapement must supply exactly this part. The smaller the loss, the less the escapement disturbs the rate.

Energy of the Oscillating Balance

(3.29) MASTER

$$W = \frac{D \cdot A^2}{2}$$

At the turning point the entire energy is in the tensioned balance spring, at the rest position in the motion of the balance: $W = \frac{1}{2} \cdot J \cdot \omega_{\max}^2$. The energy increases with the square of the amplitude. It is the energy of a torsion spring according to formula (XL 2.26).

Valid: insert the amplitude in radians: degrees times $\frac{\pi}{180}$.

W	energy (of the oscillation)	$\mu\mathrm{J}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
A	amplitude (in radians)	rad

EXAMPLE What energy is contained in a balance with $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ at 270° amplitude (4.71 rad)?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $A = 4.71\,\mathrm{rad}$

$$\begin{aligned} W &= \frac{D \cdot A^2}{2} \\ &= \frac{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \cdot (4.71\,\mathrm{rad})^2}{2} \\ &\approx 7.01\,\mu\mathrm{J} \end{aligned}$$

About 7 µJ. At 220° it would be only 4.7 µJ, two thirds of this.

Quality Factor

(3.30) ADVANCED

$$Q = \frac{2 \cdot \pi \cdot W}{\Delta W}$$

Q	quality factor	1
W	energy (of the oscillation)	μJ
ΔW	energy loss per full oscillation	μJ

The quality factor compares the stored energy with the loss per oscillation. A high quality factor means: the oscillator needs little replenishment and is little disturbed by the escapement.

Valid: weakly damped oscillator (Q much greater than 1). For the balance, Q depends somewhat on amplitude and position.

EXAMPLE The balance from the previous example loses 0.176 µJ per oscillation.

Given: $W = 7.01\,\mu J$; $\Delta W = 0.176\,\mu J$

$$\begin{aligned} Q &= \frac{2 \cdot \pi \cdot W}{\Delta W} \\ &= \frac{2 \cdot \pi \cdot 7.01\,\mu J}{0.176\,\mu J} \\ &\approx 250 \end{aligned}$$

A quality factor of around 250 is usual for a wristwatch. A good seconds pendulum reaches a few thousand to over ten thousand, a watch quartz several tens of thousands.

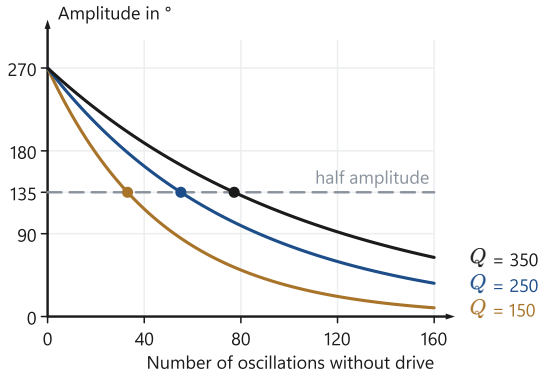


Fig. 3.13 Free decay without drive: the amplitude halves after about $0.22 \cdot Q$ oscillations

Free Decay without Drive

(3.31) APPROXIMATION ADVANCED

$$A_N \approx A_0 \cdot e^{-\frac{\pi \cdot N_S}{Q}}$$

REARRANGED $$Q \approx \frac{\pi \cdot N_S}{\ln\left(\frac{A_0}{A_N}\right)}$$

A_N	amplitude after N_S oscillations	°
A_0	amplitude at the start	°
N_S	number of full oscillations (without drive)	1
Q	quality factor	1

Without drive the oscillation decays. The amplitude halves after about $0.22 \cdot Q$ oscillations. Conversely, a decay test gives the quality factor.

Valid: friction proportional to speed (air friction), Q constant during the test. Bearing friction makes the amplitude fall faster towards the end.

EXAMPLE A balance with $Q = 250$ oscillates at 270° and is no longer driven. What is the amplitude after 55 oscillations?

Given: $A_0 = 270°$; $N_S = 55$; $Q = 250$

$$\begin{aligned} A_N &\approx A_0 \cdot e^{-\frac{\pi \cdot N_S}{Q}} \\ &= 270° \cdot e^{-\frac{\pi \cdot 55}{250}} \\ &\approx 135° \end{aligned}$$

After 55 oscillations, i.e. after just under 14 s at 4 Hz, the amplitude has fallen to half.

Power Requirement of the Balance

(3.32) MASTER

$$P = \Delta W \cdot f$$

P	power (that the escapement must deliver to the balance)	µW
ΔW	energy loss per full oscillation	µJ
f	frequency (full oscillations per second)	Hz

In equilibrium the escapement replaces exactly the loss in each oscillation. Written with the quality factor: $P = \frac{2 \cdot \pi \cdot f \cdot W}{Q}$. Here P is the power, not the thread pitch.

Valid: steady state with constant amplitude.

EXAMPLE The balance from the previous examples loses 0.176 µJ per oscillation and oscillates at 4 Hz.

Given: $\Delta W = 0.176\,\mu J$; $f = 4\,Hz$

$$\begin{aligned} P &= \Delta W \cdot f \\ &= 0.176\,\mu J \cdot 4\,Hz \\ &= 0.704\,\mu W \end{aligned}$$

The balance needs about 0.7 µW. The barrel must supply several times this, because gear train and escapement pass on only part of it (Chapters 1 and 5).

Key point: Double the amplitude, four times the energy.

3.7 Isochronism

Isochronous means of equal duration: an ideal oscillator needs the same time for large and small oscillations. Formula (3.1) satisfies this, the real watch only approximately. Because the amplitude decreases as the mainspring runs down and in the vertical positions, every isochronism error shows up as a rate difference between the fully wound and the almost run-down watch and between the positions.

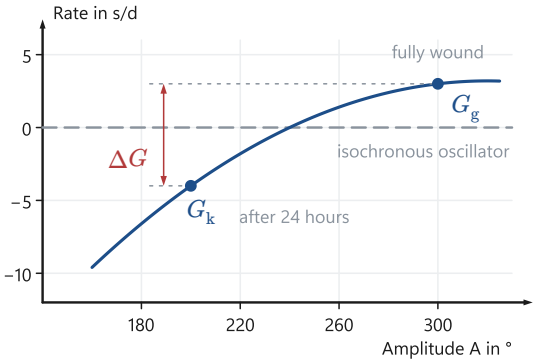


Fig. 3.14 Rate–amplitude diagram (principle): the isochronism error ΔG is the rate difference between small and large amplitude

The **isochronism error** is the rate difference between small and large amplitude, measured in the same position and at the same temperature (Fig. 3.14). The simplest way is to measure fully wound and after 24 hours of running; the formula and an example are given in Section 6.3, formula (6.14). If the watch becomes slower at small amplitude, this is often due to too much play of the balance spring between the regulator pins. Further causes are listed in the table.

What Disturbs Isochronism

Cause	Effect at small amplitude	Remedy
play of the balance spring between the regulator pins	losing: the balance spring rests against the pin for a smaller part of the oscillation and acts longer on average	keep play small, place the balance spring centrally between the pins
escapement (unlocking before, impulse around the rest position)	usually losing, because the engagement takes up a larger part of the oscillation	free-running escapement, high amplitude, high quality factor
out-of-poise of the balance	in the vertical positions gaining or losing depending on the position of the heavy point (Section 3.3)	poise
eccentric breathing of the flat balance spring	centre of gravity of the balance spring shifts, the pivots are loaded laterally; the rate depends on amplitude and position	Breguet terminal curve or balance spring with adapted shape
position of the inner pinning point at the collet	gaining or losing in the vertical positions depending on the position of the pinning point	pinning point according to manufacturer's specification; used deliberately in fine adjustment

The effects are superimposed. In fine adjustment they are matched so that they cancel each other as far as possible.

The Breguet terminal curve leads the outermost coil upwards and in a tighter arc to the stud. With the correct shape the balance spring opens and closes evenly on all sides, its centre of gravity remains on the axis and the pivots are not pushed sideways. There are design rules for the shape of the terminal curve, but no simple formula; it is bent according to a template and checked against the rate.

Key point: An oscillator is isochronous if the rate does not depend on the amplitude.

4 Pendulum

The pendulum is the oscillator of the clock. Its period depends only on its length and on gravitational acceleration, not on the mass of the bob, and at small amplitudes hardly on the amplitude. That is why a pendulum clock is regulated by its length: bob higher, clock faster.

The formulas apply to small amplitudes of a few degrees, as usual in clocks. The effects of larger amplitudes, temperature and location are covered in Sections 4.4 and 4.5.

4.1 Simple pendulum

The simple pendulum is the model we calculate with: a point mass on a weightless thread. Real pendulums deviate only slightly from it; how to calculate them exactly is shown in Section 4.3.

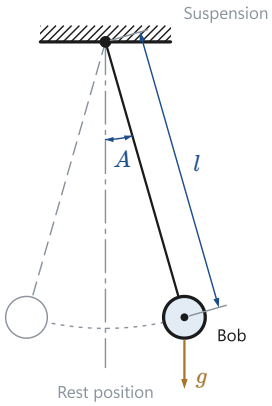


Fig. 4.1 Simple pendulum: length l to the centre of gravity of the bob, amplitude A (exaggerated for clarity)

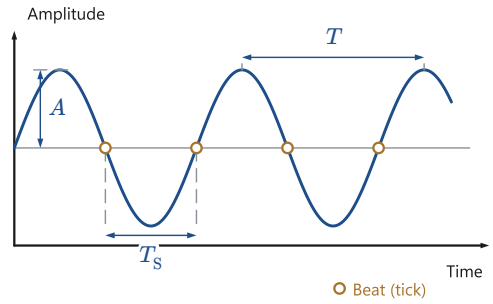


Fig. 4.2 A beat is a vibration (half-oscillation): $T_S = \frac{1}{2} T$

Period

(4.1) JOURNEYMAN

$$T = 2 \cdot \pi \cdot \sqrt{\frac{l}{g}}$$

REARRANGED $l = \frac{g \cdot T^2}{4 \cdot \pi^2}$

T	Period (full oscillation, there and back)	s
l	Pendulum length (point of suspension to centre of gravity of the bob)	m
g	Gravitational acceleration (in Central Europe $9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$

The period grows with the square root of the length: a pendulum four times as long swings half as fast.

Valid: small amplitudes (up to about 5°), point-shaped bob, no friction. For a real pendulum, replace l by the equivalent pendulum length l_r (Section 4.3).

Good to know: The mass of the bob does not appear in the formula. A heavier bob does not change the period; it only makes the pendulum less sensitive to disturbances from the escapement and the gear train.

EXAMPLE A pendulum measures 0.5 m from the point of suspension to the centre of the bob. What is its period?

Given: $l = 0.5\,\mathrm{m}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{l}{g}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{0.5\,\mathrm{m}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}} \\ &\approx 1.42\,\mathrm{s} \end{aligned}$$

One full oscillation, there and back, takes about 1.42 s; each beat lasts 0.71 s.

Beat duration (vibration)

(4.2) JOURNEYMAN

$$T_\mathrm{S} = \pi \cdot \sqrt{\frac{l}{g}}$$

T_S	Beat duration (vibration, from one end point of the swing to the other)	s
l	Pendulum length	m
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$

Watchmakers count beats. A beat is a vibration (half-oscillation), i.e. half the period; each beat can be heard as a tick.

Valid: as for formula (4.1).

EXAMPLE How long does one beat last for a pendulum 994 mm long?

Given: $l = 0.994\,\mathrm{m}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} T_\mathrm{S} &= \pi \cdot \sqrt{\frac{l}{g}} \\ &= \pi \cdot \sqrt{\frac{0.994\,\mathrm{m}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}} \\ &\approx 1.00\,\mathrm{s} \end{aligned}$$

Exactly one second: a pendulum of 994 mm is a seconds pendulum, as found in regulators and many longcase clocks.

Beat rate from the period

(4.3) JOURNEYMAN

$$n_{\mathrm{h}} = \frac{2}{T}$$

REARRANGED $T = \frac{2}{n_{\mathrm{h}}}$

n_{h}	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
T	Period	s

The pendulum beats twice per full oscillation. For clocks the beat rate is often given per minute, for watches in vibrations per hour (A/h).

EXAMPLE A pendulum has a period of 1.5 s. What is its beat rate per hour?

Given: $T = 1.5\,\mathrm{s}$

$$\begin{aligned} n_{\mathrm{h}} &= \frac{2}{T} \\ &= \frac{2}{1.5\,\mathrm{s}} \\ &= \frac{2}{1{,}5\,\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &= 4800\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

4800 beats per hour, i.e. 80 beats per minute.

Pendulum length from the beat rate

(4.4) JOURNEYMAN

$$l = \frac{g}{\pi^2 \cdot n_{\mathrm{h}}^2}$$

l	Pendulum length	m
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
n_{h}	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$

This is how the length of a missing pendulum is found: calculate the beat rate from the tooth counts of the gear train (Chapter 2), then the length.

Valid: as for the period, formula (4.1); insert n_{h} in beats per second when g is in $\frac{\mathrm{m}}{\mathrm{s}^2}$.

EXAMPLE A movement is designed for 90 beats per minute, i.e. 1.5 beats per second. How long must the pendulum be?

Given: $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$; $\quad n_{\mathrm{h}} = 1.5\,\frac{1}{\mathrm{s}}$

$$\begin{aligned} l &= \frac{g}{\pi^2 \cdot n_{\mathrm{h}}^2} \\ &= \frac{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{\pi^2 \cdot \left(1.5\,\frac{1}{\mathrm{s}}\right)^2} \\ &\approx 0.442\,\mathrm{m} \end{aligned}$$

The pendulum must be about 442 mm long from the point of suspension to the centre of the bob.

Pendulum length from beats per minute (workshop formula)

(4.5) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$l = \frac{3\,578\,400}{n_{\mathrm{h}}^2}$$

l	Pendulum length in mm	mm
n_{h}	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$

The number 3 578 400 is the length of the seconds pendulum (994 mm) times 60^2. Insert values only in the units stated.

Valid: $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; l in mm, n_{h} in beats per minute.

EXAMPLE How long is the pendulum for 90 beats per minute?

Given: $n_{\mathrm{h}} = 90\ \frac{1}{\mathrm{min}}$

$$\begin{aligned} l &= \frac{3\,578\,400}{n_{\mathrm{h}}^2} \\ &= \frac{3\,578\,400}{90^2} \\ &\approx 442\,\mathrm{mm} \end{aligned}$$

About 442 mm, as with the previous formula.

Key point: A pendulum of 994 mm beats seconds. A pendulum four times as long beats half as often.

4.2 Regulating by the pendulum length

If a pendulum clock gains, the pendulum is lengthened; if it loses, it is shortened. Because the period grows with the square root of the length, 1 ‰ more length makes the oscillation 0.5 ‰ slower, which is 43 s per day.

Rate change from a change in length

(4.6) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{\Delta l}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta l \approx -\dfrac{2 \cdot l \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δl	Change in length (+ means: pendulum longer, bob lower)	mm
l	Pendulum length	mm

A longer pendulum swings more slowly and the clock loses; hence the minus sign. The factor $86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$ converts the relative change into seconds per day.

Valid: small changes (Δl much smaller than l); pendulum without a heavy rod.

Good to know: On pendulums with a heavy rod, the centre of oscillation moves less than the bob. The bob must then be moved further than calculated. Calculating with the pendulum length from the beat rate, a rod with 10 %, 20 % and 30 % of the bob mass needs about 3 %, 7 % and 10 % more travel. For the real pendulum, the factor 2.2 is therefore often used instead of 2, see formula (4.7).

EXAMPLE A longcase clock with a pendulum length of 994 mm gains 30 s per day. It must therefore become 30 s per day slower. By how much must the bob be lowered?

Given: $\Delta G = -30\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad l = 994\,\mathrm{mm}$

$$\begin{aligned} \Delta l &\approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &= -\frac{2 \cdot 994\,\mathrm{mm} \cdot \left(-30\ \frac{\mathrm{s}}{\mathrm{d}}\right)}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &\approx 0.69\,\mathrm{mm} \end{aligned}$$

The bob must go about 0.7 mm lower, i.e. the pendulum must become longer.

Bob adjustment from rate and beat rate (workshop formula)

(4.7) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$\Delta l \approx \frac{90 \cdot G}{n_{\mathrm{h}}^2}$$

Δl	Bob travel in mm (+ means: bob lower)	mm
G	Rate of the clock in seconds per day (+ means: clock gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
n_{h}	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$

If the clock gains, the bob goes lower; if it loses, it goes higher. The number 90 contains the pendulum length from the beat rate and an allowance for the real pendulum. For the simple pendulum, formulas (4.5) and (4.6) together give $\frac{2 \cdot 3\,578\,400}{86\,400} \approx 83$. With the factor 2.2 instead of 2 this becomes 91, rounded to 90.

Valid: Δl in mm, G in seconds per day, n_{h} in beats per minute. Guide value for pendulums whose rod swings with the bob; for a very light rod use 83 instead of 90. After the adjustment the rate is measured again.

EXAMPLE The longcase clock from the example for formula (4.6) beats 60 times a minute and gains 30 s per day. By how much is the bob lowered?

Given: $G = 30\ \frac{\mathrm{s}}{\mathrm{d}}$; $n_{\mathrm{h}} = 60\ \frac{1}{\mathrm{min}}$

$$\begin{aligned}\Delta l &\approx \frac{90 \cdot G}{n_{\mathrm{h}}^2} \\ &= \frac{90 \cdot 30}{60^2} \\ &= 0.75\,\mathrm{mm}\end{aligned}$$

The bob goes 0.75 mm lower. For the simple pendulum it was 0.69 mm; the difference is the allowance for the rod swinging with the bob.

Rate change with the rating nut

(4.8) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{N \cdot P}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $$N \approx -\frac{2 \cdot l \cdot \Delta G}{P \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	Turns (of the nut; + means: bob lower)	1
P	Thread pitch (of the rating nut)	mm
l	Pendulum length	mm

One full turn of the nut moves the bob by one thread pitch.

Valid: Bob rests on the nut; pendulum without a heavy rod.

EXAMPLE How much does one full turn of the rating nut (pitch 0.5 mm) change the rate of a seconds pendulum when the bob is lowered?

Given: $N = 1$; $P = 0.5\,\mathrm{mm}$; $l = 994\,\mathrm{mm}$

$$\begin{aligned}\Delta G &\approx -\frac{N \cdot P}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{1 \cdot 0.5\,\mathrm{mm}}{2 \cdot 994\,\mathrm{mm}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx -21.7\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The clock becomes about 22 s per day slower. Finer corrections are made with partial turns; many nuts carry a graduation for this.

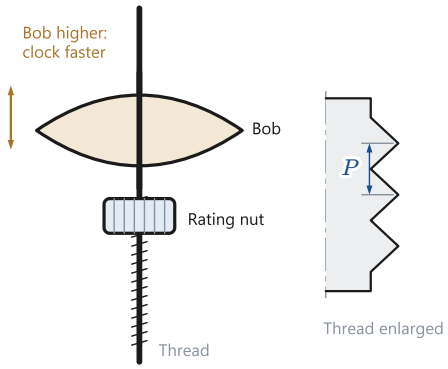


Fig. 4.3 The rating nut raises or lowers the bob by the pitch P per turn

Key point: Bob higher, clock faster. With the seconds pendulum, 1 mm gives about 43 s per day.

Timing weights. Precision pendulum clocks are finally regulated not with the nut but with small weights placed on a little tray on the pendulum rod. The pendulum keeps swinging meanwhile. Each weight added makes the clock faster, each weight removed makes it slower.

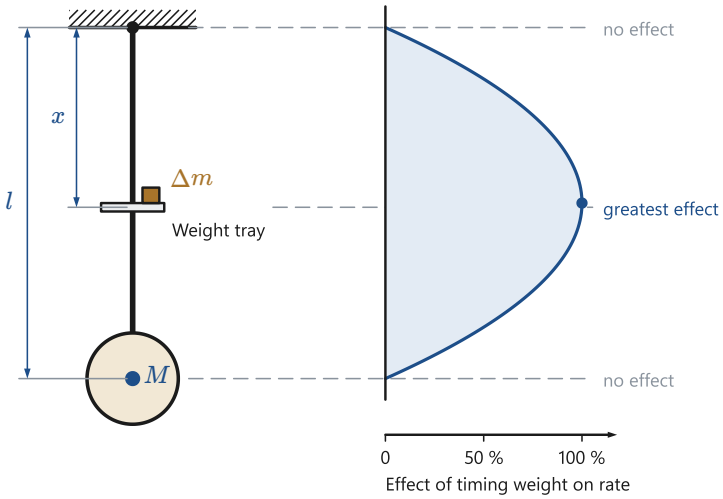


Fig. 4.4 Timing weight Δm at distance x below the suspension: the effect on the rate is greatest at half the pendulum length and vanishes at the suspension and at the centre of oscillation M

Timing weight at half the pendulum length

(4.9) APPROXIMATION JOURNEYMAN

$$\Delta G \approx \frac{\Delta m}{8 \cdot m} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $$\Delta m \approx \frac{8 \cdot m \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δm	Mass of the timing weight (+ means: added)	g
m	Mass (of the pendulum)	kg

The weight sits above the centre of oscillation and slightly shortens the effective pendulum length. Because 86 400 divided by 8 equals 10 800, the workshop uses the shorter form $\Delta m = \frac{m \cdot \Delta G}{10\,800}$ with ΔG in seconds per day.

Valid: Tray at half the pendulum length, i.e. midway between the suspension and the centre of oscillation; weight much lighter than the pendulum; mass of the pendulum essentially in the bob. With a heavy rod the weight acts somewhat more strongly.

EXAMPLE A precision pendulum clock with a pendulum mass of 6 kg loses 0.5 s per day. Which timing weight compensates for this?

Given: $m = 6\,\mathrm{kg}$; $\Delta G = 0.5\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta m &\approx \frac{8 \cdot m \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &= \frac{8 \cdot 6\,\mathrm{kg} \cdot 0.5\ \frac{\mathrm{s}}{\mathrm{d}}}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &\approx 0.28\,\mathrm{g}\end{aligned}$$

About 0.28 g. Rule of thumb: the pendulum mass divided by 10 800, here 0.56 g, gives 1 s per day.

Timing weight at any height

(4.10) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{\Delta m \cdot x}{2 \cdot m \cdot l} \cdot \left(1 - \frac{x}{l}\right) \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

At the suspension and at the centre of oscillation an added weight has no effect. In between it makes the clock faster, most strongly in the middle. Below the centre of oscillation it makes the clock slower.

Valid: Weight much lighter than the pendulum; pendulum treated as a point mass at length l.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δm	Mass of the timing weight (+ means: added)	g
x	Distance (suspension to timing weight)	mm
m	Mass (of the pendulum)	kg
l	Pendulum length (suspension to centre of oscillation)	mm

Good to know: Around half the pendulum length the effect hardly changes with height. That is why the tray belongs there: a few millimetres do not matter. A sliding regulating weight, on the other hand, does not belong in the middle, because moving it there has almost no effect.

EXAMPLE The pendulum from the previous example (6 kg) is a seconds pendulum 994 mm long. A 1 g weight is placed on it not in the middle but 331 mm below the suspension, i.e. at one third of the length. How much does the rate change?

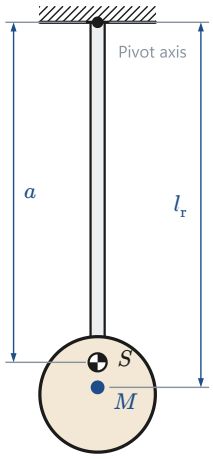
Given: $\Delta m = 1\,\mathrm{g}$; $x = 331\,\mathrm{mm}$; $m = 6\,\mathrm{kg}$; $l = 994\,\mathrm{mm}$

$$\begin{aligned}\Delta G &\approx \frac{\Delta m \cdot x}{2 \cdot m \cdot l} \cdot \left(1 - \frac{x}{l}\right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{1\,\mathrm{g} \cdot 331\,\mathrm{mm}}{2 \cdot 6\,\mathrm{kg} \cdot 994\,\mathrm{mm}} \cdot \left(1 - \frac{331\,\mathrm{mm}}{994\,\mathrm{mm}}\right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 1.60\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The clock becomes about 1.6 s per day faster. At half the length it would be 1.8 s.

4.3 Compound pendulum

A real pendulum has a rod with its own mass and a bob of finite size. It swings like a simple pendulum of equivalent length l_r. This length reaches to the centre of oscillation, which lies slightly below the centre of gravity.



S centre of gravity · M centre of oscillation

Fig. 4.5 Compound pendulum: centre of gravity S at distance a, centre of oscillation M at the equivalent length l_r

Period of the compound pendulum

(4.11) MASTER

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J_\mathrm{A}}{m \cdot g \cdot a}}$$

Valid: small amplitudes, no friction, rigid pendulum.

T	Period	s
J_A	Moment of inertia about the pivot axis (suspension)	$\mathrm{kg \cdot m^2}$
m	Mass (of the whole pendulum)	kg
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s^2}}$
a	Distance (pivot axis to centre of gravity)	m

EXAMPLE A uniform rod (length 1 m, mass 0.6 kg) swings about its upper end. Its moment of inertia about the end is $J_\mathrm{A} = \frac{1}{3} \cdot m \cdot L^2 = 0.2\,\mathrm{kg \cdot m^2}$; the centre of gravity lies in the middle.

Given: $J_\mathrm{A} = 0.2\,\mathrm{kg \cdot m^2}$; $m = 0.6\,\mathrm{kg}$; $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s^2}}$; $a = 0.5\,\mathrm{m}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{J_\mathrm{A}}{m \cdot g \cdot a}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{0.2\,\mathrm{kg \cdot m^2}}{0.6\,\mathrm{kg} \cdot 9.81\,\frac{\mathrm{m}}{\mathrm{s^2}} \cdot 0.5\,\mathrm{m}}} \\ &\approx 1.64\,\mathrm{s} \end{aligned}$$

1.64 s; a simple pendulum of 1 m would have 2.01 s. The rod swings faster because part of its mass lies close to the pivot axis.

Equivalent pendulum length

(4.12) MASTER

$$l_\mathrm{r} = \frac{J_\mathrm{A}}{m \cdot a}$$

The equivalent pendulum length is the length of the simple pendulum with the same period. With l_r instead of l, all formulas from Sections 4.1 and 4.2 apply.

l_r	Equivalent pendulum length (pivot axis to centre of oscillation)	m
J_A	Moment of inertia about the pivot axis	$\mathrm{kg \cdot m^2}$
m	Mass (of the whole pendulum)	kg
a	Distance (pivot axis to centre of gravity)	m

EXAMPLE How long is the equivalent simple pendulum for the rod from the previous example?

Given: $J_\mathrm{A} = 0.2\,\mathrm{kg \cdot m^2}$; $m = 0.6\,\mathrm{kg}$; $a = 0.5\,\mathrm{m}$

$$\begin{aligned} l_\mathrm{r} &= \frac{J_\mathrm{A}}{m \cdot a} \\ &= \frac{0.2\,\mathrm{kg \cdot m^2}}{0.6\,\mathrm{kg} \cdot 0.5\,\mathrm{m}} \\ &\approx 0.667\,\mathrm{m} \end{aligned}$$

The rod swings like a simple pendulum of 667 mm, i.e. like a pendulum two thirds of its length.

Moment of inertia about the suspension (parallel axis theorem)

(4.13) MASTER

$$J_A = J_S + m \cdot a^2$$

This converts the moment of inertia of a component from its centre of gravity to the pivot axis of the pendulum. For several components the contributions are added.

J_A	Moment of inertia about the pivot axis	$\mathrm{kg \cdot m^2}$
J_S	Moment of inertia about the parallel axis through the centre of gravity	$\mathrm{kg \cdot m^2}$
m	Mass (of the component)	kg
a	Distance (between the two axes)	m

EXAMPLE A bob (disc, mass 1.2 kg, radius 60 mm) hangs with its centre 0.9 m below the suspension. About its own centre it has $J_S = \frac{1}{2} \cdot m \cdot r^2 = 0.002\,16\,\mathrm{kg \cdot m^2}$.

Given: $J_S = 0.002\,16\,\mathrm{kg \cdot m^2}$; $m = 1.2\,\mathrm{kg}$; $a = 0.9\,\mathrm{m}$

$$\begin{aligned} J_A &= J_S + m \cdot a^2 \\ &= 0.002\,16\,\mathrm{kg \cdot m^2} + 1.2\,\mathrm{kg} \cdot (0.9\,\mathrm{m})^2 \\ &\approx 0.974\,\mathrm{kg \cdot m^2} \end{aligned}$$

Almost the entire moment of inertia comes from the distance to the suspension. That is why a pendulum with a heavy bob and a light rod behaves almost like a simple pendulum.

4.4 Amplitude and circular error

The formulas from Section 4.1 are exact only for infinitely small amplitudes. The further the pendulum swings, the longer an oscillation lasts. The clock only feels this circular error when the amplitude changes, for example because the driving force drops or the oil in the escapement ages.

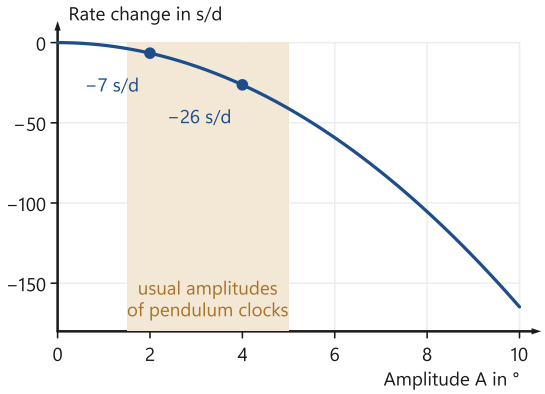


Fig. 4.6 Circular error: the larger the amplitude, the slower the pendulum swings (rate compared with very small amplitude)

Period at larger amplitude

(4.14) APPROXIMATION ADVANCED

$$T \approx T_0 \cdot \left(1 + \frac{A^2}{16}\right)$$

Valid: Amplitudes up to about 20°. Insert A in radians: degrees times $\frac{\pi}{180}$.

T	Period (at amplitude A)	s
T_0	Period at very small amplitude (formula (4.1))	s
A	Amplitude (swing of the pendulum, in radians)	rad

EXAMPLE A seconds pendulum ($T_0 = 2\,\mathrm{s}$) swings with an amplitude of 4°, i.e. 0.0698 rad.

Given: $T_0 = 2\,\mathrm{s}$; $A = 0.0698\,\mathrm{rad}$

$$\begin{aligned} T &\approx T_0 \cdot \left(1 + \frac{A^2}{16}\right) \\ &= 2\,\mathrm{s} \cdot \left(1 + \frac{(0.0698\,\mathrm{rad})^2}{16}\right) \\ &\approx 2.0006\,\mathrm{s} \end{aligned}$$

Each oscillation lasts 0.6 ms longer; compared with a very small amplitude, the clock therefore loses about 26 s per day.

Rate change when the amplitude changes

(4.15) APPROXIMATION ADVANCED

$$\Delta G \approx -\frac{A_2^2 - A_1^2}{16} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

If the amplitude decreases, the clock becomes faster.

Valid: as above.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
A_2	Amplitude after (radians)	rad
A_1	Amplitude before (radians)	rad

EXAMPLE The amplitude drops from 2.5° (0.0436 rad) to 2.0° (0.0349 rad).

Given: $A_2 = 0.0349\,\mathrm{rad}$; $A_1 = 0.0436\,\mathrm{rad}$

$$\begin{aligned} \Delta G &\approx -\frac{A_2^2 - A_1^2}{16} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{(0.0349\,\mathrm{rad})^2 - (0.0436\,\mathrm{rad})^2}{16} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 3.7\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The clock gains about 3.7 s per day. A constant drive keeps the amplitude, and with it the rate, constant; this is one reason why precision pendulum clocks are weight-driven.

4.5 Temperature and location

Heat expands the pendulum rod and the clock loses. And because gravitational acceleration is not the same everywhere, a pendulum clock must be regulated again after it has been moved to another place.

Rate change due to temperature

(4.16) APPROXIMATION MASTER

$$\Delta G \approx -\frac{\alpha \cdot \Delta\vartheta}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

Warmer means longer, longer means slower.

Valid: Pendulum without temperature compensation; expansion of bob and suspension neglected.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
α	Coefficient of linear expansion (of the pendulum rod)	$\frac{1}{\mathrm{K}}$
$\Delta\vartheta$	Temperature change	K

EXAMPLE A pendulum clock with a steel rod ($\alpha = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) stands 10 K warmer in summer than in winter.

Given: $\alpha = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$; $\Delta\vartheta = 10\,\mathrm{K}$

$$\begin{aligned}\Delta G &\approx -\frac{\alpha \cdot \Delta\vartheta}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}} \cdot 10\,\mathrm{K}}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx -4.97\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

In summer the clock becomes about 5 s per day slower. With a wooden rod the error shrinks to about one third, with an Invar rod to about one tenth to one sixth.

Coefficients of linear expansion for pendulum rods (guide values at 20 °C)

Material	α in $10^{-6}\ \frac{1}{\mathrm{K}}$
Zinc	30
Aluminium	23
Brass	18 … 21
Steel, unalloyed	11 … 13
Wood (spruce, along the grain)	3 … 5
Invar (FeNi36)	1.2 … 2
Fused silica	0.5

The values vary with alloy, heat treatment and, for wood, with moisture.

Temperature compensation with two materials

(4.17) ADVANCED

$$\alpha_1 \cdot l_1 = \alpha_2 \cdot l_2$$

REARRANGED $$l_2 = \frac{\alpha_1 \cdot l_1}{\alpha_2}$$

In the gridiron pendulum and the zinc-steel pendulum, a material with high expansion raises the bob by as much as the steel lowers it.

Valid: simplified, rods only. The expansion of the bob and the position of the centre of oscillation shift the result slightly; the pendulum is therefore fine-adjusted after it has been built.

α_1	Coefficient of linear expansion of the parts acting downwards (steel)	$\frac{1}{\mathrm{K}}$
l_1	Total length of the parts acting downwards	m
α_2	Coefficient of linear expansion of the parts acting upwards (zinc, brass)	$\frac{1}{\mathrm{K}}$
l_2	Total length of the parts acting upwards	m

EXAMPLE A zinc-steel pendulum has a total of 1.2 m of steel ($\alpha_1 = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) expanding downwards. How long must the zinc tube ($\alpha_2 = 30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) be?

Given: $\alpha_1 = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$; $\quad l_1 = 1.2\,\mathrm{m}$; $\quad \alpha_2 = 30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$

$$\begin{aligned} l_2 &= \frac{\alpha_1 \cdot l_1}{\alpha_2} \\ &= \frac{11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}} \cdot 1.2\,\mathrm{m}}{30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}} \\ &\approx 0.46\,\mathrm{m} \end{aligned}$$

The zinc tube must be about 46 cm long.

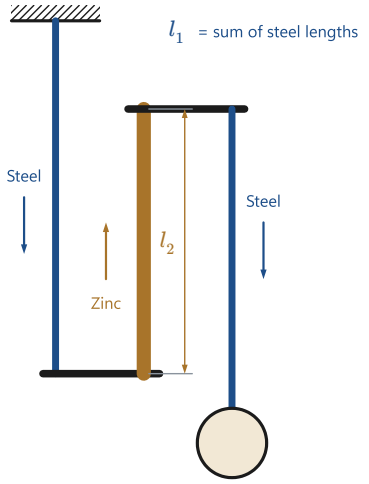


Fig. 4.7 Zinc-steel pendulum, drawn apart for clarity: steel lowers the bob, zinc raises it

Rate change due to a different location

(4.18) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{\Delta g}{2 \cdot g} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

Where gravitational acceleration is greater, the pendulum swings faster.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δg	Change in gravitational acceleration (new location minus old location)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
g	Gravitational acceleration at the old location	$\frac{\mathrm{m}}{\mathrm{s}^2}$

EXAMPLE A longcase clock regulated in Hamburg ($g = 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$) is moved to Munich ($g = 9.8074\ \frac{\mathrm{m}}{\mathrm{s}^2}$), i.e. $\Delta g = -0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

Given: $\Delta g = -0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $g = 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned}\Delta G &\approx \frac{\Delta g}{2 \cdot g} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{-0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx -28.2\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

In Munich the clock loses about 28 s per day and must be regulated again there.

Gravitational acceleration from latitude and altitude

(4.19) APPROXIMATION ADVANCED

$$g \approx 9.780\,327\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2 \varphi - 5.8 \cdot 10^{-6} \cdot \sin^2 (2 \cdot \varphi)\right) - 3.086 \cdot 10^{-6}\ \frac{1}{\mathrm{s}^2} \cdot h$$

The formula gives the normal gravity of an ideal Earth. Local ground conditions cause deviations of up to about $0.0005\ \frac{\mathrm{m}}{\mathrm{s}^2}$, more in high mountains; this is accurate enough for regulating a clock.

Valid: International gravity formula of 1980 with free-air correction for altitude.

g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
φ	Geographic latitude	$^\circ$
h	Altitude (above sea level)	m

EXAMPLE Gravitational acceleration in Munich: latitude 48.14°, altitude 520 m.

Given: $\varphi = 48.14^\circ$; $h = 520\,\mathrm{m}$

$$\begin{aligned}g &\approx 9.780\,327\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2 \varphi - 5.8 \cdot 10^{-6} \cdot \sin^2 (2 \cdot \varphi)\right) - 3.086 \cdot 10^{-6}\ \frac{1}{\mathrm{s}^2} \cdot h \\ &= 9.780\,327\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2 (48.14^\circ) - 5.8 \cdot 10^{-6} \cdot \sin^2 (2 \cdot 48.14^\circ)\right) - 3.086 \cdot 10^{-6}\ \frac{1}{\mathrm{s}^2} \cdot 520\,\mathrm{m} \\ &\approx 9.8074\ \frac{\mathrm{m}}{\mathrm{s}^2}\end{aligned}$$

About $9.8074\ \frac{\mathrm{m}}{\mathrm{s}^2}$; Hamburg at sea level has $9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

5 Escapements

The escapement sits between the gear train and the oscillator. It stops the gear train, releases it tooth by tooth in step with the balance or pendulum, and in doing so supplies the oscillator with the energy it loses through friction. The escapement therefore counts the oscillations. How accurately the timepiece keeps time also depends on how little the escapement disturbs the oscillator.

The chapter begins with the quantities common to all escapements. It continues with the geometry of the Swiss lever escapement, the energy balance, the escapement noises evaluated by a timing machine, and clock escapements.

5.1 Basic quantities of all escapements

Every escapement releases the gear train in equal steps and drives the oscillator in the process. The terms are the same for all types. In this chapter the angles are uniformly named γ at the escape wheel, β at the pallet fork, λ and A at the balance. In the lever escapement the escape wheel is also called the lever escape wheel; older books sometimes call it the scape wheel.

Terms of the escapement

Term	Meaning	Angle
Lock	A tooth rests on the locking face and the wheel is stationary. Measured as the travel of the pallet fork up to the start of impulse.	β_{R} at the pallet fork
Unlocking	The oscillator pushes the pallet out of lock and releases the tooth.	Part of λ
Impulse	The tooth slides over the impulse face and drives the oscillator via the pallet fork.	γ_{H} at the wheel, β_{H} at the pallet fork
Drop	The wheel turns freely until the next tooth lands on its locking face.	γ_{F} at the wheel
Draw	Inclination of the locking face by which the wheel pulls the pallet fork against the banking.	ψ at the pallet
Run	Travel of the pallet fork after the drop up to the banking; a safety margin for the lock.	β_{V} at the pallet fork
Lift angle	Travel of the balance while the impulse pin is in the fork.	λ at the balance
Supplementary arc	Travel of the oscillator outside the impulse, up to the point of reversal.	φ_{E} at the balance or pendulum

The balance needs its lift angle for unlocking and impulse together.

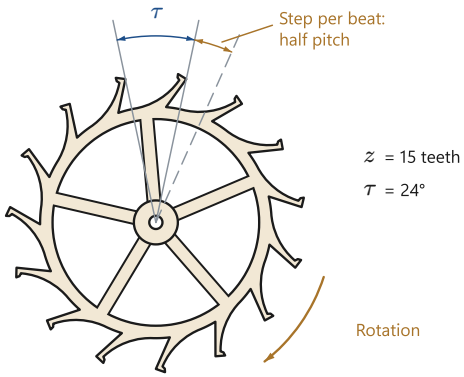


Fig. 5.1 Angular pitch τ of the escape wheel; with each beat the wheel advances by half the pitch

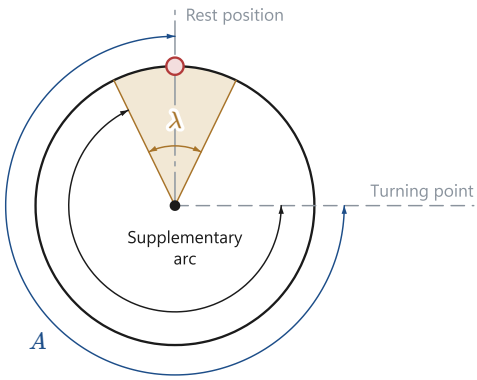


Fig. 5.2 Travel of the impulse pin from the rest position to the point of reversal: amplitude A, lift angle λ and supplementary arc

Angular pitch of the escape wheel

(5.1) JOURNEYMAN

$$\tau = \frac{360^\circ}{z}$$

τ	Angular pitch (of the escape wheel; angle from tooth to tooth)	°
z	Number of teeth (of the escape wheel)	1

The angular pitch is the angle from tooth to tooth. With each beat the wheel advances by half the pitch, because each tooth works on both pallets in turn.

Valid: Lever, cylinder, Graham and recoil escapements. In the chronometer escapement the wheel advances only on every second beat, and then by the full pitch.

EXAMPLE What is the angular pitch of an escape wheel with 15 teeth?

Given: $z = 15$

$$\begin{aligned}\tau &= \frac{360^\circ}{z} \\ &= \frac{360^\circ}{15} \\ &= 24^\circ\end{aligned}$$

The angular pitch is 24°, so the step per beat is 12°.

Beat rate and rotational speed of the escape wheel

(5.2) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot z \cdot n_\mathrm{H}$$

REARRANGED $\quad n_\mathrm{H} = \dfrac{n_\mathrm{h}}{2 \cdot z}$

n_h	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z	Number of teeth (of the escape wheel)	1
n_H	Rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$

In one revolution of the escape wheel each tooth passes once and allows two beats. The formula links gear train and oscillator: from beat rate and number of teeth it follows how fast the escape wheel must turn (Chapter 2). The same calculation using the numbers of teeth of the gear train is given in Section 2.2.

Valid: all escapements in which one tooth passes per oscillation, including the cylinder and chronometer escapements.

Good to know: With a seconds pendulum and 30 teeth, the escape wheel turns exactly once a minute. Its arbor can therefore carry the seconds hand.

EXAMPLE A pendulum movement makes 4800 beats per hour, i.e. 80 per minute. The escape wheel has 32 teeth. How fast does it turn?

Given: $n_\mathrm{h} = 4800\ \frac{\mathrm{A}}{\mathrm{h}}; \quad z = 32$

$$\begin{aligned}n_\mathrm{H} &= \frac{n_\mathrm{h}}{2 \cdot z} \\ &= \frac{4800\ \frac{\mathrm{A}}{\mathrm{h}}}{2 \cdot 32} \\ &= \frac{80\ \frac{1}{\mathrm{min}}}{2 \cdot 32} \\ &\approx 1.25\ \frac{1}{\mathrm{min}}\end{aligned}$$

The escape wheel turns 1.25 times per minute, so one revolution takes 48 s. An example for the watch is given at formula (2.8).

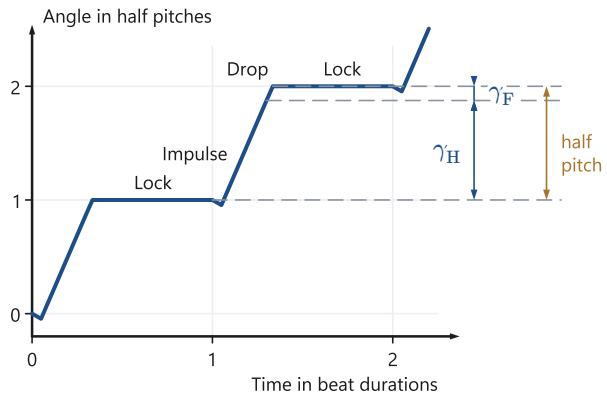


Fig. 5.3 Travel of the escape wheel in two beats: impulse γ_H and drop γ_F make up half the pitch. The small dip before each impulse is the recoil during unlocking of an escapement with draw. The time axis is not to scale: in the watch, impulse and drop last only a few hundredths of the beat duration

Impulse and drop at the wheel

(5.3) MASTER

$$\gamma_H + \gamma_F = \frac{180^\circ}{z}$$

REARRANGED $$\gamma_F = \frac{180^\circ}{z} - \gamma_H$$

γ_H	Impulse at the wheel (angle turned by the wheel between two drops)	°
γ_F	Drop (free angle of rotation of the wheel)	°
z	Number of teeth (of the escape wheel)	1

The step of the wheel consists of two parts. During impulse the tooth drives the pallet; during the drop the wheel turns freely until the next tooth lands on its locking face. The drop is a necessary safety margin, but it is lost energy.

Valid: γ_H counts the whole travel of the wheel between two drops. In escapements with draw, the wheel turns back by a fraction of a degree during unlocking and immediately advances again; this recoil cancels out in the total.

EXAMPLE On an escape wheel with 15 teeth, the wheel turns through 10.5° during impulse. How large is the drop?

Given: $z = 15$; $\gamma_H = 10.5^\circ$

$$\begin{aligned}\gamma_F &= \frac{180^\circ}{z} - \gamma_H \\ &= \frac{180^\circ}{15} - 10.5^\circ \\ &= 1.5^\circ\end{aligned}$$

The drop is 1.5°, a usual value for the Swiss lever escapement.

Drop as travel of the tooth tip

(5.4) MASTER

$$s_{\mathrm{F}} = R \cdot \gamma_{\mathrm{F}}$$

Under the loupe the drop appears as a small travel of the tooth tip. The formula converts the drop angle into this travel and shows how little clearance the escapement works with.

Valid: Angle in radians: $1^\circ = 0.01745\,\mathrm{rad}$.

s_{F}	Drop as travel (at the circumference of the wheel)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ_{F}	Drop (insert as angle in radians)	rad

EXAMPLE An escape wheel has a radius of 2.40 mm and the drop is 1.5°. How far does the tooth tip drop?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma_{\mathrm{F}} = 1.5^\circ$

$$\begin{aligned} s_{\mathrm{F}} &= R \cdot \gamma_{\mathrm{F}} \\ &= 2.40\,\mathrm{mm} \cdot 1.5^\circ \\ &= 2{,}40\,\mathrm{mm} \cdot 0{,}02618 \\ &\approx 0.063\,\mathrm{mm} \end{aligned}$$

The tooth tip drops freely by 0.063 mm. That is about the thickness of a hair.

Supplementary arc of the balance

(5.5) MASTER

$$\varphi_{\mathrm{E}} = A - \frac{\lambda}{2}$$

Outside the lift angle the balance swings freely, without contact with the pallet fork. It passes through this supplementary arc twice in each vibration: descending towards the impulse and, after the impulse, ascending to the point of reversal.

Valid: detached escapements (lever and chronometer escapements); lift angle symmetrical to the rest position, i.e. the watch is correctly in beat.

φ_{E}	Supplementary arc (from the end of impulse to the point of reversal)	°
A	Amplitude (from the rest position to the point of reversal)	°
λ	Lift angle (of the balance; unlocking and impulse together)	°

EXAMPLE A balance swings with an amplitude of 270°; the lift angle is 52°. How large is the supplementary arc?

Given: $A = 270^\circ$; $\lambda = 52^\circ$

$$\begin{aligned} \varphi_{\mathrm{E}} &= A - \frac{\lambda}{2} \\ &= 270^\circ - \frac{52^\circ}{2} \\ &= 244^\circ \end{aligned}$$

On each side the balance swings freely through 244°. Of the 540° of one vibration only 52° fall to the impulse; for about 90 % of its travel the balance is undisturbed.

Key point: With each beat the escape wheel advances by half the angular pitch: first the impulse, then the drop.

5.2 Swiss lever escapement: geometry

The Swiss lever escapement works in almost all mechanical wristwatches and pocket watches. Its pallet fork spans 2.5 pitches of the wheel. The pivot of the pallet fork lies at the intersection of the two tangents that touch the wheel circle at the contact points. As a result, the pressure of the tooth during lock acts along the pallet arm and does not turn the pallet fork. Only the draw gives the locking face a slight inclination.

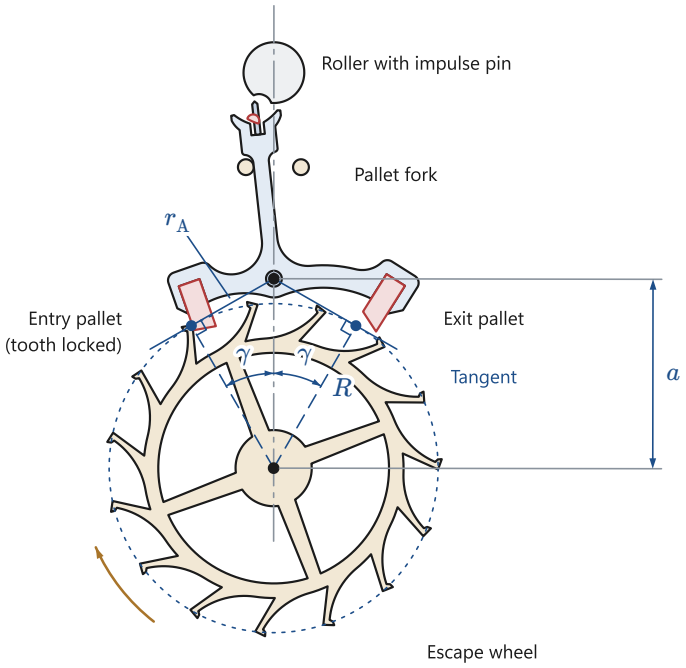


Fig. 5.4 Swiss lever escapement according to the model, tooth locked on the entry pallet: the pallet fork pivot lies at the intersection of the tangents; with 15 teeth $\gamma = 30^\circ$

Span of the pallet fork

(5.6) MASTER

$$\gamma = \frac{k \cdot 180^\circ}{z}$$

γ	Half span angle (measured at the wheel arbor)	°
k	Span in pitches (2.5 in the Swiss lever escapement)	1
z	Number of teeth (of the escape wheel)	1

The pallet fork spans k pitches of the wheel. Because one pallet is always engaged with the wheel while the other is free, the span ends on a half pitch: 2.5 pitches in the watch, 6.5 to 10.5 in pendulum clocks. Calculations use the half span angle γ.

Valid: Contact points symmetrical to the line joining the wheel and pallet arbors.

EXAMPLE What is the half span angle with 15 teeth and a span of 2.5 pitches?

Given: $k = 2.5$; $z = 15$

$$\begin{aligned}\gamma &= \frac{k \cdot 180^\circ}{z} \\ &= \frac{2.5 \cdot 180^\circ}{15} \\ &= 30^\circ\end{aligned}$$

The pallet fork spans 60°, 30° on each side of the centre line.

Centre distance of wheel and pallet fork

(5.7) MASTER

$$a = \frac{R}{\cos \gamma}$$

a	Centre distance (of escape wheel and pallet fork)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

Wheel arbor, contact point and pallet arbor form a right-angled triangle with the right angle at the contact point. The larger the span, the further the pallet fork moves away from the wheel.

Valid: tangential engagement: the line from the pallet fork pivot to the contact point is tangent to the wheel circle. R extends to the points used for the construction, depending on the design the locking corners or the centres of the impulse faces.

EXAMPLE An escape wheel with 15 teeth has a radius of 2.40 mm; the pallet fork spans 2.5 pitches. What is the centre distance?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30^\circ$

$$\begin{aligned} a &= \frac{R}{\cos \gamma} \\ &= \frac{2.40\,\mathrm{mm}}{\cos\left(30^\circ\right)} \\ &\approx 2.77\,\mathrm{mm} \end{aligned}$$

The arbors are 2.77 mm apart. With 15 teeth and a span of 2.5 pitches, $a = 1.155 \cdot R$ always applies.

Length of the pallet arm

(5.8) ADVANCED

$$r_\mathrm{A} = R \cdot \tan \gamma$$

r_A	Pallet arm (from the pallet fork pivot to the contact point)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

The pallet arm is the tangent segment from the pallet fork pivot to the contact point. It converts an angle of the pallet fork into the travel of the pallet.

Valid: tangential engagement as for the centre distance.

EXAMPLE How long is the pallet arm for a wheel of 2.40 mm radius and a half span angle of 30°?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30^\circ$

$$\begin{aligned} r_\mathrm{A} &= R \cdot \tan \gamma \\ &= 2.40\,\mathrm{mm} \cdot \tan\left(30^\circ\right) \\ &\approx 1.39\,\mathrm{mm} \end{aligned}$$

The pallet arm measures 1.39 mm. A lock of 1.5° at the pallet fork corresponds to a travel of the pallet of only 0.036 mm.

Distance between the contact points

(5.9) ADVANCED

$$l_\mathrm{P} = 2 \cdot R \cdot \sin\gamma$$

The chord between the contact points is the width with which the pallet fork embraces the wheel. It can be used to check whether a pallet fork matches the wheel.

Valid: Contact points on the wheel circle of radius R, symmetrical to the centre line.

l_P	Distance between the two contact points (chord in the wheel circle)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

EXAMPLE How far apart are the contact points for a wheel radius of 2.40 mm and a half span angle of 30°?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30^\circ$

$$\begin{aligned} l_\mathrm{P} &= 2 \cdot R \cdot \sin\gamma \\ &= 2 \cdot 2.40\,\mathrm{mm} \cdot \sin(30^\circ) \\ &= 2.40\,\mathrm{mm} \end{aligned}$$

The distance equals the wheel radius: with a span of 60°, the wheel arbor and the two contact points form an equilateral triangle.

Key point: With 15 teeth and a span of 2.5 pitches, the centre distance is 1.155 times the wheel radius, and the contact points are exactly one wheel radius apart.

Angles of the Swiss lever escapement with 15 teeth (guide values)

Quantity	Symbol	Guide value
Angular pitch of the wheel	τ	24°
Span of the pallet fork	2γ	2.5 pitches = 60°
Drop	γ_F	1° … 1.5°
Impulse at the wheel	γ_H	10.5° … 11°
Lock at the pallet fork	β_R	1.5°
Run	β_V	0.25° … 0.5°
Impulse at the pallet fork	β_H	8.5°
Draw angle	ψ	12° … 15°
Lift angle of the balance	λ	usually 49° … 53°; older wristwatch and pocket watch movements often smaller, about 38° … 46°

With the club tooth, the impulse at the pallet fork is shared between the impulse face of the pallet and that of the tooth. The manufacturer's data always take precedence, for the lift angle also on the timing machine (Section 6.2).

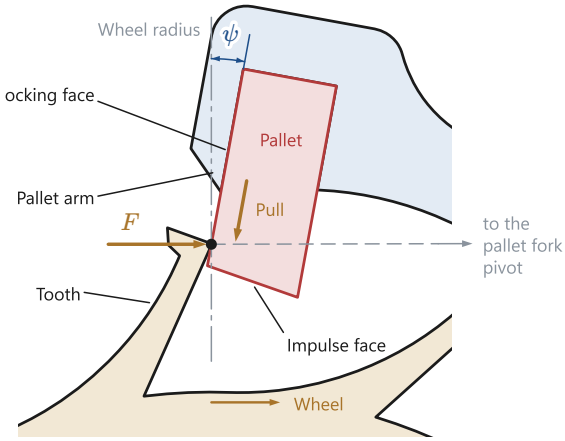


Fig. 5.5 Entry pallet locked: the locking face is inclined by the draw angle ψ to the wheel radius; the pressure F of the tooth pulls the pallet into the wheel

Draw: angle of friction as the limit

(5.10) MASTER

$$\rho = \arctan \mu$$

ρ	Angle of friction (the draw angle must be larger)	°
μ	Coefficient of friction (between tooth and locking face)	1

The locking face is inclined by the draw angle ψ. As a result, the pressure of the tooth pulls the pallet deeper into the wheel and holds the pallet fork against the banking, even under shocks. This only works if ψ is larger than the angle of friction ρ. Otherwise friction holds the pallet wherever it happens to be. The angle of friction in general is given in formula (XL 2.33).

Valid: tangential engagement; ψ measured between the locking face and the wheel radius at the contact point.

Caution: Too much draw costs amplitude: during unlocking the balance has to turn the wheel back a little against its torque.

EXAMPLE Steel on oiled ruby has a coefficient of friction of about 0.15. What is the angle of friction?

Given: $\mu = 0.15$

$$\begin{aligned}\rho &= \arctan \mu \\ &= \arctan 0.15 \\ &\approx 8.5^\circ\end{aligned}$$

The angle of friction is 8.5°. The usual 12° to 15° of draw leaves a reserve, even as the oil ages.

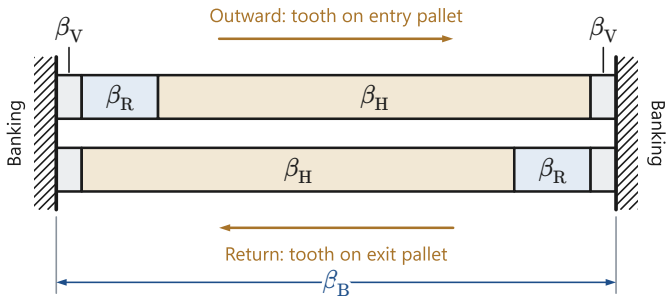


Fig. 5.6 Travel of the pallet fork between the bankings, drawn for $\beta_V = 0.5^\circ$, $\beta_R = 1.5^\circ$ and $\beta_H = 8.5^\circ$

Travel of the pallet fork between the bankings

(5.11) MASTER

$$\beta_B = \beta_R + \beta_H + 2 \cdot \beta_V$$

β_B	Travel of the pallet fork from banking to banking	°
β_R	Lock (immediately after the drop)	°
β_H	Impulse at the pallet fork	°
β_V	Run	°

Starting from the banking, the pallet fork first travels through the whole lock, then the impulse. After the drop, the draw pulls it on by the run to the other banking. The run ensures that the tooth lands safely on the locking face; it counts on both sides.

Valid: The total lock before unlocking is $\beta_R + \beta_V$. Both pallets set equally.

EXAMPLE A pallet fork has 1.5° lock, 8.5° impulse and 0.5° run. How far does it turn from banking to banking?

Given: $\beta_R = 1.5°$; $\beta_H = 8.5°$; $\beta_V = 0.5°$

$$\begin{aligned}\beta_B &= \beta_R + \beta_H + 2 \cdot \beta_V \\ &= 1.5° + 8.5° + 2 \cdot 0.5° \\ &= 11°\end{aligned}$$

The pallet fork turns through 11°. Without run it would be the frequently quoted 10°.

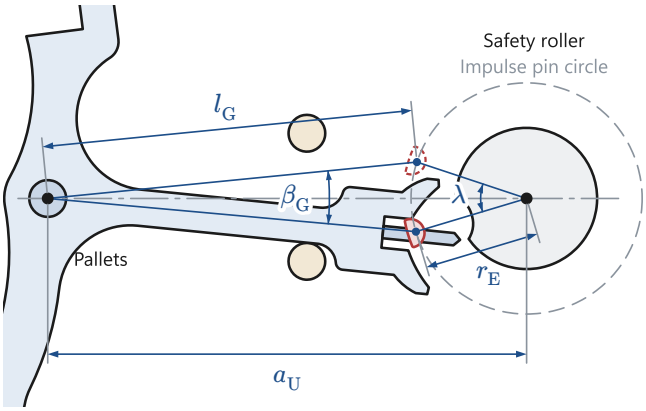


Fig. 5.7 Fork, safety roller and impulse pin according to the model ($\beta_G \approx 11^\circ$, $\lambda \approx 35^\circ$): impulse pin at entry (solid) and at exit (dashed)

Fork and impulse pin: lengths and angles

(5.12) MASTER

$$l_\mathrm{G} \cdot \sin\left(\frac{\beta_\mathrm{G}}{2}\right) = r_\mathrm{E} \cdot \sin\left(\frac{\lambda}{2}\right)$$

REARRANGED $$r_\mathrm{E} = \frac{l_\mathrm{G} \cdot \sin\left(\frac{\beta_\mathrm{G}}{2}\right)}{\sin\left(\frac{\lambda}{2}\right)}$$

At entry and at exit the impulse pin lies both on its circle about the balance arbor and in the notch of the fork. Its distance from the centre line is the same seen from either arbor; this is what the equation states. A long fork and a small impulse pin radius give a large lift angle of the balance.

Valid: Impulse pin treated as a point on the centre line of the fork notch; width of the jewel and shake in the fork neglected. Then β_G equals the travel of the pallet fork between the bankings.

l_G	Fork length (pallet arbor to impulse pin at entry)	mm
β_G	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°
r_E	Radius of the impulse pin about the balance arbor	mm
λ	Lift angle (of the balance; unlocking and impulse together)	°

EXAMPLE A fork is 2.89 mm long. The pallet fork turns 10° while the impulse pin is in the fork; the balance is to have a lift angle of 52°. At what radius must the impulse pin be placed?

Given: $l_\mathrm{G} = 2.89\,\mathrm{mm}$; $\beta_\mathrm{G} = 10°$; $\lambda = 52°$

$$r_\mathrm{E} = \frac{l_\mathrm{G} \cdot \sin\left(\frac{\beta_\mathrm{G}}{2}\right)}{\sin\left(\frac{\lambda}{2}\right)}$$

$$= \frac{2.89\,\mathrm{mm} \cdot \sin\left(\frac{10°}{2}\right)}{\sin\left(\frac{52°}{2}\right)}$$

$$\approx 0.575\,\mathrm{mm}$$

The impulse pin sits 0.575 mm from the balance arbor. The fork is about five times as long as this radius.

Impulse pin radius from the centre distance

(5.13) ADVANCED

$$r_{\mathrm{E}} = \frac{a_{\mathrm{U}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda + \beta_{\mathrm{G}}}{2}\right)}$$

r_{E}	Radius of the impulse pin about the balance arbor	mm
a_{U}	Centre distance of pallet fork and balance	mm
β_{G}	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°
λ	Lift angle (of the balance; unlocking and impulse together)	°

In the design, the centre distance and the angles are given. The law of sines in the triangle formed by pallet arbor, balance arbor and impulse pin gives the radius at which the impulse pin must be placed.

Valid: as above: impulse pin treated as a point, without shake in the fork.

EXAMPLE Pallet fork and balance are 3.40 mm apart. The pallet fork turns 10°; the balance is to have a lift angle of 52°. What is the impulse pin radius?

Given: $a_{\mathrm{U}} = 3.40\,\mathrm{mm}$; $\beta_{\mathrm{G}} = 10°$; $\lambda = 52°$

$$r_{\mathrm{E}} = \frac{a_{\mathrm{U}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda + \beta_{\mathrm{G}}}{2}\right)}$$

$$= \frac{3.40\,\mathrm{mm} \cdot \sin\left(\frac{10°}{2}\right)}{\sin\left(\frac{52° + 10°}{2}\right)}$$

$$\approx 0.575\,\mathrm{mm}$$

The impulse pin sits at a radius of 0.575 mm.

Fork length from the centre distance

(5.14) ADVANCED

$$l_G = \frac{a_U \cdot \sin\left(\frac{\lambda}{2}\right)}{\sin\left(\frac{\lambda + \beta_G}{2}\right)}$$

l_G	Fork length (pallet arbor to impulse pin at entry)	mm
a_U	Centre distance of pallet fork and balance	mm
λ	Lift angle (of the balance; unlocking and impulse together)	°
β_G	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°

The same triangle gives the fork length. Fork length and impulse pin radius together add up to slightly more than the centre distance, because at entry both are inclined to the centre line.

Valid: as above: impulse pin treated as a point, without shake in the fork.

EXAMPLE How long must the fork be for a centre distance of 3.40 mm, a pallet fork angle of 10° and a lift angle of 52°?

Given: $a_U = 3.40\,\text{mm}$; $\lambda = 52°$; $\beta_G = 10°$

$$\begin{aligned} l_G &= \frac{a_U \cdot \sin\left(\frac{\lambda}{2}\right)}{\sin\left(\frac{\lambda + \beta_G}{2}\right)} \\ &= \frac{3.40\,\text{mm} \cdot \sin\left(\frac{52°}{2}\right)}{\sin\left(\frac{52° + 10°}{2}\right)} \\ &\approx 2.89\,\text{mm} \end{aligned}$$

The fork is 2.89 mm long. Check along the centre line: $2.89 \cdot \cos 5° + 0.575 \cdot \cos 26° = 3.40$.

Ratio between fork and balance

(5.15) ADVANCED

$$i_G = \frac{a_U - r_E}{r_E}$$

i_G	Ratio in the central position (angular velocity of the balance divided by that of the pallet fork)	1
a_U	Centre distance of pallet fork and balance	mm
r_E	Radius of the impulse pin about the balance arbor	mm

In the central position the fork touches the impulse pin on the line joining the arbors. There the balance turns i_G times as fast as the pallet fork, and, neglecting friction, the balance receives the i_G-th part of the pallet fork torque. Averaged over the whole impulse, the ratio is $\frac{\lambda}{\beta_G}$.

Valid: central position; at entry and at exit the ratio is larger.

EXAMPLE What is the ratio for a centre distance of 3.40 mm and an impulse pin radius of 0.575 mm?

Given: $a_U = 3.40\,\text{mm}$; $r_E = 0.575\,\text{mm}$

$$\begin{aligned} i_G &= \frac{a_U - r_E}{r_E} \\ &= \frac{3.40\,\text{mm} - 0.575\,\text{mm}}{0.575\,\text{mm}} \\ &\approx 4.91 \end{aligned}$$

In the central position the ratio is 4.91; averaged over the impulse it is 52° to 10°, i.e. 5.2.

5.3 Force and energy

With each beat the escape wheel delivers work. Only part of it reaches the balance; the rest is lost in the drop, in friction on the impulse faces and in the impacts. Unlocking also costs energy: the balance must pull the pallet

out of the wheel against friction and draw.

In the steady state the escapement supplies the balance in each oscillation with exactly as much energy as the balance loses through friction. This determines the amplitude. As in Chapter 2, M_{H} is the torque at the escape wheel.

Work of the escape wheel per beat

(5.16) MASTER

$$W_{\mathrm{S}} = \frac{M_{\mathrm{H}} \cdot \pi}{z}$$

W_{S}	Work of the escape wheel per beat	$\mu\mathrm{J}$
M_{H}	Torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
z	Number of teeth (of the escape wheel)	1

Work is torque times angle of rotation. With each beat the wheel turns by half the angular pitch, in radians $\frac{\pi}{z}$. The escapement can never pass on more than this work in any beat.

Valid: Torque constant during the step. M_{H} is the torque that reaches the escape wheel after the losses of the gear train (Chapter 2).

EXAMPLE A torque of 1.6 µN·m reaches an escape wheel with 20 teeth. What work does the wheel deliver per beat?

Given: $M_{\mathrm{H}} = 1.6\,\mu\mathrm{N{\cdot}m}; \quad z = 20$

$$\begin{aligned} W_{\mathrm{S}} &= \frac{M_{\mathrm{H}} \cdot \pi}{z} \\ &= \frac{1.6\,\mu\mathrm{N{\cdot}m} \cdot \pi}{20} \\ &\approx 0.251\,\mu\mathrm{J} \end{aligned}$$

0.25 µJ is available per beat. At 8 beats per second that is 2 µW. The power at the escape wheel of a wristwatch is of this order of magnitude.

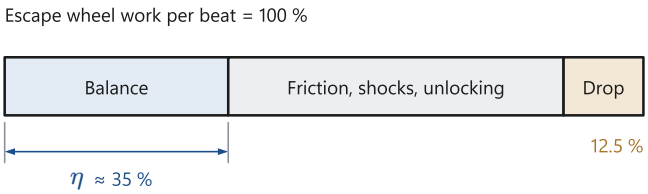


Fig. 5.8 Where the work of the escape wheel goes (guide values for the Swiss lever escapement)

Loss due to the drop

(5.17) MASTER

$$v_{\mathrm{F}} = \frac{\gamma_{\mathrm{F}} \cdot z}{180^\circ}$$

v_{F}	Proportion of the wheel's work lost in the drop	%
γ_{F}	Drop (free angle of rotation of the wheel)	°
z	Number of teeth (of the escape wheel)	1

During the drop the wheel turns without driving. The share of the drop in half the angular pitch is therefore the share of the wheel's work that is wasted when the tooth strikes the locking face.

Valid: constant torque at the wheel.

EXAMPLE What proportion of the wheel's work is lost with the wheel from Section 5.1 (15 teeth, 1.5° drop)?

Given: $\gamma_{\mathrm{F}} = 1.5^\circ; \quad z = 15$

$$\begin{aligned} v_{\mathrm{F}} &= \frac{\gamma_{\mathrm{F}} \cdot z}{180^\circ} \\ &= \frac{1.5^\circ \cdot 15}{180^\circ} \\ &= 12.5\,\% \end{aligned}$$

One eighth of the work is lost through the drop alone. This is why the drop is kept as small as safety allows. With the 20-tooth wheel from the previous example, the same drop would already cost 16.7 %.

Efficiency of the escapement

(5.18) MASTER

$$\eta = \frac{W_{\mathrm{U}}}{W_{\mathrm{S}}}$$

The efficiency states what proportion of the wheel's work reaches the balance. In the Swiss lever escapement it is about 30 % to 40 % in operation (guide value). The rest is lost through the drop, sliding friction on the impulse faces, the impacts and the acceleration of wheel and pallet fork.

Valid: Steady state, i.e. constant amplitude.

η	Efficiency (of the escapement)	1
W_{U}	Work delivered to the balance per beat (impulse minus unlocking)	$\mu\mathrm{J}$
W_{S}	Work of the escape wheel per beat	$\mu\mathrm{J}$

EXAMPLE The escape wheel delivers 0.251 µJ per beat; 0.088 µJ reaches the balance. What is the efficiency?

Given: $W_{\mathrm{U}} = 0.088\,\mu\mathrm{J}$; $W_{\mathrm{S}} = 0.251\,\mu\mathrm{J}$

$$\begin{aligned}\eta &= \frac{W_{\mathrm{U}}}{W_{\mathrm{S}}} \\ &= \frac{0.088\,\mu\mathrm{J}}{0.251\,\mu\mathrm{J}} \\ &\approx 0.35\end{aligned}$$

The efficiency is 0.35, i.e. 35 %. That is a usual value.

Amplitude from the energy balance

(5.19) APPROXIMATION ADVANCED

$$A \approx \sqrt{\frac{2 \cdot \eta \cdot M_{\mathrm{H}} \cdot Q}{z \cdot D}}$$

In the steady state, the energy that two beats supply to the balance equals its loss in one oscillation. This loss grows with the square of the amplitude. The amplitude therefore rises only with the square root of the torque: double the torque gives 1.4 times the amplitude, not double.

Valid: Q and η assumed constant; in reality both change somewhat with the amplitude. The quality factor is 2π times the oscillation energy divided by the loss per oscillation (Chapter 3). The formula gives A in radians.

A	Amplitude (in the steady state)	°
η	Efficiency (of the escapement)	1
M_{H}	Torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
Q	Quality factor (of the balance with balance spring)	1
z	Number of teeth (of the escape wheel)	1
D	Restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

EXAMPLE A movement with 28 800 vibrations per hour has an escape wheel with 20 teeth that receives 1.6 µN·m. The restoring torque coefficient of the balance spring is 0.632 µN·m per radian, the quality factor 250, the efficiency of the escapement 0.35. What amplitude results?

Given: $\eta = 0.35$; $M_{\mathrm{H}} = 1.6\,\mu\mathrm{N{\cdot}m}$; $Q = 250$; $z = 20$; $D = 0.632\,\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

$$\begin{aligned}A &\approx \sqrt{\frac{2 \cdot \eta \cdot M_{\mathrm{H}} \cdot Q}{z \cdot D}} \\ &= \sqrt{\frac{2 \cdot 0.35 \cdot 1.6\,\mu\mathrm{N{\cdot}m} \cdot 250}{20 \cdot 0.632\,\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}} \\ &\approx 4{,}71\,\mathrm{rad} \\ &\approx 270^{\circ}\end{aligned}$$

The balance swings with about 270°. If the torque falls to half towards the end of the power reserve, the amplitude drops to about 190°.

5.4 The escapement on the timing machine

The timing machine listens to the escapement. Each beat of the lever escapement consists of three noises. At the first, the impulse pin strikes the fork and unlocking begins. At the second, the tooth strikes the impulse face and the fork strikes the impulse pin; impulse begins. At the third, the next tooth drops onto the locking face and the pallet fork strikes the banking. From the times between these noises the instrument calculates amplitude and beat error. The formulas, with examples and guide values, are given in Section 6.2; only the part that depends on the escapement follows here.

Amplitude. Between the first and the third noise the balance passes through its lift angle λ, half before and half after the rest position. The larger the amplitude, the faster the balance moves there and the shorter this impulse time. The timing machine measures the impulse time and, using the set lift angle, calculates back to the amplitude, formula (6.9). The lift angle is a design quantity of the escapement (Section 5.2). If it is set incorrectly on the instrument, the displayed amplitude is wrong in the same proportion, formula (6.10).

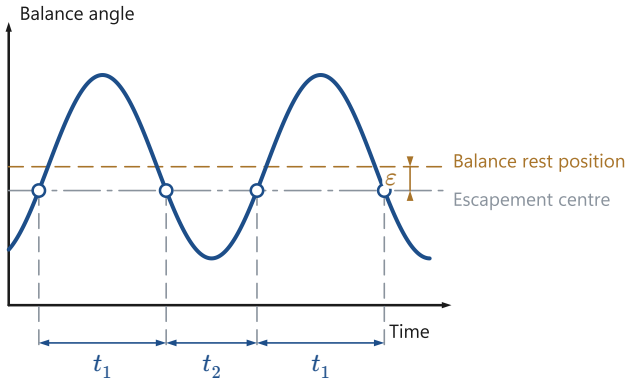


Fig. 5.9 Beat error: if the rest position of the balance lies at ε beside the centre of the escapement, long and short vibrations alternate (greatly exaggerated)

Beat error. If the rest position of the balance lies at an angle ε beside the centre of the escapement, the vibration to one side lasts longer than to the other (Fig. 5.9): the watch is out of beat. The timing machine shows half the time difference as the beat error in milliseconds, formula (6.7). How to calculate the angle ε from it is given in formula (6.8). The correction is made at the balance: the collet or the movable stud carrier is turned by ε until, with the balance at rest, the impulse pin lies on the line joining the pallet arbor and the balance arbor.

5.5 Clock escapements

Pendulum clocks use dead-beat and recoil anchor escapements. In the Graham escapement the locking faces are circular arcs about the anchor pivot: as long as a tooth rests on them, the wheel stands still. In the recoil escapement the locking faces are inclined; in the supplementary arc the pendulum pushes the wheel back a little, hence the name recoil. The pin-wheel escapement of large clocks carries pins on the wheel rim instead of teeth and, like the Graham escapement, is a dead-beat escapement.

Angular pitch, step, span, centre distance and pallet arm are calculated for all these escapements with the formulas from Sections 5.1 and 5.2.

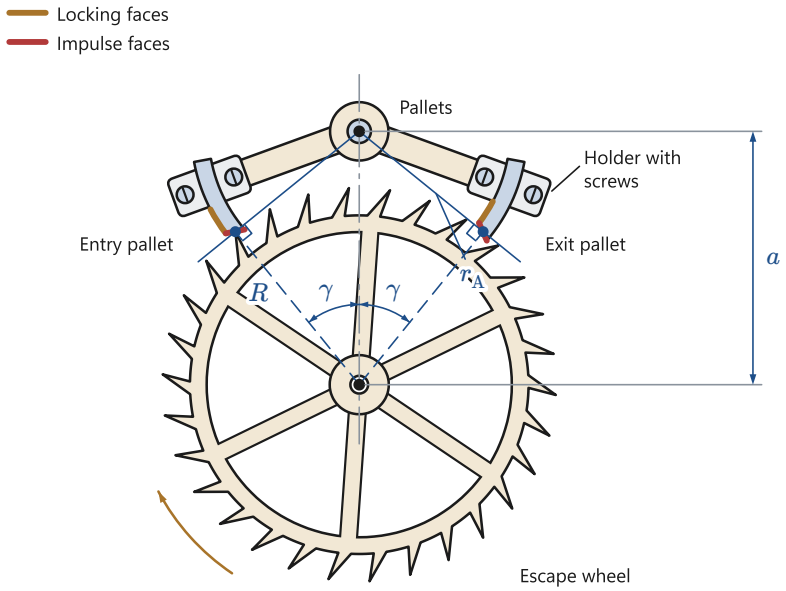


Fig. 5.10 Graham escapement with 30 teeth and a span of 6.5 pitches: the locking faces are circular arcs about the anchor pivot

Centre distance and pallet arm with tangential engagement

Number of teeth z	Span k in pitches	Span angle 2γ	Centre distance a in multiples of R	Pallet arm r_A in multiples of R
15	2.5	60°	1.155	0.577
30	6.5	78°	1.287	0.810
30	7.5	90°	1.414	1.000
30	8.5	102°	1.589	1.235
30	9.5	114°	1.836	1.540
30	10.5	126°	2.203	1.963

First row: Swiss lever escapement. The other rows apply to pendulum clocks with 30 teeth; the larger the span, the longer the pallet arm and the smaller the angle of the anchor for the same impulse face.

Pallet width of the Graham escapement

(5.20) MASTER

$$b_K = R \cdot \left(\frac{\pi}{z} - \gamma_F \right)$$

A pallet must fit alternately between two tooth tips. The pallet therefore gets half the angular pitch minus the drop. This is the width of the ring about the anchor pivot from which both pallets are cut.

Valid: pointed teeth; tangential engagement. Angles in radians: half the angular pitch is $\frac{\pi}{z}$, and $1^\circ = 0.01745\,\text{rad}$.

b_K	Width of the pallet, measured at the circumference of the wheel	mm
R	Radius (of the escape wheel; to the tooth tips)	mm
z	Number of teeth (of the escape wheel)	1
γ_F	Drop (insert as angle in radians)	rad

EXAMPLE An escape wheel with 30 teeth has a radius of 20 mm. The drop is to be 1°. How wide will the pallets be?

Given: $R = 20\,\text{mm}; \quad z = 30; \quad \gamma_F = 1^\circ$

$$\begin{aligned} b_K &= R \cdot \left(\frac{\pi}{z} - \gamma_F \right) \\ &= 20\,\text{mm} \cdot \left(\frac{\pi}{30} - 1^\circ \right) \\ &= 20\,\text{mm} \cdot (0{,}1047 - 0{,}0175) \\ &\approx 1.75\,\text{mm} \end{aligned}$$

The pallets will be 1.75 mm wide. On this wheel one degree of drop is 0.35 mm of clearance.

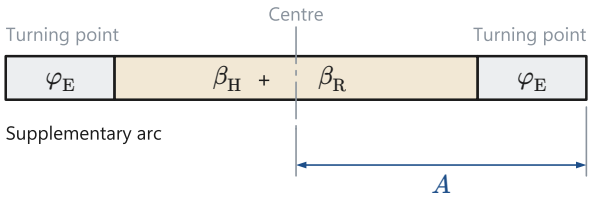


Fig. 5.11 Swing of the pendulum in the Graham escapement: impulse and lock in the middle, supplementary arc φ_E on the outside

Supplementary arc of the pendulum

(5.21) MASTER

$$\varphi_E = A - \frac{\beta_H + \beta_R}{2}$$

The anchor follows every movement of the pendulum. Impulse and lock lie in the middle of the swing, together $\beta_H + \beta_R$ wide. Whatever the pendulum swings beyond this on each side is the supplementary arc. During it the tooth slides on the locking face.

Valid: Anchor set centrally (clock in beat), crutch without play. The pendulum must swing further than $\frac{1}{2}(\beta_H + \beta_R)$, otherwise no tooth drops and the clock stops.

φ_E	Supplementary arc (on each side)	°
A	Amplitude (swing of the pendulum, from the centre to the point of reversal)	°
β_H	Impulse at the anchor	°
β_R	Lock (at the anchor)	°

Good to know: Guide values for Graham escapements with seconds pendulums: impulse at the anchor 1° to 2°, lock 0.5° to 1°. A large supplementary arc costs friction on the locking faces.

EXAMPLE A pendulum swings 1.75° to each side. The Graham anchor has 1.5° impulse and 0.5° lock. How large is the supplementary arc?

Given: $A = 1.75°$; $\beta_H = 1.5°$; $\beta_R = 0.5°$

$$\varphi_E = A - \frac{\beta_H + \beta_R}{2}$$
$$= 1.75° - \frac{1.5° + 0.5°}{2}$$
$$= 0.75°$$

On each side the pendulum swings 0.75° beyond impulse and lock. This reserve absorbs fluctuations in the driving force.

5.6 Other escapements

For the other escapements a few key quantities are sufficient. Angular pitch and beat rate are calculated for all of them as in Section 5.1; the table shows how the types differ.

Escapements at a glance

Escapement	Type	Impulses per oscillation	Step of the wheel per impulse
Swiss lever escapement	detached	2	half pitch
Pin pallet escapement	detached	2	half pitch
Chronometer escapement	detached	1	full pitch
Cylinder escapement	dead-beat	2	half pitch
Verge escapement	recoil	2	half pitch
Graham escapement	dead-beat	2	half pitch
Pin-wheel escapement (clock)	dead-beat	2	half pitch
Recoil escapement	recoil	2	half pitch

Detached: outside the impulse the oscillator swings without contact with the escapement. Dead-beat: during the supplementary arc a tooth rests against the oscillator or the anchor and the wheel stands still. Recoil: the wheel is turned back a little in the process.

The chronometer escapement drives the balance in only one direction of oscillation, but over the full pitch of the wheel. In the opposite direction the discharging pallet passes the passing spring (gold spring) without releasing the wheel. Formula (5.2) still applies to the beat rate, because here too one tooth passes per oscillation.

In the cylinder escapement the wheel works without a pallet fork directly on the balance staff, a hollow cylinder cut open at the side. The tooth rests alternately on the outside and the inside of the cylinder wall; the balance never swings freely.

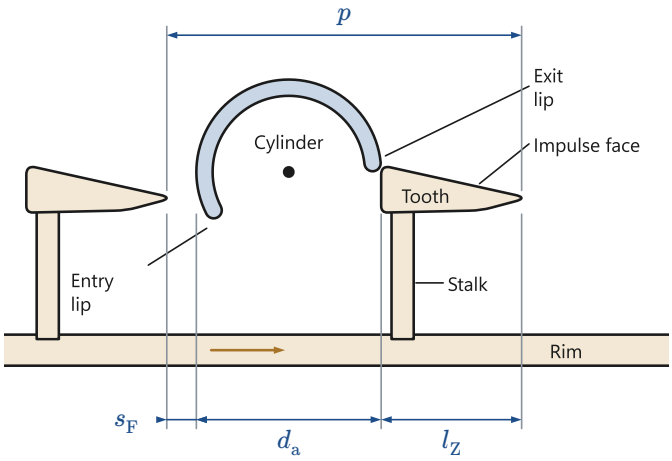


Fig. 5.12 Cylinder escapement with the pitch circle straightened, at the moment the right-hand tooth leaves the cylinder: the pitch p is made up of drop, cylinder diameter and tooth length

Cylinder escapement: pitch, tooth length and cylinder

(5.22) APPROXIMATION ADVANCED

$$\frac{2 \cdot \pi \cdot R}{z} \approx l_Z + d_a + s_F$$

REARRANGED $$d_a \approx \frac{2 \cdot \pi \cdot R}{z} - l_Z - s_F$$

The left-hand side is the pitch p of the wheel at its circumference. Between two teeth there must be room for the outside diameter of the cylinder plus the outside drop. Inside the cylinder there must be room for the tooth: the inside diameter exceeds the tooth length by the inside drop.

Valid: Pitch circle treated as a straight line (arc instead of chord).

R	Radius (of the escape wheel; cylinder wheel, to the tooth tips)	mm
z	Number of teeth (of the escape wheel)	1
l_Z	Length of a tooth	mm
d_a	Outside diameter of the cylinder	mm
s_F	Drop as travel (outside drop)	mm

EXAMPLE A cylinder wheel with 15 teeth has a radius of 2.50 mm, the teeth are 0.45 mm long, and the outside drop is to be 0.05 mm. What outside diameter may the cylinder have?

Given: $R = 2.50\,\text{mm}$; $z = 15$; $l_Z = 0.45\,\text{mm}$; $s_F = 0.05\,\text{mm}$

$$\begin{aligned} d_a &\approx \frac{2 \cdot \pi \cdot R}{z} - l_Z - s_F \\ &= \frac{2 \cdot \pi \cdot 2.50\,\text{mm}}{15} - 0.45\,\text{mm} - 0.05\,\text{mm} \\ &= 1{,}047\,\text{mm} - 0{,}45\,\text{mm} - 0{,}05\,\text{mm} \\ &\approx 0.55\,\text{mm} \end{aligned}$$

The cylinder may measure 0.55 mm on the outside. With a wall thickness of 0.04 mm, 0.47 mm remains inside, i.e. an inside drop of 0.02 mm.

6 Rate Testing and Measurement

Whether a watch keeps good time is decided by measurement. Two terms carry the whole chapter: the time error states how far the watch deviates from the correct time; the rate states how fast the time error changes. The time error is read off; the rate is calculated from it or measured in less than a minute on the timing machine.

The timing machine also gives amplitude and beat error. Measurements are taken in several positions and at different states of wind, because only the differences show where a fault lies. In this chapter, S stands for the time error and D for the difference between vertical and horizontal.

6.1 Time error and rate

The time error is a span of time in seconds; the rate is a change in time error per day. The same sign rule applies to both: plus means the watch is ahead or gaining.

Time error

(6.1) JOURNEYMAN

$$S = t_{\mathrm{U}} - t_{\mathrm{N}}$$

The time error is the reading of the watch minus the correct time. A radio-controlled clock or a time signal serves as the reference.

Valid: both times read at the same instant.

S	Time error (+ means: watch is ahead)	s
t_{U}	Reading of the watch	s
t_{N}	Reference time (correct time at the same instant)	s

EXAMPLE At the full-minute time signal, the seconds hand of the watch already stands at 12 s.

Given: $t_{\mathrm{U}} = 12\,\mathrm{s}$; $t_{\mathrm{N}} = 0\,\mathrm{s}$

$$\begin{aligned} S &= t_{\mathrm{U}} - t_{\mathrm{N}} \\ &= 12\,\mathrm{s} - 0\,\mathrm{s} \\ &= 12\,\mathrm{s} \end{aligned}$$

The watch is 12 s ahead; its time error is +12 s.

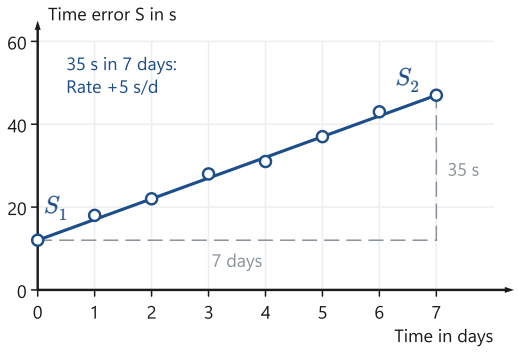


Fig. 6.1 Time error over time: the rate is the slope of the line through the read time errors

Rate from two time errors

(6.2) JOURNEYMAN

$$G = \frac{S_2 - S_1}{t_2 - t_1}$$

The rate is the change in time error per day. In the time-error/time diagram it is the slope of the line.

Valid: mean rate between the two readings; the watch is not set in between.

G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
S_2	Time error at the second reading	s
S_1	Time error at the first reading	s
t_2	Time of the second reading	d
t_1	Time of the first reading	d

Good to know: The longer the interval, the more accurate the rate: the reading error is spread over more days (Section 6.7).

EXAMPLE On day 0 the watch is 12 s ahead, on day 7 it is 47 s ahead. What is its rate?

Given: $S_2 = 47\,\mathrm{s}$; $S_1 = 12\,\mathrm{s}$; $t_2 = 7\,\mathrm{d}$; $t_1 = 0\,\mathrm{d}$

$$\begin{aligned} G &= \frac{S_2 - S_1}{t_2 - t_1} \\ &= \frac{47\,\mathrm{s} - 12\,\mathrm{s}}{7\,\mathrm{d} - 0\,\mathrm{d}} \\ &= 5.0\,\tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The watch gains 5 s per day.

Change in time error over a running period

(6.3) JOURNEYMAN

$$\Delta S = G \cdot t$$

REARRANGED $t = \dfrac{\Delta S}{G}$

At a constant rate, the time error grows uniformly. This is how a rate is extrapolated to a month or a year, or conversely how a figure in seconds per month is converted back to the day.

Valid: constant rate over the whole running period.

ΔS	Change in time error (+ means: watch has gained)	s
G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
t	Time (running period)	d

EXAMPLE A quartz watch gains 0.5 s per day. How much does it gain in a month of 30 days?

Given: $G = 0.5\,\frac{\mathrm{s}}{\mathrm{d}}$; $t = 30\,\mathrm{d}$

$$\begin{aligned} \Delta S &= G \cdot t \\ &= 0.5\,\tfrac{\mathrm{s}}{\mathrm{d}} \cdot 30\,\mathrm{d} \\ &= 15\,\mathrm{s} \end{aligned}$$

15 s per month, a usual value for a simple quartz watch. Per year it would be about 3 min.

Mean rate

(6.4) JOURNEYMAN

$$\bar{G} = \frac{\sum G_i}{n}$$

Over several days, the mean daily rate is also the total change in time error divided by the number of days. Averaged over several positions, it shows how the watch is regulated.

$\bar{G}$	Mean rate (over days or positions)	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum G_i$	Sum of the individual rates, with sign	$\frac{\mathrm{s}}{\mathrm{d}}$
n	Number (of individual rates)	1

Valid: all individual rates measured over equal periods; observe the signs.

EXAMPLE On five days the rates are found to be +6, +4, +7, +3 and +5 seconds per day. The sum is +25 seconds per day.

Given: $\sum G_i = 25\ \frac{\mathrm{s}}{\mathrm{d}}; \quad n = 5$

$$\begin{aligned}\bar{G} &= \frac{\sum G_i}{n} \\ &= \frac{25\ \frac{\mathrm{s}}{\mathrm{d}}}{5} \\ &= 5.0\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

On average +5 s per day. Over the five days the watch has gained 25 s.

Converting the rate

Rate in $\frac{\mathrm{s}}{\mathrm{d}}$	Seconds per month (30 days)	Seconds per year (365 days)	ppm
0.1	3	36.5	1.16
0.5	15	182.5	5.79
1	30	365	11.6
5	150	1825	57.9
10	300	3650	116
30	900	10 950	347

ppm means parts per million. 1 s per day is an error of 1 in 86 400, i.e. 11.6 ppm (Section XL 1.1).

Rate from the deviation of the beat rate

(6.5) JOURNEYMAN

$$G = \frac{n_{\text{ist}} - n_{\text{soll}}}{n_{\text{soll}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

Every watch counts oscillations. If the oscillator swings faster by a certain fraction, the watch gains by the same fraction. The formula with frequencies instead of beat rates is given in Section 3.4, for quartz in Section 7.2.

G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
n_{ist}	Measured beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$
n_{soll}	Nominal beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$

Valid: beat rate averaged over the measuring time.

EXAMPLE A movement with 28 800 vibrations per hour makes 28 803 vibrations in one hour.

Given: $n_{\text{ist}} = 28\,803\ \frac{\mathrm{A}}{\mathrm{h}}; \quad n_{\text{soll}} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$

$$\begin{aligned}G &= \frac{n_{\text{ist}} - n_{\text{soll}}}{n_{\text{soll}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{28\,803\ \frac{\mathrm{A}}{\mathrm{h}} - 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}}{28\,800\ \frac{\mathrm{A}}{\mathrm{h}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= 9.0\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The watch gains 9 s per day. Three beats too many per hour amount to three eighths of a second on the dial, and 24 times as much per day.

Key point: The time error is a state; the rate is a change. For both, plus means: the watch is ahead.

6.2 Timing machine

The timing machine picks up the beats of the watch with a microphone and compares their sequence with a quartz frequency. From a measuring time of less than a minute it calculates rate, beat error and amplitude. All three values follow from measured times.

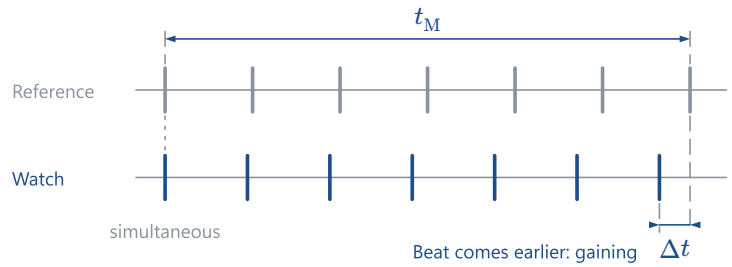


Fig. 6.2 Principle of the timing machine: during the measuring time t_M the beats of the watch shift by Δt relative to the reference frequency

Rate from the time shift of the beats

(6.6) MASTER

$$G = \frac{\Delta t}{t_M} \cdot 86\,400\ \frac{s}{d}$$

REARRANGED $$\Delta t = \frac{G \cdot t_M}{86\,400\ \frac{s}{d}}$$

G	Rate (+ means: watch gains)	$\frac{s}{d}$
Δt	Time shift of the beats relative to the reference frequency during the measuring time (+ means: beats arrive earlier)	ms
t_M	Measuring time	s

If each beat arrives slightly earlier than the reference frequency dictates, the lead accumulates. The rate follows from the lead and the measuring time. In the timing machine's diagram it is the slope of the dotted line.

Valid: uniform rate during the measuring time; averaged over tick and tock so that the beat error does not enter the rate.

Caution: The interval between beats varies from beat to beat, mainly due to the teeth of the escape wheel and the meshing of the gear train. A few seconds are therefore not enough for a statement to within 1 s per day; measuring times of 10 s to 60 s are usual, longer if the trace is unsteady.

EXAMPLE In 30 s of measuring time, the beats of a watch advance by 2.0 ms relative to the reference frequency.

Given: $\Delta t = 2.0\,\text{ms}$; $t_M = 30\,\text{s}$

$$G = \frac{\Delta t}{t_M} \cdot 86\,400\ \frac{s}{d}$$
$$= \frac{2.0\,\text{ms}}{30\,\text{s}} \cdot 86\,400\ \frac{s}{d}$$
$$= 5.76\ \frac{s}{d}$$

The watch gains 5.76 s per day. The timing machine displays +5.8 after rounding.



Fig. 6.3 Timing machine diagram: the slope of the dotted line shows the rate, the distance between two lines the beat error

Time shift of the beats at a rate of 1 $\frac{\mathrm{s}}{\mathrm{d}}$

Measuring time	Time shift
2 s	23 µs
4 s	46 µs
10 s	116 µs
30 s	347 µs
60 s	694 µs

For comparison: at 28 800 vibrations per hour, two beats are 125 000 µs apart.

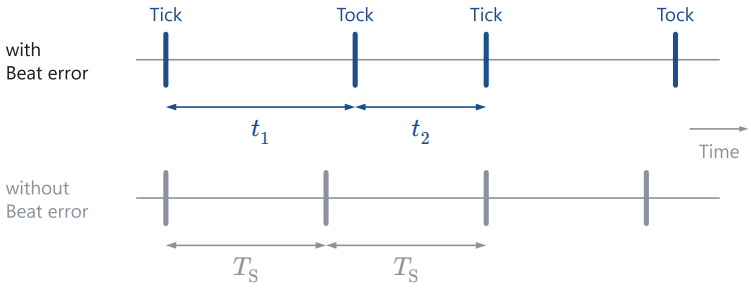


Fig. 6.4 Beat error: more time passes from tick to tock than from tock to tick; T_S is the beat duration without beat error

Beat error

(6.7) JOURNEYMAN

$$t_A = \frac{|t_1 - t_2|}{2}$$

If the balance at rest does not lie exactly in the centre of the escapement, tick and tock follow at unequal intervals. The beat error is the deviation of each interval from the mean, i.e. half the difference between the two intervals. Section 5.4 shows how beat error arises at the escapement.

Valid: times measured in each case from the first sound of one beat to the first sound of the next; displayed as an absolute value.

t_A	Beat error	ms
t_1	Duration of one vibration (tick to tock)	ms
t_2	Duration of the following vibration (tock to tick)	ms

Good to know: Guide values: with the movement fully wound and in the dial-up position, movement manufacturers usually allow at most 0.6 ms to 0.8 ms (ETA 2892A2 and ETA 7750: 0.6 ms; Sellita SW200 0.8 ms or 0.6 ms depending on grade). From about 1 ms onwards the watch should in any case be put in beat. The movement manufacturer's specification is decisive. A large beat error makes it harder for the watch to start by itself.

EXAMPLE On a movement with 28 800 vibrations per hour, the timing machine measures 125.6 ms from tick to tock and 124.4 ms from tock to tick.

Given: $t_1 = 125.6\,\text{ms}$; $t_2 = 124.4\,\text{ms}$

$$\begin{aligned} t_A &= \frac{|t_1 - t_2|}{2} \\ &= \frac{|125.6\,\text{ms} - 124.4\,\text{ms}|}{2} \\ &= 0.6\,\text{ms} \end{aligned}$$

0.6 ms: one interval is 0.6 ms longer, the other 0.6 ms shorter than the nominal beat duration of 125 ms. In the diagram, two lines run side by side.

Beat error as an angle

(6.8) APPROXIMATION ADVANCED

$$\varepsilon \approx \pi \cdot f \cdot A \cdot t_A$$

The same offset of the rest position appears as a larger beat error in milliseconds at a small amplitude, because the balance then passes through the centre more slowly. Beat errors are therefore compared at the same amplitude.

Valid: offset small compared with the amplitude; sinusoidal oscillation.

ε	Angle (between the rest position of the balance and the centre of the escapement)	°
f	Frequency (of the balance; full oscillations per second; 28 800 vibrations per hour are 4 Hz)	Hz
A	Amplitude (from the rest position to the turning point)	°
t_A	Beat error	ms

EXAMPLE A watch with 28 800 vibrations per hour shows 0.6 ms beat error at 270° amplitude.

Given: $f = 4\,\text{Hz}$; $A = 270°$; $t_A = 0.6\,\text{ms}$

$$\begin{aligned} \varepsilon &\approx \pi \cdot f \cdot A \cdot t_A \\ &= \pi \cdot 4\,\text{Hz} \cdot 270° \cdot 0.6\,\text{ms} \\ &\approx 2.0° \end{aligned}$$

The rest position of the balance lies about 2° off the centre. The collet or the movable stud carrier must be turned by this angle. At 180° amplitude the same watch showed 0.9 ms.

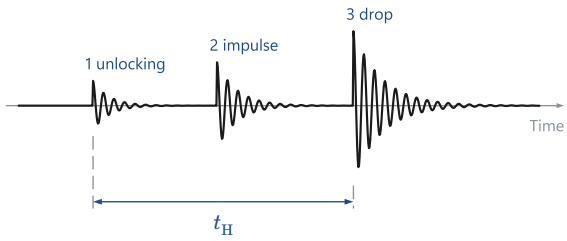


Fig. 6.5 Sound of one beat of the lever escapement: the lift time t_H extends from the first to the third pulse

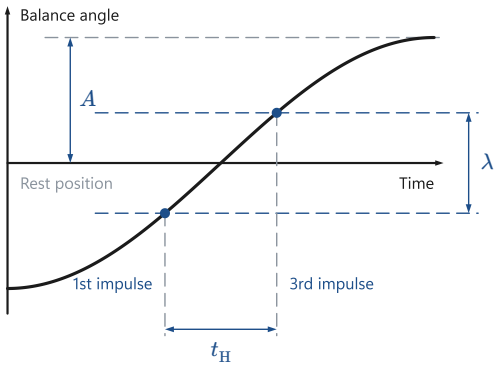


Fig. 6.6 The balance passes through the lift angle λ on both sides of the rest position in the lift time t_H (angle drawn enlarged)

Amplitude from lift angle and lift time

(6.9) MASTER

$$A = \frac{\lambda}{2 \cdot \sin\left(\pi \cdot f \cdot t_H\right)}$$

REARRANGED $$t_H = \frac{\arcsin\left(\frac{\lambda}{2 \cdot A}\right)}{\pi \cdot f}$$

A	Amplitude (from the rest position to the turning point)	°
λ	Lift angle (of the balance; manufacturer's specification, set on the timing machine)	°
f	Frequency (of the balance; full oscillations per second; 28 800 vibrations per hour are 4 Hz)	Hz
t_H	Lift time (from the first to the third sound of a beat)	ms

The greater its amplitude, the faster the balance passes through the lift angle. The timing machine measures the lift time between the first and the third sound of a beat and calculates the amplitude from it.

Valid: sinusoidal oscillation, lift angle symmetrical about the rest position. The product $\pi \cdot f \cdot t_H$ is an angle in radians; the calculator must be set to rad for the sine.

Good to know: Derivation: the balance oscillates according to $\varphi = A \cdot \sin(2\pi \cdot f \cdot t)$. The lift angle lies symmetrically about the rest position, so the balance reaches half the lift angle after half the lift time: $\frac{\lambda}{2} = A \cdot \sin(\pi \cdot f \cdot t_H)$.

Good to know: For usual amplitudes the approximation $A \approx \frac{\lambda}{2\pi \cdot f \cdot t_H}$ is sufficient. In the example it gives 268.7°, i.e. 0.4° too little.

EXAMPLE A movement with 28 800 vibrations per hour has a lift angle of 52°. The timing machine measures a lift time of 7.7 ms.

Given: $\lambda = 52°$; $f = 4\,\text{Hz}$; $t_H = 7.7\,\text{ms}$

$$\begin{aligned} A &= \frac{\lambda}{2 \cdot \sin\left(\pi \cdot f \cdot t_H\right)} \\ &= \frac{52°}{2 \cdot \sin\left(\pi \cdot 4\,\text{Hz} \cdot 7.7\,\text{ms}\right)} \\ &= \frac{52°}{2 \cdot \sin(0{,}0968)} \\ &\approx 269° \end{aligned}$$

The balance swings at about 269°. At 300° the lift time would have shrunk to 6.9 ms.

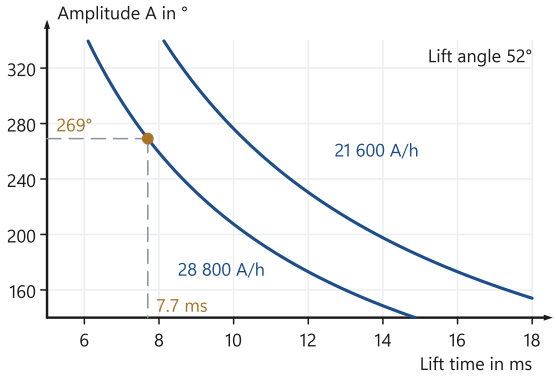


Fig. 6.7 Amplitude against lift time at a lift angle of 52°: the shorter the lift time, the greater the amplitude

Lift time in ms at a lift angle of 52°

Amplitude	28 800 A/h (4 Hz)	21 600 A/h (3 Hz)	18 000 A/h (2.5 Hz)
200°	10.4	13.8	16.6
220°	9.4	12.6	15.1
240°	8.6	11.5	13.8
260°	8.0	10.6	12.8
280°	7.4	9.9	11.8
300°	6.9	9.2	11.0
320°	6.5	8.6	10.4

Calculated with the rearranged formula. Between 280° and 300° the lift time at 4 Hz changes by only 0.5 ms: the timing machine must measure it to within hundredths of a millisecond.

Amplitude with incorrectly set lift angle

(6.10) MASTER

$$A = \frac{A_Z \cdot \lambda}{\lambda_Z}$$

A	Amplitude (true value)	°
A_Z	Amplitude displayed by the timing machine	°
λ	Lift angle (of the balance; true value of the movement)	°
λ_Z	Lift angle set on the timing machine	°

The timing machine only measures the lift time; the lift angle has to be entered. If it is set incorrectly, the displayed amplitude is wrong in the same proportion: 4 % too much lift angle gives 4 % too much amplitude. For the errors, $\frac{\Delta A}{A} = \frac{\Delta \lambda}{\lambda}$ applies.

Valid: follows from the amplitude formula: for the same lift time, the amplitude is proportional to the lift angle.

Caution: If the lift angle is unknown, check the amplitude at the balance. A balance with two arms shows the same picture at the turning points at 180° amplitude as at the rest position; at 270° the arms stand in the same place at both turning points, at right angles to the rest position. Set the lift angle on the timing machine so that the display matches.

EXAMPLE The timing machine is set to 52° and shows 280°. The movement, however, has a lift angle of 50°.

Given: $A_Z = 280°$; $\lambda = 50°$; $\lambda_Z = 52°$

$$A = \frac{A_Z \cdot \lambda}{\lambda_Z} = \frac{280° \cdot 50°}{52°} \approx 269°$$

The balance swings at only about 269°. Here each degree of error in the lift angle amounts to just over 5° of amplitude.

Usual lift angles (guide values)

Escapement and period of construction	Lift angle of the balance
Swiss lever escapement, more recent wristwatch movements	mostly 49° … 53°
Swiss lever escapement, older wristwatch and pocket watch movements	often smaller, about 38° … 46°
Co-Axial escapement	about 30° … 38°

The lift angle is a design parameter of the calibre. The manufacturer's data sheet is decisive; if it is missing, a visual check at the balance helps. Examples: ETA 2824-2 50°, ETA 2892A2 51°, ETA 7750 49°, Sellita SW200 50°, ETA 6497-1 44°.

Key point: The timing machine measures times. Rate, beat error and amplitude are calculated from them, and the amplitude is correct only with the right lift angle.

6.3 Positions and characteristic values

A watch does not run equally in every position: pivot friction, the centre of gravity of the balance and the balance spring act differently depending on the position. Testing is therefore carried out in up to six positions, once fully wound and once after 24 hours of running.

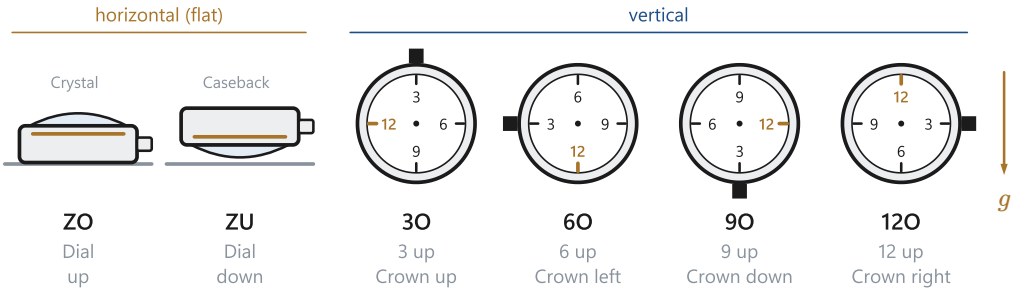


Fig. 6.8 The six test positions of a wristwatch with the crown at 3

The six test positions and their abbreviations

Position	German	International	Wristwatch crown
Dial up	ZO	CH	horizontal
Dial down	ZU	FH (also CB)	horizontal
3 up	3O	3H	up
6 up	6O	6H	left
9 up	9O	9H	down
12 up	12O	12H	right

The abbreviations state what is up. Internationally the French abbreviations are usual: H for haut (up), C for cadran (dial), F for fond (back), B for bas (down). In English the horizontal positions are called DU (dial up) and DD (dial down); the vertical ones crown up, crown down, crown left and crown right. The crown details apply to wristwatches with the crown at 3; on pocket watches it sits at 12.

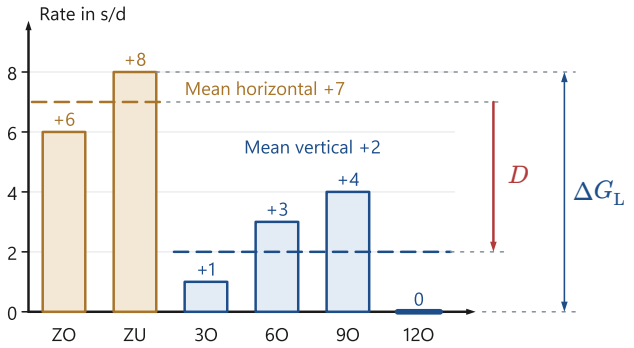


Fig. 6.9 Rates in six positions: greatest positional variation ΔG_L and vertical–horizontal difference D

Greatest positional variation

(6.11) JOURNEYMAN

$$\Delta G_\mathrm{L} = G_\mathrm{max} - G_\mathrm{min}$$

The greatest positional variation is the simplest measure of the quality of the adjustment: the smaller it is, the less the rate depends on how the watch is worn and laid down.

Valid: all positions measured at the same state of wind and the same temperature.

ΔG_L	Greatest positional variation	$\frac{\mathrm{s}}{\mathrm{d}}$
G_max	Highest rate of all positions (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_min	Lowest rate of all positions (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE Fully wound, a watch shows: ZO +6, ZU +8, 3O +1, 6O +3, 9O +4 and 12O 0 seconds per day.

Given: $G_\mathrm{max} = 8\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad G_\mathrm{min} = 0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G_\mathrm{L} &= G_\mathrm{max} - G_\mathrm{min} \\ &= 8\ \tfrac{\mathrm{s}}{\mathrm{d}} - 0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 8\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

8 s per day between the fastest position (dial down) and the slowest (12 up). The mean rate of the six positions is +3.7 s per day.

Difference between vertical and horizontal positions

(6.12) MASTER

$$D = G_{\mathrm{h}} - G_{\mathrm{f}}$$

The vertical positions are averaged and compared with the mean of the horizontal positions. A negative result means: the watch runs slower vertical than horizontal.

Valid: same state of wind in all positions. Chronometer testing forms D from only two positions, 6 up and dial up (Section 6.4).

D	Vertical–horizontal difference (+ means: rate higher vertical than horizontal)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{h}	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{f}	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$

Good to know: In the mean of the four vertical positions, the effect of poise error of the balance largely cancels out. What remains comes from the smaller amplitude and the different bearing of the pivots. Poise error shows in the differences between the vertical positions (Section 3.3).

EXAMPLE The watch from the previous example: mean of the four vertical positions +2.0, mean of the two horizontal positions +7.0 seconds per day.

Given: $G_{\mathrm{h}} = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_{\mathrm{f}} = 7.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} D &= G_{\mathrm{h}} - G_{\mathrm{f}} \\ &= 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} - 7.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -5.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

Vertical, the watch runs 5 s per day slower than horizontal. The amplitude is smaller in the vertical positions; the watch therefore loses at small amplitude, a sign of an isochronism error (Section 3.7).

What positional differences show (guide for fault finding)

Observation	Common cause
Dial up and dial down differ	Pivot ends or cap jewels, oil, balance spring not flat or rubbing
Vertical slower or faster than horizontal	Isochronism error: play of the balance spring between the regulator pins, shape of the balance spring, escapement
Vertical positions differ from one another	Poise error of the balance, balance spring not concentric
Amplitude much smaller vertical than horizontal	Friction at the balance pivots: pivots, hole jewels, oil

The table gives indications, not reliable diagnoses; often several causes act together.

Rate when worn from two groups of positions

(6.13) ADVANCED

$$G_{\mathrm{T}} = a \cdot G_{\mathrm{f}} + (1 - a) \cdot G_{\mathrm{h}}$$

On the wrist and on the bedside table the positions change. The rate when worn lies between the horizontal mean and the vertical mean, depending on the proportion of time.

Valid: simplified model with two groups of positions; temperature on the wrist, movement and state of wind are not taken into account.

G_{T}	Expected rate when worn	$\frac{\mathrm{s}}{\mathrm{d}}$
a	Proportion of time in horizontal positions	1
G_{f}	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{h}	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE The watch from the previous examples lies flat at night and at times during the day, 40 % of the time in total.

Given: $a = 0.4$; $G_{\mathrm{f}} = 7.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_{\mathrm{h}} = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} G_{\mathrm{T}} &= a \cdot G_{\mathrm{f}} + (1 - a) \cdot G_{\mathrm{h}} \\ &= 0.4 \cdot 7.0\ \tfrac{\mathrm{s}}{\mathrm{d}} + (1 - 0.4) \cdot 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 4.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

About +4 s per day. Laying the watch down flat at night makes it run faster here; laid down vertically it runs slower.

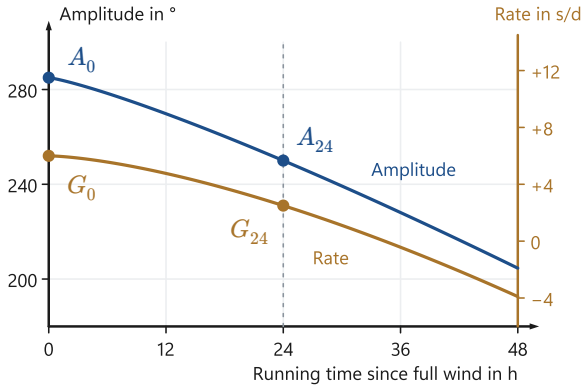


Fig. 6.10 Amplitude and rate over the running time (example): the values at 0 h and 24 h give the amplitude drop and the isochronism error

Isochronism error from 0 h and 24 h

(6.14) MASTER

$$\Delta G = G_{24} - G_0$$

As the mainspring runs down, the amplitude falls. If the rate changes in the process, the oscillator is not isochronous. The formula is the same as in Section 3.7, here with the two measuring times used in the workshop.

Valid: same position and same temperature for both measurements.

ΔG	Isochronism error (+ means: watch gains as the mainspring runs down)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{24}	Rate after 24 hours of running	$\frac{\mathrm{s}}{\mathrm{d}}$
G_0	Rate fully wound	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE Fully wound and dial up, a watch gains 6.0 s per day; after 24 hours of running in the same position, 2.5 s per day.

Given: $G_{24} = 2.5\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_0 = 6.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G &= G_{24} - G_0 \\ &= 2.5\ \tfrac{\mathrm{s}}{\mathrm{d}} - 6.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -3.5\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

After 24 hours the watch has become 3.5 s per day slower. If it is wound daily, its rate lies between the two values.

Amplitude drop in 24 hours

(6.15) JOURNEYMAN

$$\Delta A = A_0 - A_{24}$$

The amplitude drop shows how evenly the mainspring and gear train deliver the force over the day.

Valid: same position for both measurements, lift angle set correctly.

ΔA	Amplitude drop	°
A_0	Amplitude fully wound	°
A_{24}	Amplitude after 24 hours of running	°

EXAMPLE Dial up, the balance swings at 285° when fully wound and at 250° after 24 hours.

Given: $A_0 = 285°$; $A_{24} = 250°$

$$\begin{aligned}\Delta A &= A_0 - A_{24} \\ &= 285° - 250° \\ &= 35°\end{aligned}$$

35° in 24 hours. More important than the drop itself is that the amplitude after 24 hours is still sufficient in the vertical positions as well; movement manufacturers specify minimum values for this, often 190° to 200° in the 6-up position (ETA 2892A2 and ETA 7750: 200°).

Key point: Horizontal against vertical shows the influence of amplitude, vertical against vertical shows poise error, fully wound against 24 hours shows isochronism.

6.4 Chronometer testing

According to ISO 3159, a chronometer is a watch whose movement has been tested by an independent body over several days in different positions and at different temperatures; in Switzerland this is the COSC. The test lasts 15 days, with five positions and three temperatures. Seven criteria are calculated from the 15 daily rates. Other tests, such as that of METAS or to DIN 8319, apply to the finished watch and in some cases demand more.

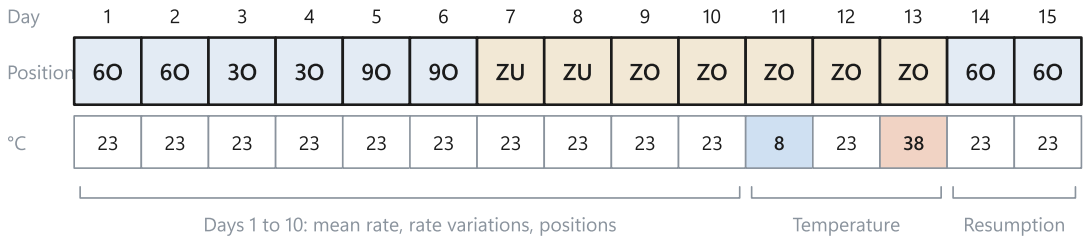


Fig. 6.11 Test programme of chronometer testing to ISO 3159: position and temperature on the 15 test days

The seven criteria of chronometer testing to ISO 3159 (watches over 20 mm in diameter)

No.	Criterion	Calculation from the daily rates G_1 to G_{15}	Limit to ISO 3159 in $\frac{\text{s}}{\text{d}}$
1	Mean daily rate	$\bar{G} = \frac{1}{10} \cdot (G_1 + G_2 + \cdots + G_{10})$	−4 … +6
2	Mean variation in rates	$\frac{1}{5} \cdot (\lvert G_2 - G_1 \rvert + \lvert G_4 - G_3 \rvert + \cdots + \lvert G_{10} - G_9 \rvert)$	at most 2
3	Greatest variation in rates	largest of the five absolute values from row 2	at most 5
4	Difference between horizontal and vertical positions D	$\frac{1}{2} \cdot (G_1 + G_2) - \frac{1}{2} \cdot (G_9 + G_{10})$	−6 … +8
5	Largest deviation from the mean daily rate	largest of the ten absolute values $\lvert G_i - \bar{G} \rvert$ of days 1 to 10	at most 10
6	Rate variation per °C C	$\frac{1}{30\,\text{K}} \cdot (G_{13} - G_{11})$	±0.6 per K
7	Resumption of rate	$G_{15} - \frac{1}{2} \cdot (G_1 + G_2)$	±5

Each of the seven limits must be met. For watches up to 20 mm in diameter the standard allows somewhat wider limits. The standard writes the daily rates as M_i; here they are written G_i, as throughout the book.

Mean variation in rates

(6.16) MASTER

$$\overline{\Delta G} = \frac{\sum |G_{2k} - G_{2k-1}|}{5}$$

$\overline{\Delta G}$	Mean variation in rates	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum \lvert G_{2k} - G_{2k-1} \rvert$	Sum of the five absolute values: difference between the two daily rates in each position	$\frac{\mathrm{s}}{\mathrm{d}}$

In each of the five positions, measurements are taken on two consecutive days. The criterion shows how well the watch repeats its rate from one day to the next.

Valid: to ISO 3159: days 1 to 10 at 23 °C; limit 2 s per day (watches over 20 mm).

EXAMPLE In the five positions, the two daily rates differ by 1.0, 0.8, 2.1, 0.7 and 1.4 seconds per day. The sum is 6.0 seconds per day.

Given: $\sum |G_{2k} - G_{2k-1}| = 6.0 \ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\overline{\Delta G} &= \frac{\sum |G_{2k} - G_{2k-1}|}{5} \\ &= \frac{6.0 \ \frac{\mathrm{s}}{\mathrm{d}}}{5} \\ &= 1.2 \ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

1.2 s per day; the limit of 2 s per day is met. The largest individual value, 2.1 s per day, is also below its limit of 5 s per day.

Temperature coefficient in chronometer testing (rate variation per °C)

(6.17) MASTER

$$C = \frac{G_{38} - G_8}{30\,\mathrm{K}}$$

C	Temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}}$
G_{38}	Rate at 38 °C (day 13)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_8	Rate at 8 °C (day 11)	$\frac{\mathrm{s}}{\mathrm{d}}$

Both rates are measured in the same position, dial up. The general form for any test temperatures is given in Section 3.5.

Valid: to ISO 3159: limit ±0.6 s per day per kelvin (watches over 20 mm).

EXAMPLE A watch gains 2.4 s per day at 8 °C and 7.2 s per day at 38 °C.

Given: $G_{38} = 7.2 \ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad G_8 = 2.4 \ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}C &= \frac{G_{38} - G_8}{30\,\mathrm{K}} \\ &= \frac{7.2 \ \frac{\mathrm{s}}{\mathrm{d}} - 2.4 \ \frac{\mathrm{s}}{\mathrm{d}}}{30\,\mathrm{K}} \\ &= 0.16 \ \frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}}\end{aligned}$$

+0.16 s per day per kelvin: the watch gains more in the warmth. The limit of 0.6 is clearly met.

Resumption of rate

(6.18) MASTER

$$\Delta G_{\mathrm{W}} = G_{15} - \frac{G_1 + G_2}{2}$$

ΔG_{W}	Resumption of rate	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{15}	Rate on the last test day	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	Rate on the first test day	$\frac{\mathrm{s}}{\mathrm{d}}$
G_2	Rate on the second test day	$\frac{\mathrm{s}}{\mathrm{d}}$

On the last day the watch is again in the position of the first two days. The criterion shows whether, after two weeks of testing with cold and heat, the watch returns to its initial rate.

Valid: to ISO 3159: 6-up position at 23 °C; limit ±5 s per day (watches over 20 mm).

EXAMPLE The watch from the example on the mean variation in rates gains 3.0 and 4.0 s per day on the first two days, and 4.5 s per day on the last day.

Given: $G_{15} = 4.5\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 3.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_2 = 4.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G_{\mathrm{W}} &= G_{15} - \frac{G_1 + G_2}{2} \\ &= 4.5\ \tfrac{\mathrm{s}}{\mathrm{d}} - \frac{3.0\ \frac{\mathrm{s}}{\mathrm{d}} + 4.0\ \frac{\mathrm{s}}{\mathrm{d}}}{2} \\ &= 1.0\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

+1.0 s per day, well within the limit of ±5 s per day. The watch has almost returned to its initial rate.

Key point: Chronometer means: tested for 15 days, in five positions and at three temperatures, with a mean daily rate between −4 and +6 s per day.

6.5 Water resistance and pressure

With every metre of depth, water presses harder on the crystal, caseback and gaskets. The pressure rating on a watch states the test overpressure. It applies to the new watch and to static pressure, in water or in air. The requirements for water-resistant watches are laid down in ISO 22810.

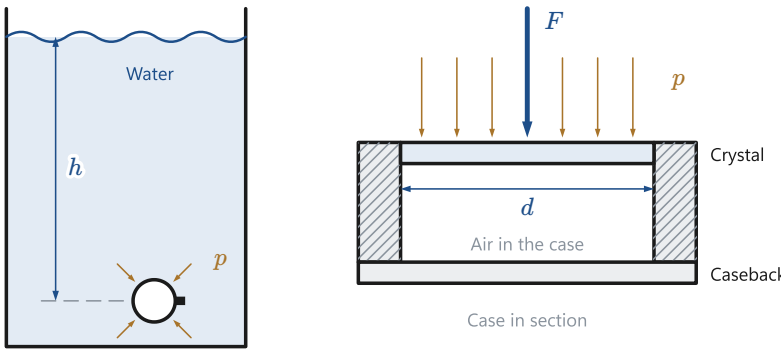


Fig. 6.12 Left: water pressure increases with depth h. Right: the pressure p loads the crystal of diameter d with the force F

Pressure of a water column

(6.19) JOURNEYMAN

$$p = \varrho \cdot g \cdot h$$

REARRANGED $h = \dfrac{p}{\varrho \cdot g}$

The pressure increases uniformly with depth, in fresh water by just under 1 bar per 10 m.

Valid: still water. Fresh water has $1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, sea water about $1025\ \frac{\mathrm{kg}}{\mathrm{m}^3}$.

p	Pressure (overpressure relative to the water surface)	bar
ϱ	Density (of the water)	$\frac{\mathrm{kg}}{\mathrm{m}^3}$
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	Height (of the water column above the watch, i.e. the diving depth)	m

EXAMPLE What overpressure acts at a depth of 100 m in fresh water?

Given: $\varrho = 1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 100\,\mathrm{m}$

$$\begin{aligned} p &= \varrho \cdot g \cdot h \\ &= 1000\ \tfrac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 100\,\mathrm{m} \\ &= 981\,000\,\mathrm{Pa} \\ &= 9.81\,\mathrm{bar} \end{aligned}$$

9.81 bar, rounded 10 bar. Hence the rule of thumb: 10 m of water column correspond to 1 bar.

Pressure force on crystal or caseback

(6.20) JOURNEYMAN

$$F = \frac{p \cdot \pi \cdot d^2}{4}$$

The force is pressure times area (Section XL 2.2), here written out with the area of a circle. It increases with the square of the diameter: large watches need thicker crystals and stiffer casebacks.

Valid: pressure uniform over a circular area; $1\,\mathrm{bar} = 0.1\ \frac{\mathrm{N}}{\mathrm{mm}^2}$.

F	Force (on the loaded area)	N
p	Pressure (overpressure outside relative to inside)	bar
d	Diameter (of the loaded area, here of the crystal)	mm

EXAMPLE A diver's watch is tested at 30 bar. The crystal is 28 mm in diameter. What force acts on it?

Given: $p = 30\,\mathrm{bar}$; $d = 28\,\mathrm{mm}$

$$\begin{aligned} F &= \frac{p \cdot \pi \cdot d^2}{4} \\ &= \frac{30\,\mathrm{bar} \cdot \pi \cdot (28\,\mathrm{mm})^2}{4} \\ &= 3\,\frac{\mathrm{N}}{\mathrm{mm}^2} \cdot \frac{\pi \cdot 784\,\mathrm{mm}^2}{4} \\ &\approx 1847\,\mathrm{N} \end{aligned}$$

About 1850 N, as much as the weight of 188 kg. This force presses the crystal into its gasket and bows it inwards. The example for 10 bar is given at formula (XL 2.11).

Testing is usually done dry: the tester places the watch in a chamber under overpressure or vacuum and senses how the case deforms. A tight case is compressed or bulged and stays so as long as the pressure is applied. If it leaks, the pressure inside and outside equalises and the deformation recedes. In the wet test, the watch is first pressurised above the water, then immersed and the pressure released; air bubbles rise from a leaking watch.

Dynamic pressure during movement in water

(6.21) ADVANCED

$$p_{\text{dyn}} = \frac{\varrho \cdot v^2}{2}$$

p_{dyn}	Dynamic pressure (additional pressure due to the flow)	bar
ϱ	Density (of the water)	$\frac{\text{kg}}{\text{m}^3}$
v	Velocity (of the watch relative to the water)	$\frac{\text{m}}{\text{s}}$

When the watch moves through the water, the dynamic pressure adds to the pressure of the depth. It remains small.

Valid: uniform flow; the water is brought completely to rest at the watch. This is the greatest pressure a uniform movement produces; the impact on hitting the water surface is not covered by it.

EXAMPLE In a very fast arm movement, the wrist travels through the water at 5 m per second.

Given: $\varrho = 1000\ \frac{\text{kg}}{\text{m}^3}$; $v = 5\ \frac{\text{m}}{\text{s}}$

$$\begin{aligned} p_{\text{dyn}} &= \frac{\varrho \cdot v^2}{2} \\ &= \frac{1000\ \frac{\text{kg}}{\text{m}^3} \cdot \left(5\ \frac{\text{m}}{\text{s}}\right)^2}{2} \\ &= 12\,500\,\text{Pa} \\ &= 0.125\,\text{bar} \end{aligned}$$

0.125 bar, equivalent to 1.3 m more depth. Swimming movements therefore hardly increase the pressure. If a watch leaks in the water, the cause is usually aged gaskets, changes in temperature, or the crown and pushers having been operated under water.

Key point: 10 m of water column is about 1 bar. The force on the crystal is pressure times area.

6.6 Magnetism and shock

Magnetic fields and shocks are the most frequent disturbances in everyday life. Standards specify what a watch must withstand to be called antimagnetic or shock-resistant: ISO 764 requires a field of $4800\ \frac{\text{A}}{\text{m}}$, ISO 1413 a shock equivalent to a fall from a height of 1 m.

Flux density and field strength in air

(6.22) MASTER

$$B = \mu_0 \cdot H$$

B	Magnetic flux density ($1\ \frac{\text{V·s}}{\text{m}^2} = 1\,\text{T} = 10\,000\,\text{G}$)	$\frac{\text{V·s}}{\text{m}^2}$
μ_0	Magnetic constant	$\frac{\text{V·s}}{\text{A·m}}$
H	Magnetic field strength	$\frac{\text{A}}{\text{m}}$

The standards state the field strength H; magnets and measuring instruments mostly give the flux density B in millitesla or gauss. In air the two are firmly linked: 1 mT corresponds to $796\ \frac{\text{A}}{\text{m}}$ and 10 gauss.

Valid: in air and in non-magnetic materials.

EXAMPLE What flux density corresponds to the field strength of $4800\ \frac{\text{A}}{\text{m}}$ that an antimagnetic watch must withstand according to ISO 764?

Given: $\mu_0 = 1.2566 \cdot 10^{-6}\ \frac{\text{V·s}}{\text{A·m}}$; $H = 4800\ \frac{\text{A}}{\text{m}}$

$$\begin{aligned} B &= \mu_0 \cdot H \\ &= 1.2566 \cdot 10^{-6}\ \frac{\text{V·s}}{\text{A·m}} \cdot 4800\ \frac{\text{A}}{\text{m}} \\ &\approx 6.03 \cdot 10^{-3}\ \frac{\text{V·s}}{\text{m}^2} \end{aligned}$$

6.03 mT or 60 gauss. A small permanent magnet, for example in the clasp of a bag or in a loudspeaker, is many times stronger at its surface.

Converting magnetic fields in air

Field strength in $\frac{\mathrm{A}}{\mathrm{m}}$	Flux density in mT	Flux density in gauss	Example
40	0.05	0.5	Earth's magnetic field (order of magnitude)
796	1	10	Conversion value
4800	6.03	60.3	Test field for antimagnetic watches to ISO 764
79 600	100	1000	close to small permanent magnets (order of magnitude)

The gauss is an older unit: 1 G = 0.1 mT. Apart from the standard value, the examples are orders of magnitude.

Impact velocity in a fall

(6.23) MASTER

$$v = \sqrt{2 \cdot g \cdot h}$$

v	Velocity (on impact)	$\frac{\mathrm{m}}{\mathrm{s}}$
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	Height (height of fall)	m

The shock test to ISO 1413 simulates a watch falling from a height of 1 m onto a wooden floor. The formula gives the velocity at which the watch hits the floor.

Valid: free fall without air resistance.

EXAMPLE A watch falls 1 m from the workbench onto the floor.

Given: $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 1\,\mathrm{m}$

$$v = \sqrt{2 \cdot g \cdot h}$$
$$= \sqrt{2 \cdot 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot 1\,\mathrm{m}}$$
$$\approx 4.43\ \frac{\mathrm{m}}{\mathrm{s}}$$

$4.43\ \frac{\mathrm{m}}{\mathrm{s}}$, i.e. about 16 km per hour. In the shock test, the hammer strikes the watch at this velocity.

6.7 Reading error and scatter

Every measurement scatters. The mean, range and standard deviation of a series of measurements are given in Section XL 12.1; the example there is a series of rate values. For the timing machine this means, for example: if ten measurements in the same position scatter with a standard deviation of 0.6 s per day, their mean is reliable to about 0.2 s per day according to formula (XL 12.4). For the rate from two read time errors, the reading error must be added.

Error of the rate due to reading

(6.24) MASTER

$$u_G = \frac{2 \cdot u_S}{t}$$

Two readings enter into the rate. In the worst case their errors add up; divided by the running period, this gives the maximum error of the rate.

Valid: maximum error; the watch runs undisturbed between the readings.

u_G	Maximum error of the rate due to reading	$\frac{\mathrm{s}}{\mathrm{d}}$
u_S	Maximum reading error of one time error	s
t	Time (running period between the two readings)	d

EXAMPLE The time error can be read on the seconds hand to within 0.5 s. There are 10 days between the two readings.

Given: $u_S = 0.5\,\mathrm{s}$; $t = 10\,\mathrm{d}$

$$\begin{aligned} u_G &= \frac{2 \cdot u_S}{t} \\ &= \frac{2 \cdot 0.5\,\mathrm{s}}{10\,\mathrm{d}} \\ &= 0.10\,\tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

After ten days the rate is reliable to 0.1 s per day, after one day only to 1 s per day. When an accurate result is needed more quickly, the timing machine is used.

Key point: Measuring for longer achieves more than reading more precisely: doubling the running period halves the reading error in the rate.

7 Quartz watch and battery

The quartz watch replaces mainspring, escapement and balance with a battery, a circuit and a quartz crystal. Testing and repairing it requires only a little electrical engineering: Ohm's law, power and charge, plus the behaviour of the battery, the capacitor and the coil.

The formulas of the watch itself build on this: battery life, frequency division, rate and temperature behaviour of the quartz crystal, current consumption of the stepper motor. In this chapter, U stands for voltage, A for the conductor cross-section, P for power and C for capacitance.

7.1 Battery

The capacity of a battery is the charge that can be drawn from it, given in milliampere-hours (mAh). It has only the name in common with the capacitance of a capacitor. Together with the current consumption of the movement it gives the battery life.

Battery life from capacity and current consumption

(7.1) JOURNEYMAN

$$t = \frac{C_\mathrm{N}}{I}$$

REARRANGED $I = \frac{C_\mathrm{N}}{t}$

t	Time (battery life)	h
C_N	Nominal capacity of the battery (available charge)	mAh
I	Current (mean current consumption of the movement)	mA

Capacity divided by current gives the battery life. Conversely, the formula shows the maximum current consumption a movement may have for the battery to last the promised time.

Valid: constant mean current consumption; self-discharge not taken into account. Use consistent units: mAh and mA give hours.

EXAMPLE An SR626SW cell has 28 mAh. The movement draws an average of 1.2 µA. How long does the watch run?

Given: $C_\mathrm{N} = 28\,\mathrm{mAh}$; $I = 1.2\,\mu\mathrm{A}$

$$\begin{aligned} t &= \frac{C_\mathrm{N}}{I} \\ &= \frac{28\,\mathrm{mAh}}{1.2\,\mu\mathrm{A}} \\ &= 23\,333\,\mathrm{h} \\ &\approx 2.66\,\mathrm{a} \end{aligned}$$

About 23 300 hours, a good two and a half years. If the movement draws 3 µA because of contamination, the cell is exhausted after only one year.

Energy content of the battery

(7.2) APPROXIMATION MASTER

$$W \approx U_\mathrm{N} \cdot C_\mathrm{N}$$

Nominal voltage times capacity gives the stored energy, usually in milliwatt-hours. It allows cells of different voltages to be compared.

Valid: Voltage approximately constant during discharge, as with silver oxide and lithium cells.

W	Energy (stored in the battery)	J
U_N	Nominal voltage of the battery	V
C_N	Nominal capacity of the battery (available charge)	mAh

EXAMPLE How much energy does a silver oxide cell of 1.55 V and 28 mAh contain?

Given: $U_\mathrm{N} = 1.55\,\mathrm{V}$; $C_\mathrm{N} = 28\,\mathrm{mAh}$

$$\begin{aligned} W &\approx U_\mathrm{N} \cdot C_\mathrm{N} \\ &= 1.55\,\mathrm{V} \cdot 28\,\mathrm{mAh} \\ &= 43.4\,\mathrm{mWh} \end{aligned}$$

43.4 mWh, which is about 156 J. A fully wound wristwatch mainspring stores less than 1 J (Chapter 1).

Battery life with self-discharge

(7.3) APPROXIMATION ADVANCED

$$t \approx \frac{C_\mathrm{N}}{I + s_\mathrm{E} \cdot C_\mathrm{N}}$$

Every cell slowly loses charge even without a load. Self-discharge acts like a small additional current $s_\mathrm{E} \cdot C_\mathrm{N}$ alongside the current consumption of the movement.

Valid: Self-discharge calculated as a constant loss. Guide value for silver oxide cells at room temperature: a few per cent per year, considerably more in warm conditions.

t	Time (battery life)	h
C_N	Nominal capacity of the battery (available charge)	mAh
I	Current (mean current consumption of the movement)	mA
s_E	Self-discharge (fraction of nominal capacity lost per unit time)	$\frac{\%}{\mathrm{a}}$

EXAMPLE How long does the watch from the battery-life example (28 mAh, 1.2 µA) run if the cell also loses 4 % of its capacity per year through self-discharge?

Given: $C_\mathrm{N} = 28\,\mathrm{mAh}$; $I = 1.2\,\mu\mathrm{A}$; $s_\mathrm{E} = 4\,\frac{\%}{\mathrm{a}}$

$$\begin{aligned} t &\approx \frac{C_\mathrm{N}}{I + s_\mathrm{E} \cdot C_\mathrm{N}} \\ &= \frac{28\,\mathrm{mAh}}{1.2\,\mu\mathrm{A} + 4\,\frac{\%}{\mathrm{a}} \cdot 28\,\mathrm{mAh}} \\ &\approx 2.41\,\mathrm{a} \end{aligned}$$

About 2.4 years instead of 2.7 years. The lower the current consumption of the movement, the more self-discharge matters.

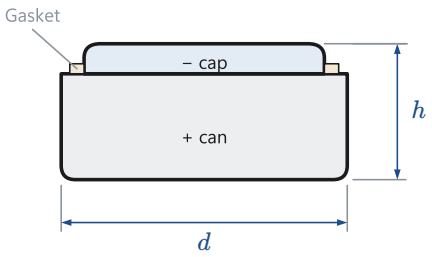


Fig. 7.1 Button cell: the can is the positive terminal, the cap the negative terminal; diameter d and height h are contained in the designation

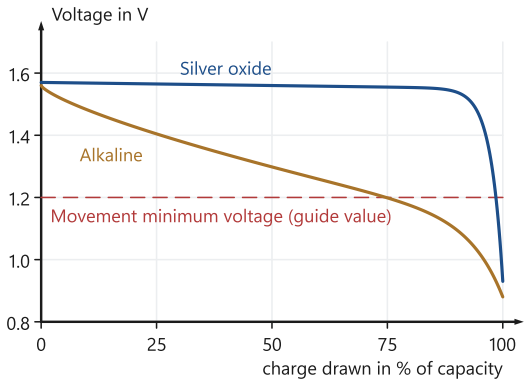


Fig. 7.2 Discharge curves in principle: the silver oxide cell holds its voltage until shortly before the end; the alkaline cell falls steadily

Battery systems for watches

System	Code letters	Nominal voltage	Discharge behaviour	Use
Silver oxide	SR	1.55 V	voltage remains almost constant until shortly before the end	standard for quartz wristwatches
Alkaline manganese	LR	1.5 V	voltage falls steadily from the start	low-cost replacement, clocks, alarm clocks
Lithium manganese dioxide	CR	3 V	voltage remains almost constant for a long time	watches with high current demand, long battery life

An alkaline cell of the same size fits mechanically but has less capacity and a falling voltage. A wristwatch takes a silver oxide cell.

IEC designation of button cells

Part of the designation	Meaning	Example SR626SW
first letter	electrochemical system: S silver oxide, L alkaline manganese, C lithium manganese dioxide	S: silver oxide
second letter	shape: R round	R: round cell
first digit or first two digits	diameter in mm, rounded down to whole millimetres	6: diameter 6.8 mm
last two digits	height in tenths of a millimetre	26: height 2.6 mm
manufacturer's suffix	SW: for small continuous currents (analogue watches); W: for high current peaks (light, alarm)	SW: for small currents

A CR2032 is thus a round lithium cell 20 mm in diameter and 3.2 mm high. Older designations such as SR44 are serial numbers without dimensional information.

Common silver oxide button cells (1.55 V)

Designation	Trade number	Diameter in mm	Height in mm	Capacity in mAh (typical)
SR521SW	379	5.8	2.15	14 to 16
SR616SW	321	6.8	1.65	14 to 16
SR621SW	364	6.8	2.15	19 to 23
SR626SW	377	6.8	2.6	24 to 28
SR721SW	362	7.9	2.1	21 to 25
SR920SW	371	9.5	2.05	35 to 40
SR927SW	395	9.5	2.7	50 to 55
SR1130SW	390	11.6	3.05	60 to 85
SR44W	357	11.6	5.4	150 to 180

Capacity depends on the manufacturer; the cell's data sheet is authoritative.

7.2 Quartz crystal and frequency divider

The oscillator of the quartz watch is a tiny quartz crystal in the shape of a tuning fork. It oscillates at 32 768 Hz, that is 2^{15} oscillations per second. A divider halves this frequency 15 times and so obtains exactly one pulse per second.

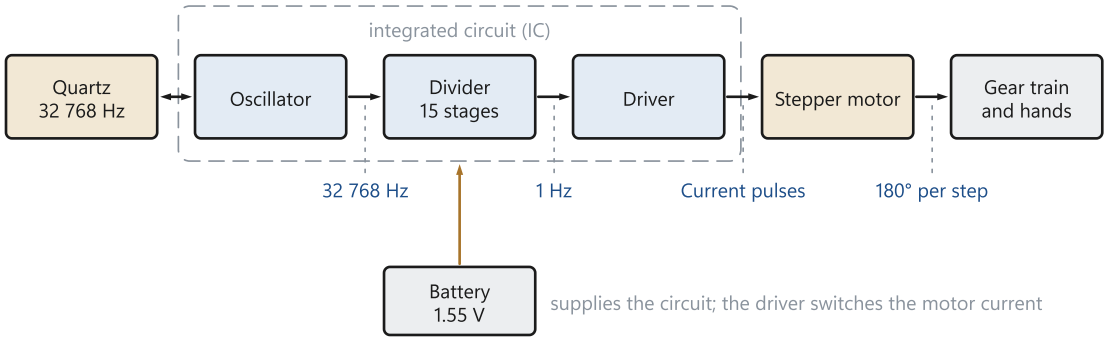


Fig. 7.3 Block diagram of the analogue quartz watch

Frequency divider

(7.4) JOURNEYMAN

$$f_{\text{aus}} = \frac{f}{2^n}$$

Each stage is a flip-flop that emits one output pulse for every second input pulse. This is why a power of two is chosen as the quartz frequency.

f_{aus}	Frequency at the divider output	Hz
f	Frequency (of the quartz crystal)	Hz
n	Number (of divider stages; each halves the frequency)	1

EXAMPLE What frequency does a 15-stage divider deliver from the quartz frequency of 32 768 Hz?

Given: $f = 32\,768\,\text{Hz}; \quad n = 15$

$$\begin{aligned} f_{\text{aus}} &= \frac{f}{2^n} \\ &= \frac{32\,768\,\text{Hz}}{2^{15}} \\ &= 1\,\text{Hz} \end{aligned}$$

Exactly 1 Hz: one pulse per second for the stepper motor.

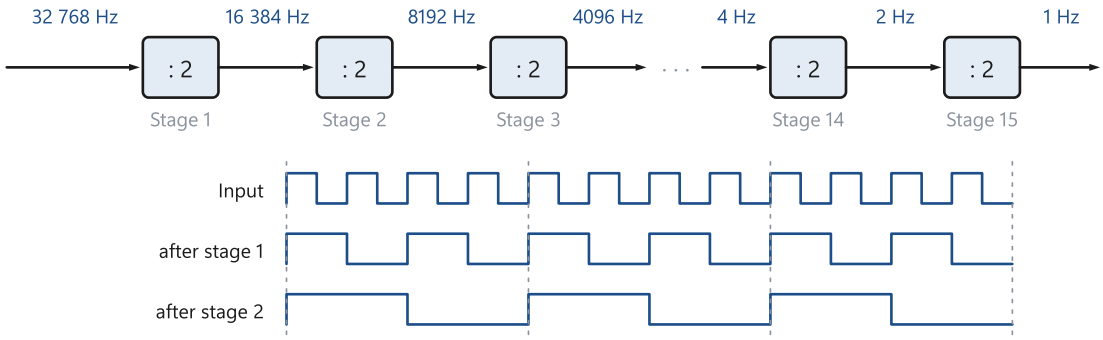


Fig. 7.4 Divider chain: each stage halves the frequency; after 15 stages 1 Hz remains

Rate of the quartz watch from the frequency deviation

(7.5) JOURNEYMAN

$$G = \frac{\Delta f}{f} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δf	Frequency deviation (+ means: crystal oscillates too fast)	Hz
f	Frequency (nominal frequency of the crystal)	Hz

If the crystal oscillates too fast, the watch gains in the same ratio. Relative deviations are given in millionths (ppm, Chapter XL 1). The same formula for the balance is given in Section 3.4.

Valid: the watch counts every quartz oscillation (no inhibition).

Good to know: 1 ppm corresponds to 0.0864 $\frac{\mathrm{s}}{\mathrm{d}}$, about 2.6 s per month and about 31.5 s per year.

EXAMPLE A crystal oscillates at 32 768.2 Hz instead of 32 768 Hz, i.e. 0.2 Hz too fast. What is the rate of the watch?

Given: $\Delta f = 0.2\,\mathrm{Hz}$; $f = 32\,768\,\mathrm{Hz}$

$$\begin{aligned} G &= \frac{\Delta f}{f} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{0.2\,\mathrm{Hz}}{32\,768\,\mathrm{Hz}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx 0.53\ \frac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The watch gains 0.53 s per day, about 16 s per month. The deviation is 6.1 ppm.

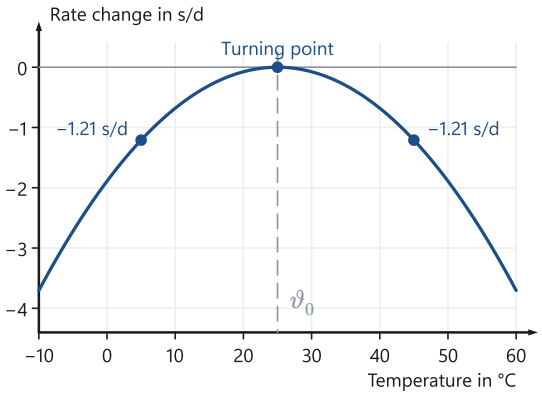


Fig. 7.5 Temperature behaviour of the tuning-fork crystal, calculated with $\beta = 0.035\ \frac{\mathrm{ppm}}{\mathrm{K}^2}$ and turnover point 25 °C

Temperature behaviour of the tuning-fork crystal

(7.6) APPROXIMATION MASTER

$$\Delta G \approx -\beta \cdot (\vartheta - \vartheta_0)^2 \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

The frequency of the tuning-fork crystal is highest at the turnover point and falls on both sides with the square of the temperature difference. The watch therefore runs slower than at the turnover point both in the cold and in the heat, never faster.

Valid: Tuning-fork crystals of 32 768 Hz; coefficient and turnover point vary from crystal to crystal, the data sheet is authoritative.

ΔG	Rate change (relative to the rate at the turnover point; – means: watch slows down)	$\frac{\mathrm{s}}{\mathrm{d}}$
β	Temperature coefficient of the crystal (guide value 0.03 to 0.04 ppm per kelvin squared)	$\frac{\mathrm{ppm}}{\mathrm{K^2}}$
ϑ	Temperature (of the crystal)	$^\circ\mathrm{C}$
ϑ_0	Turnover temperature (about 25 °C)	$^\circ\mathrm{C}$

Good to know: Ageing comes on top of this: in the first year the frequency of a crystal changes by a few ppm, thereafter ever more slowly. Thermocompensated movements measure the temperature and compensate for the parabola by calculation.

EXAMPLE In winter a quartz watch lies in a car at 5 °C. By how much does its rate change compared with 25 °C if the coefficient is 0.035 ppm per kelvin squared?

Given: $\beta = 0.035\ \frac{\mathrm{ppm}}{\mathrm{K^2}}$; $\vartheta = 5^\circ\mathrm{C}$; $\vartheta_0 = 25^\circ\mathrm{C}$

$$\begin{aligned}\Delta G &\approx -\beta \cdot (\vartheta - \vartheta_0)^2 \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -0.035\ \tfrac{\mathrm{ppm}}{\mathrm{K^2}} \cdot (5^\circ\mathrm{C} - 25^\circ\mathrm{C})^2 \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -1.21\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The watch slows down by about 1.2 s per day. On the wrist the crystal stays close to the turnover point; a watch that is worn therefore runs more evenly than one that is laid aside.

Inhibition: rate correction by suppressed pulses

(7.7) MASTER

$$\Delta G = -\frac{N}{f \cdot t_\mathrm{I}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

Movements without a trimmer adjust the rate digitally. The crystal is deliberately slightly too fast; in each inhibition period the circuit omits a stored number of pulses and thus slows the watch down to the nominal rate.

Valid: Inhibition period usually 60 s depending on the circuit, 10 s in some movements, and several minutes in thermocompensated movements (ETA 252.511: 8 min); observe the manufacturer's specification.

ΔG	Rate change (due to inhibition; – means: watch slows down)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	Number (of pulses suppressed per inhibition period)	1
f	Frequency (of the pulses at the divider stage where they are suppressed)	Hz
t_I	Inhibition period	s

Caution: Between corrections the watch runs at the uncorrected quartz frequency. The rate must therefore be measured over a whole inhibition period or a multiple of it, at the motor pulse and not at the quartz frequency. Otherwise the tester shows an apparent gain.

EXAMPLE A circuit suppresses one pulse of the 32 768 Hz quartz frequency every 60 s. How large is this smallest correction step?

Given: $N = 1$; $f = 32\,768\,\mathrm{Hz}$; $t_\mathrm{I} = 60\,\mathrm{s}$

$$\begin{aligned}\Delta G &= -\frac{N}{f \cdot t_\mathrm{I}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{1}{32\,768\,\mathrm{Hz} \cdot 60\,\mathrm{s}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -0.044\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

Each suppressed pulse slows the watch down by 0.044 s per day, which is 0.51 ppm. A crystal that oscillates 5 ppm too fast therefore needs about 10 suppressed pulses per minute.

Key point: The watch crystal oscillates at 32 768 Hz. 15 divider stages turn this into the one-second pulse. 1 ppm is about 2.6 s per month.

7.3 Stepper motor

The Lavet stepper motor consists of a coil, a stator and a rotor with a two-pole permanent magnet. Each current pulse turns the rotor on by half a revolution. The next pulse must have the opposite polarity, otherwise the rotor stays still. Between pulses the positioning notches of the stator hold it in place without current.

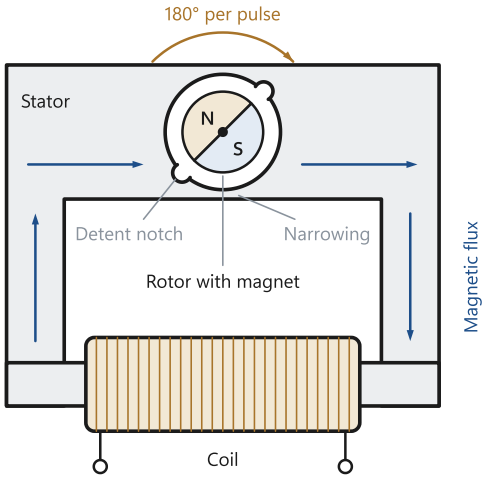


Fig. 7.6 Lavet stepper motor in principle: the narrow necks of the stator are magnetically saturated, so the flux of the coil passes through the rotor and turns it by 180° into the next detent position

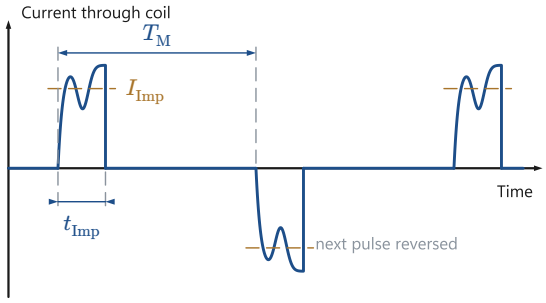


Fig. 7.7 Motor current: short pulses of duration t_{Imp} at intervals of T_M with alternating direction (pulse width greatly exaggerated)

Rotational speed of the rotor

(7.8) JOURNEYMAN

$$n_R = \frac{f_M \cdot \varphi_S}{360°}$$

With one pulse per second and half a revolution per step, the rotor turns once every two seconds.

n_R	Rotational speed (of the rotor)	$\frac{1}{\text{min}}$
f_M	Frequency of the motor pulses (steps per unit time)	Hz
φ_S	Step angle of the rotor (180° for the Lavet motor)	°

EXAMPLE How fast does the rotor of a movement with seconds hand turn if it makes one step of 180° every second?

Given: $f_M = 1\,\text{Hz}$; $\varphi_S = 180°$

$$\begin{aligned} n_R &= \frac{f_M \cdot \varphi_S}{360°} \\ &= \frac{1\,\text{Hz} \cdot 180°}{360°} \\ &= 0{,}5\,\frac{1}{\text{s}} \cdot 60\,\frac{\text{s}}{\text{min}} \\ &= 30\,\frac{1}{\text{min}} \end{aligned}$$

30 revolutions per minute.

Reduction ratio from rotor to fourth wheel

(7.9) JOURNEYMAN

$$i_{\mathrm{U}} = \frac{n_{\mathrm{R}}}{n_{\mathrm{S}}}$$

In the quartz movement the small rotor pinion drives the wheels, and the gear train gears down. The gear ratio i according to Chapter 2 is then less than 1. Its reciprocal, the reduction ratio, is more intuitive: it states how many times the rotor turns for one revolution of the fourth wheel. It is the product of the ratios of the number of teeth of the wheel to the number of leaves of the driving pinion at each stage.

i_{U}	Reduction ratio (rotational speed of the driving rotor divided by that of the fourth wheel; reciprocal of the gear ratio i from Chapter 2)	1
n_{R}	Rotational speed (of the rotor)	$\frac{1}{\mathrm{min}}$
n_{S}	Rotational speed of the fourth wheel (one revolution per minute)	$\frac{1}{\mathrm{min}}$

EXAMPLE The rotor makes 30 revolutions per minute, the fourth wheel one. What reduction ratio does the gear train between them need?

Given: $n_{\mathrm{R}} = 30\,\frac{1}{\mathrm{min}}$; $n_{\mathrm{S}} = 1\,\frac{1}{\mathrm{min}}$

$$\begin{aligned} i_{\mathrm{U}} &= \frac{n_{\mathrm{R}}}{n_{\mathrm{S}}} \\ &= \frac{30\,\frac{1}{\mathrm{min}}}{1\,\frac{1}{\mathrm{min}}} \\ &= 30 \end{aligned}$$

30, for example in two stages of 6 and 5: the seconds hand advances by 6° per step, one seconds division.

Mean current consumption of the movement

(7.10) MASTER

$$I_{\mathrm{m}} = \frac{I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}}{T_{\mathrm{M}}} + I_{\mathrm{IC}}$$

The motor draws current only during the short pulse. Averaged over the whole second, the fraction pulse duration divided by period remains; to this is added the small continuous current of the circuit.

Valid: regular sequence of identical pulses. A slow-responding meter or a measurement over several seconds shows this mean value.

I_{m}	Mean current consumption of the movement	$\mu\mathrm{A}$
I_{Imp}	Current through the coil during the pulse (mean value over the pulse duration)	mA
t_{Imp}	Pulse duration	ms
T_{M}	Period of the motor pulses (1 s for a seconds step)	s
I_{IC}	Current consumption of the circuit including oscillator	$\mu\mathrm{A}$

Good to know: In movements with a stop switch the motor stops when the crown is pulled out, and only the current of the circuit flows. In this way both components can be measured separately. If the gear train runs stiffly, many circuits lengthen the pulse, and the current consumption rises.

EXAMPLE During the pulse of 3.9 ms an average of 0.24 mA flows. The circuit needs 0.15 µA. What is the mean current consumption at one step per second?

Given: $I_{\mathrm{Imp}} = 0.24\,\mathrm{mA}$; $t_{\mathrm{Imp}} = 3.9\,\mathrm{ms}$; $T_{\mathrm{M}} = 1\,\mathrm{s}$; $I_{\mathrm{IC}} = 0.15\,\mu\mathrm{A}$

$$\begin{aligned} I_{\mathrm{m}} &= \frac{I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}}{T_{\mathrm{M}}} + I_{\mathrm{IC}} \\ &= \frac{0.24\,\mathrm{mA} \cdot 3.9\,\mathrm{ms}}{1\,\mathrm{s}} + 0.15\,\mu\mathrm{A} \\ &= 1.09\,\mu\mathrm{A} \end{aligned}$$

About 1.1 µA, a typical value for a three-hand movement. The motor accounts for a good 85 % of it.

Energy of a motor pulse

(7.11) MASTER

$$W = U \cdot I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}$$

This is the energy one step draws from the battery. Only part of it moves the rotor and the gear train; the rest is turned into heat in the coil.

Valid: Voltage approximately constant during the pulse.

W	Energy (electrical energy per pulse)	J
U	Voltage (of the battery under load)	V
I_{Imp}	Current through the coil during the pulse (mean value over the pulse duration)	mA
t_{Imp}	Pulse duration	ms

EXAMPLE How much energy does a pulse of 3.9 ms duration with an average of 0.24 mA at 1.55 V cost?

Given: $U = 1.55\,\mathrm{V}$; $I_{\mathrm{Imp}} = 0.24\,\mathrm{mA}$; $t_{\mathrm{Imp}} = 3.9\,\mathrm{ms}$

$$\begin{aligned} W &= U \cdot I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}} \\ &= 1.55\,\mathrm{V} \cdot 0.24\,\mathrm{mA} \cdot 3.9\,\mathrm{ms} \\ &\approx 1.45\,\mu\mathrm{J} \end{aligned}$$

About 1.5 µJ per step. In one day with 86 400 steps this amounts to 0.13 J.

7.4 Measurements on the quartz watch

With a few measurements almost any fault of a quartz watch can be narrowed down: battery, current consumption, minimum operating voltage, coil and rate. The values in the table are guide values for three-hand movements. The specifications in the technical documentation of the calibre are binding.

Test values for the analogue quartz watch (guide values)

Test parameter	Guide value	Meaning
Battery voltage without load (silver oxide)	1.55 V to 1.6 V	shows only the cell chemistry; says little about the condition of the cell
Battery voltage under light load (measured in the watch)	at least 1.5 V	below this, replace the cell; across a heavy load resistance of the tester (a few hundred ohms) the voltage may drop lower; the scale of the instrument applies there
Current consumption of the movement	depending on the calibre, at most 0.7 µA to 1.3 µA	measured without battery at 1.55 V; too high: stiff gear train, contamination, moisture or short circuit; chronographs and movements with additional functions need more, about 3 µA to 6.5 µA
Current consumption with crown pulled out (movements with stop switch)	at most 0.4 µA to 0.5 µA	circuit only; too high: faulty circuit or circuit board
Minimum operating voltage	at most 1.3 V	lower the supply voltage from 1.55 V until the movement stops; down to 1.3 V it must still reliably execute every step. If it stops earlier, the gear train runs too stiffly or the motor or circuit is faulty
Coil resistance	depending on the calibre, about 0.8 kΩ to 2.7 kΩ	no continuity: wire broken; clearly too low: short between turns. Measure with a low test voltage, at most 0.4 V, preferably 0.2 V
Insulation resistance (coil to mainplate)	no continuity (reading infinite)	if the meter shows a resistance: moisture, damaged insulation
Rate	depending on the manufacturer, about −10 s to +20 s per month, i.e. −0.3 $\frac{\mathrm{s}}{\mathrm{d}}$ to +0.7 $\frac{\mathrm{s}}{\mathrm{d}}$	measure at the motor pulse over the inhibition period, usually 60 s (Section 7.2)

Guide values from manufacturers' specifications, measured at 20 °C to 25 °C. Examples of the maximum permissible current consumption: ETA 902.002 0.7 µA, Ronda 1005 0.95 µA, ETA F06.111 1.1 µA, ETA 255.411 1.2 µA, ETA 805.112 1.3 µA; chronograph ETA G10.211 3.5 µA, with stopwatch function running 6.5 µA. Examples of coil resistance: ETA 805.112 0.8 kΩ to 1.1 kΩ, ETA 255.411 1.3 kΩ to 1.75 kΩ, Ronda 1005 2.5 kΩ to 2.7 kΩ. The nominal values and test conditions in the technical documentation of the calibre are binding.

Key point: First the battery under load, then the current consumption, then the minimum operating voltage. If all three are correct, the fault rarely lies in the electronics.

8 Time and Calendar

Our clocks and watches show mean solar time: an imaginary, perfectly uniform solar day of 86 400 s. The true Sun, the stars and the Moon do not keep to this measure. For a sidereal clock, a perpetual calendar, an equation of time display or a moon phase we need the figures and formulas of this chapter.

The chapter ends with the chronograph scales: tachymeter, telemeter and pulsometer. In this chapter E is the equation of time, λ the geographical longitude, s a distance and c the speed of sound. The numbers of teeth for calendar and moon phase trains are given in Chapter 2.

8.1 Day and year

A day is one rotation of the Earth, but measured against what? Relative to the Sun it lasts 24 h on average, relative to the stars about 4 min less. The reason: in one day the Earth moves almost 1° further along its orbit and must turn through this extra angle before the Sun is due south again.

Astronomical units of time (mean values for the present epoch)

Unit of time	Duration	Written out	Reference
mean solar day	86 400 s	24 h	Sun due south to Sun due south, averaged over the year
sidereal day	86 164.09 s	23 h 56 min 4.09 s	one rotation of the Earth relative to the stars
tropical year	365.242 19 d	365 d 5 h 48 min 45 s	vernal equinox to vernal equinox; year of the seasons
sidereal year	365.256 36 d	365 d 6 h 9 min 10 s	one revolution of the Earth relative to the stars
synodic month	29.530 59 d	29 d 12 h 44 min 3 s	new moon to new moon; month of the lunar phases
sidereal month	27.321 66 d	27 d 7 h 43 min 11 s	one revolution of the Moon relative to the stars

Days here are always mean solar days. The calendar follows the tropical year, the moon phase the synodic month. An individual synodic month deviates from the mean by up to about 7 h.

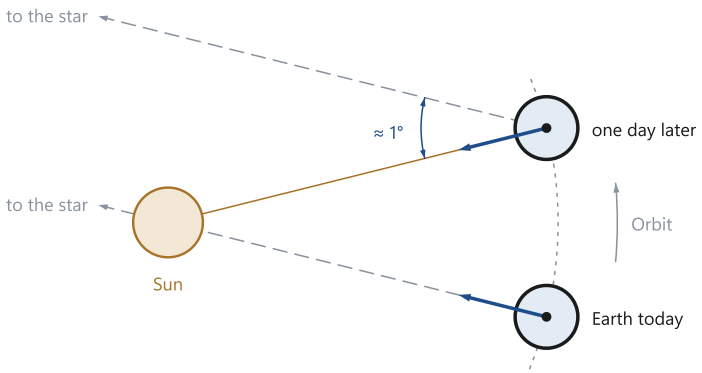


Fig. 8.1 Sidereal day and solar day: after one full rotation relative to the stars (dashed direction), the Earth must turn almost 1° further until the Sun is due south again (angle greatly exaggerated)

Sidereal day from solar day and year

(8.1) MASTER

$$T_{\mathrm{St}} = \frac{T_{\mathrm{So}} \cdot T_{\mathrm{J}}}{T_{\mathrm{J}} + T_{\mathrm{So}}}$$

T_{St}	Sidereal day	s
T_{So}	Mean solar day	s
T_{J}	Tropical year	d

In one year the Earth turns exactly once more relative to the stars than relative to the Sun: 366.2422 sidereal days correspond to 365.2422 solar days. This gives the length of the sidereal day.

Valid: Mean values for the present epoch.

EXAMPLE How long is the sidereal day if the tropical year has 365.2422 mean solar days?

Given: $T_{\mathrm{So}} = 1\,\mathrm{d}$; $T_{\mathrm{J}} = 365.2422\,\mathrm{d}$

$$\begin{aligned} T_{\mathrm{St}} &= \frac{T_{\mathrm{So}} \cdot T_{\mathrm{J}}}{T_{\mathrm{J}} + T_{\mathrm{So}}} \\ &= \frac{1\,\mathrm{d} \cdot 365.2422\,\mathrm{d}}{365.2422\,\mathrm{d} + 1\,\mathrm{d}} \\ &= 0{,}997\,269\,6\,\mathrm{d} \\ &\approx 86\,164.1\,\mathrm{s} \end{aligned}$$

The sidereal day lasts 23 h 56 min 4.1 s. It is 3 min 55.9 s shorter than the mean solar day.

Rate of a sidereal clock against mean time

(8.2) MASTER

$$G = \frac{T_{\mathrm{So}} - T_{\mathrm{St}}}{T_{\mathrm{St}}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

G	Rate (+ means: clock gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
T_{So}	Mean solar day	s
T_{St}	Sidereal day	s

A sidereal clock counts sidereal days, so its day is shorter. Measured against a mean-time clock it gains almost 4 min every day.

Good to know: For the pendulum clock this means: the seconds pendulum for sidereal time is about 5.4 mm shorter than the seconds pendulum for mean time (Chapter 4).

EXAMPLE A sidereal clock is regulated against a radio-controlled clock showing mean time. What rate must it have relative to the radio-controlled clock?

Given: $T_{\mathrm{So}} = 86\,400\,\mathrm{s}$; $T_{\mathrm{St}} = 86\,164.09\,\mathrm{s}$

$$\begin{aligned} G &= \frac{T_{\mathrm{So}} - T_{\mathrm{St}}}{T_{\mathrm{St}}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{86\,400\,\mathrm{s} - 86\,164.09\,\mathrm{s}}{86\,164.09\,\mathrm{s}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 236.6\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The sidereal clock must gain 236.6 s per day, that is 3 min 56.6 s. In one year it gains exactly one day.

Synodic month from the Moon's revolution

(8.3) ADVANCED

$$T_{\mathrm{syn}} = \frac{T_{\mathrm{sid}} \cdot T_{\mathrm{E}}}{T_{\mathrm{E}} - T_{\mathrm{sid}}}$$

During one revolution of the Moon the Earth moves on along its orbit. The Moon therefore needs a good two days longer until it stands between Earth and Sun again. The same formula describes a fast hand catching up with a slow one.

Valid: Mean values; both revolutions in the same direction.

T_{syn}	Synodic month (new moon to new moon)	d
T_{sid}	Sidereal month (one lunar revolution relative to the stars)	d
T_{E}	Sidereal year (one revolution of the Earth relative to the stars)	d

EXAMPLE The Moon orbits the Earth once in 27.321 66 days, the Earth orbits the Sun once in 365.256 36 days. How long is it from new moon to new moon?

Given: $T_{\mathrm{sid}} = 27.321\,66\,\mathrm{d}$; $T_{\mathrm{E}} = 365.256\,36\,\mathrm{d}$

$$\begin{aligned} T_{\mathrm{syn}} &= \frac{T_{\mathrm{sid}} \cdot T_{\mathrm{E}}}{T_{\mathrm{E}} - T_{\mathrm{sid}}} \\ &= \frac{27.321\,66\,\mathrm{d} \cdot 365.256\,36\,\mathrm{d}}{365.256\,36\,\mathrm{d} - 27.321\,66\,\mathrm{d}} \\ &\approx 29.5306\,\mathrm{d} \end{aligned}$$

About 29.53 days. A moon phase train must reproduce this figure with its numbers of teeth (Chapter 2).

8.2 Calendar

The calendar year has whole days, the tropical year does not. Leap days make up the difference. How good a leap-year rule is shows in the mean length of its calendar year.

Leap-year rule

Calendar	Leap year (366 days, with 29 February)	Mean length of year
Julian	every year whose number is divisible by 4	$365.25\,\mathrm{d}$
Gregorian (since 1582)	divisible by 4; full centuries only if divisible by 400	$365.2425\,\mathrm{d}$

Under the Gregorian rule 1700, 1800 and 1900 were not leap years, 2000 was one, and 2100 again will not be.

Mean length of the calendar year

(8.4) JOURNEYMAN

$$T_{\mathrm{K}} = 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot n_{\mathrm{S}}}{n_{\mathrm{Z}}}$$

Each leap year adds one day. Spread over the whole cycle, this gives the mean length of the year.

T_{K}	Mean length of the calendar year	d
n_{S}	Number of leap years in the cycle	1
n_{Z}	Number of years in the cycle	1

EXAMPLE The Gregorian calendar has 97 leap years in 400 years. What is the mean length of its year?

Given: $n_{\mathrm{S}} = 97$; $n_{\mathrm{Z}} = 400$

$$\begin{aligned} T_{\mathrm{K}} &= 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot n_{\mathrm{S}}}{n_{\mathrm{Z}}} \\ &= 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot 97}{400} \\ &= 365.2425\,\mathrm{d} \end{aligned}$$

365.2425 days. The Julian rule, with one leap year in four, gives 365.25 days.

Years until one day of error

(8.5) MASTER

$$n_{\mathrm{F}} = \frac{1\,\mathrm{d}}{T_{\mathrm{K}} - T_{\mathrm{J}}}$$

If the calendar year is longer than the tropical year, the calendar falls behind the seasons: the vernal equinox falls on an ever earlier date.

Valid: Present length of the tropical year; it changes by a few seconds over millennia.

n_{F}	Number of years until the calendar deviates by one day from the solar year	1
T_{K}	Mean length of the calendar year	d
T_{J}	Tropical year	d

EXAMPLE After how many years does the Julian calendar deviate by one day from the course of the seasons?

Given: $T_{\mathrm{K}} = 365.25\,\mathrm{d}$; $T_{\mathrm{J}} = 365.242\,19\,\mathrm{d}$

$$\begin{aligned} n_{\mathrm{F}} &= \frac{1\,\mathrm{d}}{T_{\mathrm{K}} - T_{\mathrm{J}}} \\ &= \frac{1\,\mathrm{d}}{365.25\,\mathrm{d} - 365.242\,19\,\mathrm{d}} \\ &\approx 128 \end{aligned}$$

After 128 years. From the Council of Nicaea in 325 to 1582, ten days had accumulated; they were dropped in the calendar reform. The Gregorian calendar, with 365.2425 days, takes about 3200 years to drift by one day.



Fig. 8.2 Four-year cycle of the perpetual calendar: $3 \cdot 365 + 366 = 1461$ days

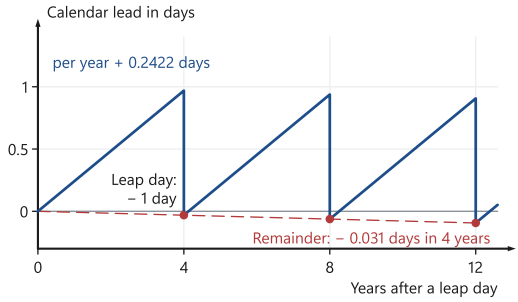


Fig. 8.3 Without a leap day the calendar year is 0.2422 days too short; every four years the leap day takes back a whole day

Days in the leap-year cycle

(8.6) JOURNEYMAN

$$N_{\mathrm{d}} = 365 \cdot n_{\mathrm{Z}} + n_{\mathrm{S}}$$

The perpetual calendar knows the lengths of the months and the leap year. Its program wheel turns once in four years.

N_{d}	Number of days in the cycle	1
n_{Z}	Number of years in the cycle	1
n_{S}	Number of leap years in the cycle	1

Caution: A perpetual calendar with a four-year cycle does not know the century rule. In 2100 it shows a 29 February and must be advanced by one day on 1 March 2100, likewise in 2200 and 2300. Only a secular calendar reproduces the 400-year cycle.

EXAMPLE A perpetual calendar reproduces four years with one leap year. How many days does this cycle have?

Given: $n_{\mathrm{Z}} = 4$; $n_{\mathrm{S}} = 1$

$$\begin{aligned} N_{\mathrm{d}} &= 365 \cdot n_{\mathrm{Z}} + n_{\mathrm{S}} \\ &= 365 \cdot 4 + 1 \\ &= 1461 \end{aligned}$$

1461 days or 48 months; this is how long the leap-year wheel takes for one turn. The Gregorian cycle of 400 years has 146 097 days. That is exactly 20 871 weeks, so after 400 years the days of the week repeat as well.

Key point: 365 days are 52 weeks and one day. A date therefore moves on by one weekday each year, and by two across a leap day.

8.3 Equation of time

The true Sun moves unevenly over the year: the Earth's orbit is an ellipse, and the Earth's axis is tilted to the orbit. A sundial therefore differs by up to a good quarter of an hour from a uniformly running clock. This difference is called the equation of time.

Equation of time

(8.7) MASTER

$$E = t_{\mathrm{W}} - t_{\mathrm{M}}$$

REARRANGED $t_{\mathrm{M}} = t_{\mathrm{W}} - E$

The equation of time is apparent solar time minus mean solar time. It depends only on the date, not on the location.

E	Equation of time (+ means: sundial is fast)	min
t_{W}	Apparent local time (sundial reading)	min
t_{M}	Mean local time	min

Caution: Older tables and some watches with an equation of time display use the opposite sign (mean minus apparent time). Before setting, check which direction is meant.

EXAMPLE On 11 February the equation of time is −14.2 min. A sundial shows 12:00, which is 720 min after midnight. What is the mean local time?

Given: $t_{\mathrm{W}} = 720\,\mathrm{min}$; $E = -14.2\,\mathrm{min}$

$$\begin{aligned} t_{\mathrm{M}} &= t_{\mathrm{W}} - E \\ &= 720\,\mathrm{min} - (-14.2\,\mathrm{min}) \\ &= 734.2\,\mathrm{min} \end{aligned}$$

734.2 min after midnight: it is 12:14 mean local time. The sundial is a good 14 min slow.

Annual angle for the equation of time

(8.8) ADVANCED

$$B = \frac{360^\circ \cdot (N - 81)}{364}$$

The annual angle divides the year into 360° and starts on 22 March, the 81st day. It is only an auxiliary quantity for the approximation formula.

B	Annual angle (auxiliary quantity; 0° on 22 March)	$^\circ$
N	Day number in the year (1 January = 1)	1

EXAMPLE In a common year, 3 November is the 307th day. What is the annual angle?

Given: $N = 307$

$$\begin{aligned} B &= \frac{360^\circ \cdot (N - 81)}{364} \\ &= \frac{360^\circ \cdot (307 - 81)}{364} \\ &\approx 223.5^\circ \end{aligned}$$

About 223.5°. The following formula works with this angle.

Equation of time from the date

(8.9) APPROXIMATION ADVANCED

$$E \approx 9.87\,\text{min} \cdot \sin(2 \cdot B) - 7.53\,\text{min} \cdot \cos B - 1.5\,\text{min} \cdot \sin B$$

The formula reproduces the two causes: the term with $2B$ comes from the tilted Earth axis and has two waves per year; the terms with B come from the elliptical orbit and have one wave per year.

E	Equation of time (+ means: sundial is fast)	min
B	Annual angle (auxiliary quantity; 0° on 22 March)	°

Valid: Approximation, accurate to about one minute (largest deviation, depending on the year, up to about 1.4 min, in early December). This is sufficient for sundials and for checking an equation of time display; exact daily values are given in astronomical almanacs.

Note: For 11 February (day 42, $B = -38.6^\circ$) the approximation gives −14.6 min, the table of key values −14.2 min. Differences of a few tenths of a minute lie within the accuracy of the approximation; for setting, the table values apply.

EXAMPLE What is the equation of time on 3 November, with the annual angle of 223.5° from the previous example?

Given: $B = 223.5^\circ$

$$\begin{aligned} E &\approx 9.87\,\text{min} \cdot \sin(2 \cdot B) - 7.53\,\text{min} \cdot \cos B - 1.5\,\text{min} \cdot \sin B \\ &= 9.87\,\text{min} \cdot \sin(2 \cdot 223.5^\circ) - 7.53\,\text{min} \cdot \cos(223.5^\circ) - 1.5\,\text{min} \cdot \sin(223.5^\circ) \\ &\approx 16.4\,\text{min} \end{aligned}$$

About +16.4 min: in early November the sundial is furthest ahead.

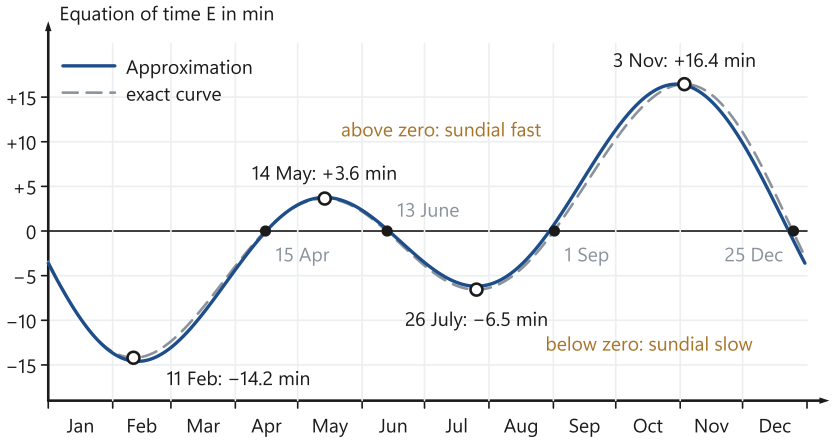


Fig. 8.4 Equation of time over the year: approximation formula and exact curve

Key values of the equation of time (rounded, for the present epoch)

Date	E	Sundial relative to mean local time
11 February	−14.2 min	furthest behind
15 April	0	agrees
14 May	+3.6 min	fast (small maximum)
13 June	0	agrees
26 July	−6.5 min	slow (small minimum)
1 September	0	agrees
3 November	+16.4 min	furthest ahead
25 December	0	agrees

Depending on the position in the leap-year cycle, the dates shift by up to one day.

8.4 Local time and zone time

Mean local time depends on the longitude: the Sun moves across 15 degrees of longitude in one hour and needs 4 min for one degree. So that not every place has its own time, the local time of the zone meridian applies throughout a time zone. For Central European Time (CET) this is the meridian 15° East.

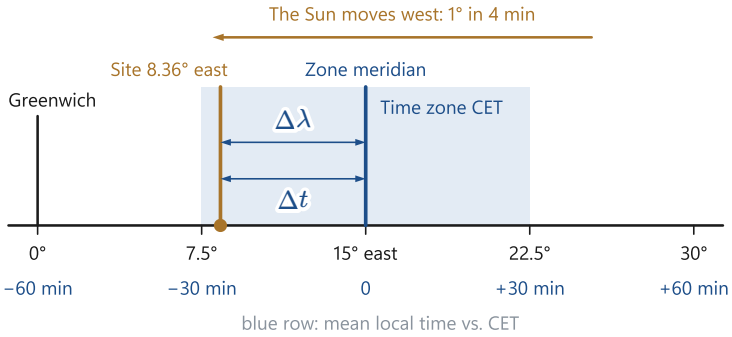


Fig. 8.5 Local time and zone time: a place west of the zone meridian has the Sun due south later

Local time from the longitude

(8.10) JOURNEYMAN

$$\Delta t = \frac{(\lambda - \lambda_Z) \cdot 4\,\text{min}}{1^\circ}$$

East of the zone meridian, local time is ahead of zone time; west of it, local time lags behind.

Valid: Longitudes east of Greenwich positive, west negative.

Δt	Local time difference (mean local time minus zone time; + means: local time is ahead)	min
λ	Geographical longitude of the place (east longitude positive)	$^\circ$
λ_Z	Geographical longitude of the zone meridian (CET: 15° East)	$^\circ$

EXAMPLE A workshop lies at 8.36° east longitude. By how much does its mean local time differ from CET?

Given: $\lambda = 8.36^\circ$; $\lambda_Z = 15^\circ$

$$\begin{aligned}\Delta t &= \frac{(\lambda - \lambda_Z) \cdot 4\,\text{min}}{1^\circ} \\ &= \frac{(8.36^\circ - 15^\circ) \cdot 4\,\text{min}}{1^\circ} \\ &\approx -26.6\,\text{min}\end{aligned}$$

Mean local time lags 26.6 min behind CET. Here the mean Sun is due south only at 12:26:34 CET.

Zone time from the sundial

(8.11) MASTER

$$t_Z = t_W - E - \Delta t$$

Two corrections lead from the sundial to the wristwatch: the equation of time for the date and the local time difference for the longitude.

Valid: Sundial showing apparent local time. Some sundials are already laid out for the longitude of their location; Δt is then omitted.

t_Z	Zone time (CET)	min
t_W	Apparent local time (sundial reading)	min
E	Equation of time (+ means: sundial is fast)	min
Δt	Local time difference (mean local time minus zone time; + means: local time is ahead)	min

EXAMPLE On 3 November the sundial at the workshop from the previous example shows 12:00, i.e. 720 min. The equation of time is +16.4 min. What is the time in CET?

Given: $t_W = 720\,\text{min}$; $E = 16.4\,\text{min}$; $\Delta t = -26.6\,\text{min}$

$$\begin{aligned} t_Z &= t_W - E - \Delta t \\ &= 720\,\text{min} - 16.4\,\text{min} - (-26.6\,\text{min}) \\ &= 730.2\,\text{min} \end{aligned}$$

730.2 min after midnight, i.e. 12:10 CET. During summer time (daylight saving time) one hour is added.

8.5 Moon phase

The lunar phase repeats on average every 29.530 59 days. The age of the Moon counts the days since the last new moon; a moon phase display is set by it. How the 29.53 days are approximated with numbers of teeth, and how large the error of the display becomes, is covered in Chapter 2.

Age of the Moon from the date

(8.12) MASTER

$$a = t - k \cdot T_{syn}$$

All complete lunations are subtracted from the interval since a known new moon. What remains is the age of the Moon.

Valid: Mean lunation; the true phase can deviate from it by up to about one day. This is sufficient for setting a moon phase display.

a	Age of the Moon (days since the last new moon)	d
t	Time (since a known new moon)	d
k	Number (of lunations completed since then; the largest whole number for which a does not become negative)	1
T_{syn}	Synodic month (new moon to new moon)	d

EXAMPLE There was a new moon on 12 August 2026. How old is the Moon on 24 December 2026, i.e. 134 days later? In 134 days, 4 lunations are completed.

Given: $t = 134\,\text{d}$; $k = 4$; $T_{syn} = 29.530\,59\,\text{d}$

$$\begin{aligned} a &= t - k \cdot T_{syn} \\ &= 134\,\text{d} - 4 \cdot 29.530\,59\,\text{d} \\ &\approx 15.9\,\text{d} \end{aligned}$$

The age of the Moon is about 15.9 days. Full moon is at 14.8 days, so the Moon has just been full.

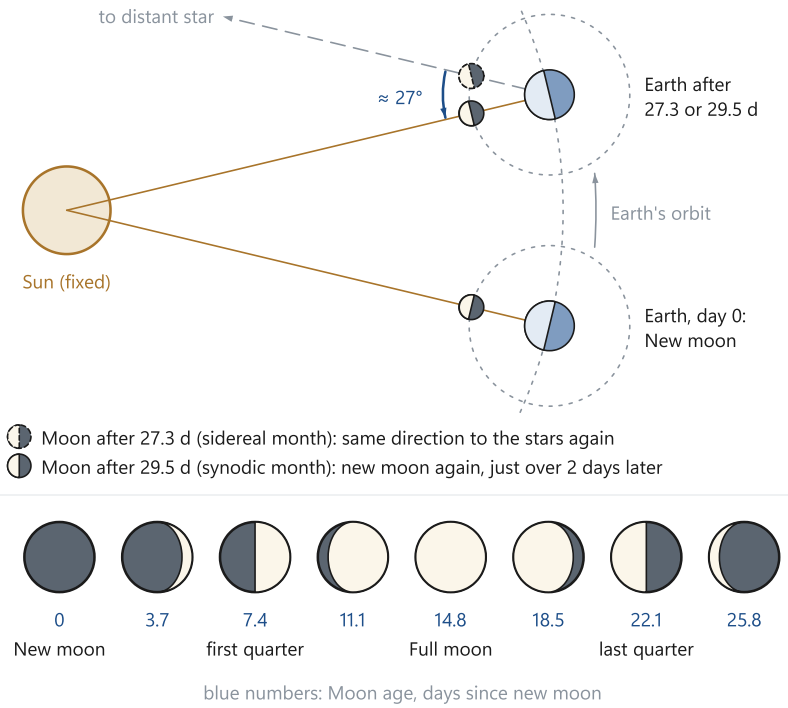


Fig. 8.6 Top: Sun fixed, Earth on its orbit, Moon around the Earth. After one sidereal month (27.3 d) the Moon is again in the same direction relative to the stars; it needs a good two days more until new moon (synodic month, 29.5 d; sizes not to scale). Bottom: moon phases as seen from the northern hemisphere, with the mean age of the Moon

8.6 Chronograph scales

Many chronographs carry a scale on the bezel or dial that immediately converts the measured time into another quantity. Behind every scale is a simple formula. With it a scale can be checked, or calculated for a new dial.

Speed from measured distance and stopped time

(8.13) JOURNEYMAN

$$v = \frac{s}{t}$$

v	Speed	$\frac{\mathrm{km}}{\mathrm{h}}$
s	Distance (measured distance)	km
t	Time (stopped)	s

The chronograph is started at the beginning of a known distance and stopped at its end.

Valid: Constant speed over the measured distance.

EXAMPLE A train needs 45 s for the distance between two kilometre posts. How fast is it travelling?

Given: $s = 1\,\mathrm{km}$; $t = 45\,\mathrm{s}$

$$\begin{aligned} v &= \frac{s}{t} \\ &= \frac{1\,\mathrm{km}}{45\,\mathrm{s}} \\ &= \frac{1\,\mathrm{km}}{45\,\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &= 80\,\tfrac{\mathrm{km}}{\mathrm{h}} \end{aligned}$$

The train travels at 80 km per hour.

Tachymeter scale

(8.14) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$v = \frac{3600}{t}$$

v	Speed (scale value in km per hour)	$\frac{\mathrm{km}}{\mathrm{h}}$
t	Time (stopped, in s)	s

The tachymeter scale is the preceding formula for a measured distance of 1 km: when stopped after t seconds, the hand points to the speed. In the same way the scale counts units per hour if the time for one unit is stopped.

Valid: Numerical values: t in s; measured distance 1 km (or 1 mile, then miles per hour). The scale only reaches 60 s. For slower events, time half the distance and halve the scale value.

EXAMPLE Which number is on a tachymeter scale at 24 s?

Given: $t = 24\,\mathrm{s}$

$$\begin{aligned} v &= \frac{3600}{t} \\ &= \frac{3600}{24} \\ &= 150\ \tfrac{\mathrm{km}}{\mathrm{h}} \end{aligned}$$

At 24 s the scale reads 150.

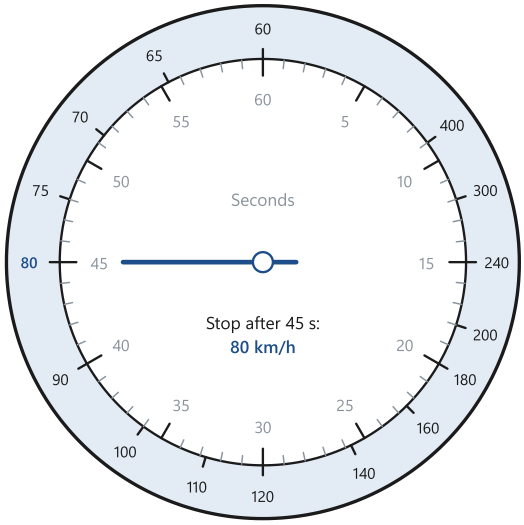


Fig. 8.7 Tachymeter scale: scale value equals 3600 divided by the seconds

Telemeter: distance from the travel time of sound

(8.15) JOURNEYMAN

$$s = c \cdot t$$

The lightning is seen practically at once; the thunder needs almost 3 s for every kilometre. The telemeter scale shows the distance in kilometres for the stopped travel time.

Valid: Speed of sound in air at 20 °C: $343\ \frac{\mathrm{m}}{\mathrm{s}}$; the travel time of light is neglected.

s	Distance	m
c	Speed of sound (in air)	$\frac{\mathrm{m}}{\mathrm{s}}$
t	Time (stopped, between light signal and sound)	s

EXAMPLE 6 s are timed between lightning and thunder. How far away is the thunderstorm?

Given: $c = 343\ \frac{\mathrm{m}}{\mathrm{s}}$; $\quad t = 6\,\mathrm{s}$

$$\begin{aligned} s &= c \cdot t \\ &= 343\ \tfrac{\mathrm{m}}{\mathrm{s}} \cdot 6\,\mathrm{s} \\ &\approx 2.06\,\mathrm{km} \end{aligned}$$

The thunderstorm is about 2 km away.

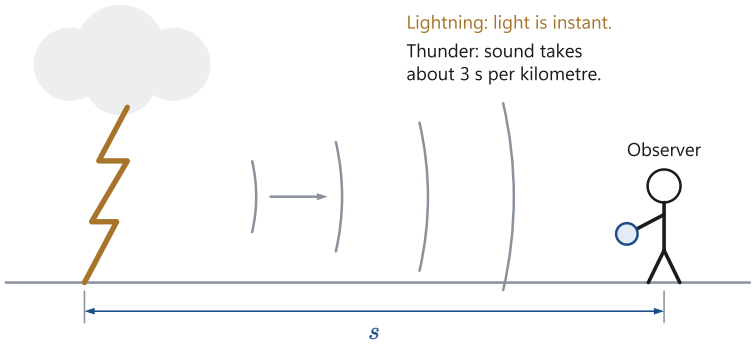


Fig. 8.8 Telemeter: the distance s follows from the time between lightning and thunder

Speed of sound and temperature

(8.16) NUMERICAL-VALUE EQUATION ADVANCED

$$c \approx 331.3 + 0.606 \cdot \vartheta$$

Sound travels faster in warm air. Telemeter scales are calculated for a fixed temperature, usually 20 °C.

Valid: Numerical values: ϑ in °C, c in $\frac{\mathrm{m}}{\mathrm{s}}$; dry air, approx. −20 °C to 40 °C.

c	Speed of sound (in air, numerical value in m per second)	$\frac{\mathrm{m}}{\mathrm{s}}$
ϑ	Temperature (of the air, in °C)	$^\circ\mathrm{C}$

EXAMPLE How fast is sound on a winter day at −10 °C?

Given: $\vartheta = -10^\circ\mathrm{C}$

$$\begin{aligned} c &\approx 331.3 + 0.606 \cdot \vartheta \\ &= 331.3 + 0.606 \cdot (-10) \\ &\approx 325.2\ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

About $325\ \frac{\mathrm{m}}{\mathrm{s}}$, which is 5 % less than at 20 °C. A telemeter scale then reads 5 % too high.

Pulsometer

(8.17) JOURNEYMAN

$$f_{\mathrm{P}} = \frac{N_{\mathrm{P}}}{t}$$

Start the chronograph, count the number of pulse beats stated on the scale, stop: the hand points to the beats per minute. The asthmometer scale counts breaths in the same way.

f_{P}	Pulse rate	$\frac{1}{\mathrm{min}}$
N_{P}	Number of pulse beats counted (graduation of the scale, usually 15 or 30)	1
t	Time (stopped)	s

EXAMPLE A pulsometer scale is graduated for 30 pulse beats. 20 s are timed for 30 beats. Which number is on the scale at that point?

Given: $N_{\mathrm{P}} = 30; \quad t = 20\,\mathrm{s}$

$$\begin{aligned} f_{\mathrm{P}} &= \frac{N_{\mathrm{P}}}{t} \\ &= \frac{30}{20\,\mathrm{s}} \\ &= 1{,}5\,\frac{1}{\mathrm{s}} \cdot 60\,\frac{\mathrm{s}}{\mathrm{min}} \\ &= 90\,\frac{1}{\mathrm{min}} \end{aligned}$$

The hand points to 90 beats per minute.

Decimal minutes

(8.18) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$t_{\mathrm{dez}} = \frac{t}{60}$$

For time studies the decimal scale divides the minute into 100 parts. One hundredth of a minute lasts 0.6 s. With decimal minutes, times can be added and multiplied by quantities without conversion.

Valid: Numerical values: t in s, result in min.

t_{dez}	Time in minutes as a decimal number	min
t	Time (stopped, in s)	s

EXAMPLE An operation takes 39 s. What does the decimal scale show?

Given: $t = 39\,\mathrm{s}$

$$\begin{aligned} t_{\mathrm{dez}} &= \frac{t}{60} \\ &= \frac{39}{60} \\ &= 0.65\,\mathrm{min} \end{aligned}$$

0.65 min, i.e. 65 hundredths of a minute.

Key point: Three seconds between lightning and thunder are about one kilometre.

Symbols

All symbols used in this formulary with their meaning, usual unit and chapter. The same symbol may mean different things in different chapters; the table of quantities given with each formula is always authoritative.

A	amplitude	$^\circ$, rad	Ch. 3
A	Amplitude	rad, $^\circ$	Ch. 4, 5, 6
ΔA	Amplitude drop	$^\circ$	Ch. 6
a	centre distance	mm	Ch. 2
a	Distance	m	Ch. 4
a	Centre distance	mm	Ch. 5
a	Proportion of time in horizontal positions	1	Ch. 6
a	Age of the Moon	d	Ch. 8
A_0	amplitude at the start	$^\circ$	Ch. 3
A_0	Amplitude fully wound	$^\circ$	Ch. 6
A_1	Amplitude before	rad	Ch. 4
A_2	Amplitude after	rad	Ch. 4
A_{24}	Amplitude after 24 hours of running	$^\circ$	Ch. 6
α	pressure angle	$^\circ$	Ch. 2
α	Coefficient of linear expansion	$\frac{1}{\mathrm{K}}$	Ch. 4
α_1	Coefficient of linear expansion of the parts acting downwards	$\frac{1}{\mathrm{K}}$	Ch. 4
α_2	Coefficient of linear expansion of the parts acting upwards	$\frac{1}{\mathrm{K}}$	Ch. 4
α_S	coefficient of linear expansion of the balance spring	$\frac{1}{\mathrm{K}}$	Ch. 3
α_U	coefficient of linear expansion of the balance	$\frac{1}{\mathrm{K}}$	Ch. 3
A_N	amplitude after N_S oscillations	$^\circ$	Ch. 3
a_U	Centre distance of pallet fork and balance	mm	Ch. 5
A_Z	Amplitude displayed by the timing machine	$^\circ$	Ch. 6
B	Magnetic flux density	$\frac{\mathrm{V \cdot s}}{\mathrm{m^2}}$	Ch. 6
B	Annual angle	$^\circ$	Ch. 8
b	blade height	mm	Ch. 1, 3
b	blows at each half hour	1	Ch. 2
b_1	blade height of the original spring	mm	Ch. 1
b_2	blade height of the replacement spring	mm	Ch. 1
β	angle	$^\circ$	Ch. 3
β	Temperature coefficient of the crystal	$\frac{\mathrm{ppm}}{\mathrm{K^2}}$	Ch. 7
β_B	Travel of the pallet fork from banking to banking	$^\circ$	Ch. 5
β_E	temperature coefficient of the modulus of elasticity of the balance spring	$\frac{1}{\mathrm{K}}$	Ch. 3
β_G	Angle of the pallet fork while the impulse pin is in the fork	$^\circ$	Ch. 5
β_H	Impulse at the pallet fork	$^\circ$	Ch. 5
β_H	Impulse at the anchor	$^\circ$	Ch. 5
β_R	Lock	$^\circ$	Ch. 5
β_V	Run	$^\circ$	Ch. 5
b_K	Width of the pallet, measured at the circumference of the wheel	mm	Ch. 5
C	temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d \cdot K}}$	Ch. 3
C	Temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d \cdot K}}$	Ch. 6
c	tip clearance between wheel tip and pinion root	mm	Ch. 2

c	Speed of sound	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 8
C_N	Nominal capacity of the battery	mAh	Ch. 7
D	restoring torque coefficient	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$	Ch. 3
D	Restoring torque coefficient	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$	Ch. 5
D	Vertical–horizontal difference	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
ΔD	change in restoring torque coefficient	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$	Ch. 3
d	diameter	mm	Ch. 1
d	pitch circle diameter	mm	Ch. 2
d	Diameter	mm	Ch. 6
d'_a	tip circle diameter of the pinion	mm	Ch. 2
d'_f	root circle diameter of the pinion	mm	Ch. 2
d_a	tip circle diameter	mm	Ch. 2
d_a	diameter of the outermost coil	mm	Ch. 3
d_a	Outside diameter of the cylinder	mm	Ch. 5
d_b	base circle diameter	mm	Ch. 2
d_f	root circle diameter	mm	Ch. 2
d_i	diameter of the innermost coil	mm	Ch. 3
d_m	measured dimension between the measuring faces	mm	Ch. 2
d_S	thickness of the cable or gut line	mm	Ch. 1
d_w	effective barrel diameter	mm	Ch. 1
E	modulus of elasticity	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 1, 3
E	Equation of time	min	Ch. 8
e	terminal radius	mm	Ch. 3
η	efficiency	1	Ch. 1, 2
η	Efficiency	1	Ch. 5
η_F	efficiency of the spring in the barrel	1	Ch. 1
η_ges	overall efficiency of the gear train	1	Ch. 2
F	Force	N	Ch. 6
Δf	Frequency deviation	Hz	Ch. 7
f	frequency	Hz	Ch. 3
f	Frequency	Hz	Ch. 6, 7
f_aus	Frequency at the divider output	Hz	Ch. 7
f_ist	measured frequency	Hz	Ch. 3
f_M	Frequency of the motor pulses	Hz	Ch. 7
f_P	Pulse rate	$\frac{1}{\mathrm{min}}$	Ch. 8
f_soll	nominal frequency	Hz	Ch. 3
G	rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
G	Rate of the clock in seconds per day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 4
G	Rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6, 7, 8
ΔG	rate change	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
ΔG	Rate change	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 4, 7
ΔG	Isochronism error	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
Δg	Change in gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 4
$\bar{G}$	Mean rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
g	gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 1, 3
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 4, 6

g	Gravitational acceleration at the old location	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 4
G_0	Rate fully wound	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_1	rate at the lower temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
G_1	Rate on the first test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_{15}	Rate on the last test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_2	rate at the higher temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
G_2	Rate on the second test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_{24}	Rate after 24 hours of running	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_{38}	Rate at 38 °C	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_8	Rate at 8 °C	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
γ	Half span angle	$^\circ$	Ch. 5
γ_F	Drop	$^\circ$, rad	Ch. 5
γ_H	Impulse at the wheel	$^\circ$	Ch. 5
G_f	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_h	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
ΔG_L	Greatest positional variation	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_m	rate at the middle temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
G_max	Highest rate of all positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_min	Lowest rate of all positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G_T	Expected rate when worn	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
ΔG_W	Resumption of rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
H	Magnetic field strength	$\frac{\mathrm{A}}{\mathrm{m}}$	Ch. 6
h	height	mm, m	Ch. 1
h	Altitude	m	Ch. 4
h	Height	m	Ch. 6
I	Current	mA	Ch. 7
i	gear ratio	1	Ch. 2
i_G	Ratio in the central position	1	Ch. 5
i_ges	overall gear ratio	1	Ch. 2
I_IC	Current consumption of the circuit including oscillator	$\mu\mathrm{A}$	Ch. 7
I_Imp	Current through the coil during the pulse	mA	Ch. 7
I_m	Mean current consumption of the movement	$\mu\mathrm{A}$	Ch. 7
i_R	reduction ratio of automatic winding	1	Ch. 1
i_U	Reduction ratio	1	Ch. 7
i_W	gear ratio from pin wheel to fly	1	Ch. 2
J	moment of inertia	$\mathrm{mg{\cdot}cm^2}$	Ch. 3
ΔJ	change in moment of inertia	$\mathrm{mg{\cdot}cm^2}$	Ch. 3
j	backlash on the pitch circle	mm	Ch. 2
J_A	Moment of inertia about the pivot axis	$\mathrm{kg{\cdot}m^2}$	Ch. 4
J_S	Moment of inertia about the parallel axis through the centre of gravity	$\mathrm{kg{\cdot}m^2}$	Ch. 4
J_Z	known moment of inertia of the attached disc	$\mathrm{mg{\cdot}cm^2}$	Ch. 3
K	CGS value of the balance spring	$\frac{\mathrm{N{\cdot}mm^3}}{\mathrm{rad}}$	Ch. 3
k	number	1	Ch. 1, 2
k	addendum coefficient	1	Ch. 2
k	last hour struck	1	Ch. 2
k	Span	1	Ch. 5

k	Number	1	Ch. 8
k'	addendum coefficient of the pinion	1	Ch. 2
k_A	amplitude factor of the out-of-poise	1	Ch. 3
k_F	filling ratio: share of the free area covered by the spring	1	Ch. 1
Δl	change	mm	Ch. 3
Δl	Change	mm	Ch. 4
Δl	Bob travel	mm	Ch. 4
l	length	mm	Ch. 1, 3
l	Pendulum length	m, mm	Ch. 4
l_1	effective length in the trial	mm	Ch. 3
l_1	Total length of the parts acting downwards	m	Ch. 4
l_2	required effective length	mm	Ch. 3
l_2	Total length of the parts acting upwards	m	Ch. 4
λ	Lift angle	$^\circ$	Ch. 5, 6
λ	Geographical longitude of the place	$^\circ$	Ch. 8
λ_Z	Lift angle set on the timing machine	$^\circ$	Ch. 6
λ_Z	Geographical longitude of the zone meridian	$^\circ$	Ch. 8
l_G	Fork length	mm	Ch. 5
l_P	Distance between the two contact points	mm	Ch. 5
l_r	Equivalent pendulum length	m	Ch. 4
l_Z	Length of a tooth	mm	Ch. 5
M	torque	$\mathrm{N{\cdot}mm}$, $\mathrm{N{\cdot}m}$, $\mathrm{\mu N{\cdot}m}$	Ch. 1, 2, 3
Δm	Mass of the timing weight	g	Ch. 4
m	mass	kg, mg	Ch. 1, 3
m	module	mm	Ch. 2
m	Mass	kg	Ch. 4
M'	torque at the arbor of the driven pinion	$\mathrm{N{\cdot}mm}$	Ch. 2
M_1	torque of the original spring	$\mathrm{N{\cdot}mm}$	Ch. 1
m_1	module of the cannon pinion and minute wheel mesh	mm	Ch. 2
M_2	torque of the replacement spring	$\mathrm{N{\cdot}mm}$	Ch. 1
m_2	module of the minute pinion and hour wheel mesh	mm	Ch. 2
M_F	torque of the barrel at the same moment	$\mathrm{N{\cdot}mm}$	Ch. 1
M_F	torque at the barrel	$\mathrm{N{\cdot}mm}$	Ch. 2
M_H	torque at the escape wheel	$\mathrm{\mu N{\cdot}m}$	Ch. 2
M_H	Torque at the escape wheel	$\mathrm{\mu N{\cdot}m}$	Ch. 5
M_m	mean torque over these turns	$\mathrm{N{\cdot}mm}$	Ch. 1
M_max	maximum torque transmissible elastically	$\mathrm{N{\cdot}mm}$	Ch. 1
M_S	torque at the fusee	$\mathrm{N{\cdot}mm}$	Ch. 1
m_S	mass of one screw	mg	Ch. 3
μ	Coefficient of friction	1	Ch. 5
m_u	excess weight	$\mathrm{\mu g}$	Ch. 3
μ_0	Magnetic constant	$\frac{\mathrm{V{\cdot}s}}{\mathrm{A{\cdot}m}}$	Ch. 6
N	turns	1	Ch. 1
N	number	1	Ch. 2, 3
N	Turns	1	Ch. 4

N	Number	1	Ch. 7
N	Day number in the year	1	Ch. 8
n	rotational speed	$\frac{1}{\mathrm{h}}$	Ch. 2
n	Number	1	Ch. 6, 7
n'	rotational speed of the driven pinion	$\frac{1}{\mathrm{h}}$	Ch. 2
n_1	beat rate in the trial	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 3
N_{12}	hour blows in 12 hours	1	Ch. 2
n_2	beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 3
N_A	blows per revolution of the driving wheel	1	Ch. 2
N_ab	coils of the run-down spring	1	Ch. 1
N_auf	coils of the wound spring	1	Ch. 1
N_d	blows in 24 hours	1	Ch. 2
N_d	Number of days in the cycle	1	Ch. 8
n_d	blows per day	$\frac{1}{\mathrm{d}}$	Ch. 2
N_E	terminal curve number	1	Ch. 3
n_F	Number of years until the calendar deviates by one day from the solar year	1	Ch. 8
n_H	rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$	Ch. 2
n_H	Rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$	Ch. 5
n_h	beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 2, 3
n_h	Beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 4, 5
n_h	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$	Ch. 4
n_ist	Measured beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 6
N_K	revolutions of the crown	1	Ch. 1
N_M	revolutions of the minute hand	1	Ch. 1
n_M	rotational speed of the centre wheel	$\frac{1}{\mathrm{h}}$	Ch. 2
N_max	maximum possible turns of the barrel	1	Ch. 1
N_P	Number of pulse beats counted	1	Ch. 8
N_R	revolutions of the rotor	1	Ch. 1
n_R	Rotational speed	$\frac{1}{\mathrm{min}}$	Ch. 7
N_S	number of full oscillations	1	Ch. 3
n_S	rotational speed of the fourth wheel	$\frac{1}{\mathrm{min}}$	Ch. 2
n_S	Rotational speed of the fourth wheel	$\frac{1}{\mathrm{min}}$	Ch. 7
n_S	Number of leap years in the cycle	1	Ch. 8
N_Sch	blows per revolution of the count wheel	1	Ch. 2
n_soll	Nominal beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 6
N_W	revolutions of the barrel	1	Ch. 1
n_Z	Number of years in the cycle	1	Ch. 8
ω	angular frequency	$\frac{\mathrm{rad}}{\mathrm{s}}$	Ch. 3
ω_max	angular velocity of the balance when passing through the rest position	$\frac{\mathrm{rad}}{\mathrm{s}}$	Ch. 3
$\overline{\Delta G}$	Mean variation	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
P	power	$\mu\mathrm{W}$	Ch. 1, 3
P	Thread pitch	mm	Ch. 4
p	circular pitch	mm	Ch. 2
p	coil pitch	mm	Ch. 3
p	Pressure	bar	Ch. 6
p_dyn	Dynamic pressure	bar	Ch. 6

P_{U}	power reaching the balance or pendulum	$\mu\mathrm{W}$	Ch. 1
Q	quality factor	1	Ch. 3
Q	Quality factor	1	Ch. 5
R	inner radius	mm	Ch. 1
R	Radius	mm	Ch. 5
Δr	shift of the centre of gravity	mm	Ch. 3
r	radius	mm	Ch. 1, 3
r_{A}	Pallet arm	mm	Ch. 5
r_{a}	radius of the outermost coil	mm	Ch. 3
r_{E}	Radius of the impulse pin about the balance arbor	mm	Ch. 5
r_{F}	radius of the barrel	mm	Ch. 1
ρ	radius	mm	Ch. 1
ρ	Angle of friction	$^\circ$	Ch. 5
r_{i}	inner radius of the run-down spring pack	mm	Ch. 1
r_{i}	radius of the innermost coil	mm	Ch. 3
r_{S}	effective radius of the fusee at the point where the chain runs off	mm	Ch. 1
r_{S}	distance of its centre of gravity from the balance axis	mm	Ch. 3
r_{u}	distance of the excess weight from the balance axis	mm	Ch. 3
S	middle temperature error	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 3
S	Time error	s	Ch. 6
ΔS	Change in time error	s	Ch. 6
Δs	change in blade thickness	$\mu\mathrm{m}$	Ch. 3
s	blade thickness	mm, $\mu\mathrm{m}$	Ch. 1, 3
s	tooth thickness	mm	Ch. 2
s	Distance	km, m	Ch. 8
s'	leaf thickness of the pinion on the pitch circle	mm	Ch. 2
S_1	Time error at the first reading	s	Ch. 6
s_1	blade thickness of the original spring	mm	Ch. 1
S_2	Time error at the second reading	s	Ch. 6
s_2	blade thickness of the replacement spring	mm	Ch. 1
s_{E}	Self-discharge	$\frac{\%}{\mathrm{a}}$	Ch. 7
s_{F}	Drop as travel	mm	Ch. 5
σ_{zul}	permissible bending stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 1
$\sum G_i$	Sum of the individual rates, with sign	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
$\sum \lvert G_{2k} - G_{2k-1} \rvert$	Sum of the five absolute values: difference between the two daily rates in each position	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
T	period	s	Ch. 3
T	Period	s	Ch. 4
Δt	deviation of the display from the true age of the moon	d	Ch. 2
Δt	Time shift of the beats relative to the reference frequency during the measuring time	ms	Ch. 6
Δt	Local time difference	min	Ch. 8
t	time	s	Ch. 3
t	Time	d, h, s	Ch. 6, 7, 8
t'_{U}	rotation period of the pinion	s	Ch. 2
T_0	Period at very small amplitude)	s	Ch. 4
T_1	period without disc	s	Ch. 3

t_1	Time of the first reading	d	Ch. 6
t_1	Duration of one vibration	ms	Ch. 6
T_2	period with disc	s	Ch. 3
t_2	Time of the second reading	d	Ch. 6
t_2	Duration of the following vibration	ms	Ch. 6
t_A	rotation period of the driving arbor	d	Ch. 2
t_A	Beat error	ms	Ch. 6
τ	Angular pitch	$^\circ$	Ch. 5
t_dez	Time in minutes as a decimal number	min	Ch. 8
T_E	Sidereal year	d	Ch. 8
t_F	rotation period of the barrel	h	Ch. 1
t_F	running time until the display deviates by Δt	d	Ch. 2
t_G	running time	h, d	Ch. 1, 2
t_H	Lift time	ms	Ch. 6
t_I	Inhibition period	s	Ch. 7
t_Imp	Pulse duration	ms	Ch. 7
T_J	Tropical year	d	Ch. 8
T_K	Mean length of the calendar year	d	Ch. 8
t_L	displayed duration of one lunation	d	Ch. 2
T_M	Period of the motor pulses	s	Ch. 7
t_M	rotation period of the centre wheel	h	Ch. 1
t_M	Measuring time	s	Ch. 6
t_M	Mean local time	min	Ch. 8
t_N	Reference time	s	Ch. 6
t_R	power reserve	h	Ch. 1
T_S	Beat duration	s	Ch. 4
t_s	time between two advances	d	Ch. 2
T_sid	Sidereal month	d	Ch. 8
T_So	Mean solar day	s	Ch. 8
T_St	Sidereal day	s	Ch. 8
T_syn	synodic month	d	Ch. 2
T_syn	Synodic month	d	Ch. 8
t_U	rotation period of the driving arbor	h	Ch. 1
t_U	rotation period of the driving wheel	s	Ch. 2
t_U	Reading of the watch	s	Ch. 6
t_W	rotation period of the barrel	h	Ch. 1
t_W	Apparent local time	min	Ch. 8
t_Z	Zone time	min	Ch. 8
U	required revolutions of the driving wheel	1	Ch. 2
U	Voltage	V	Ch. 7
u_G	Maximum error of the rate due to reading	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
U_N	Nominal voltage of the battery	V	Ch. 7
u_S	Maximum reading error of one time error	s	Ch. 6
U_W	revolutions of the fly per blow	1	Ch. 2
v	Velocity	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 6
v	Speed	$\frac{\mathrm{km}}{\mathrm{h}}$	Ch. 8

ε	Angle	$^\circ$	Ch. 6
φ	angle of twist	rad	Ch. 1
φ	angle	$^\circ$, rad	Ch. 3
φ	Geographic latitude	$^\circ$	Ch. 4
φ_E	Supplementary arc	$^\circ$	Ch. 5
φ_S	Step angle of the rotor	$^\circ$	Ch. 7
ϱ	Density	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	Ch. 6
$\Delta\vartheta$	Temperature change	K	Ch. 4
ϑ	Temperature	$^\circ\mathrm{C}$	Ch. 7, 8
ϑ_0	Turnover temperature	$^\circ\mathrm{C}$	Ch. 7
ϑ_1	lower test temperature	$^\circ\mathrm{C}$	Ch. 3
ϑ_2	higher test temperature	$^\circ\mathrm{C}$	Ch. 3
v_F	Proportion of the wheel's work lost in the drop	$\%$	Ch. 5
W	work	mJ, J	Ch. 1
W	energy	$\mu\mathrm{J}$	Ch. 3
W	Energy	J	Ch. 7
ΔW	energy loss per full oscillation	$\mu\mathrm{J}$	Ch. 3
W_ab	work delivered while running down	mJ	Ch. 1
W_auf	work expended	mJ	Ch. 1
W_max	maximum elastically storable energy	mJ	Ch. 1
W_S	Work of the escape wheel per beat	$\mu\mathrm{J}$	Ch. 5
W_U	Work delivered to the balance per beat	$\mu\mathrm{J}$	Ch. 5
x	Distance	mm	Ch. 4
z	number of teeth	1	Ch. 2
z	Number of teeth	1	Ch. 5
z'	number of leaves of the pinion	1	Ch. 2
z'_1	number of teeth of the pinion on the first reduction wheel	1	Ch. 1
z'_1	number of leaves of the first driven pinion	1	Ch. 2
z'_2	number of teeth of the pinion on the second reduction wheel	1	Ch. 1
z'_2	number of leaves of the second driven pinion	1	Ch. 2
z'_3	number of leaves of the third driven pinion	1	Ch. 2
z'_A	number of teeth of the winding pinion	1	Ch. 1
z'_B	number of teeth of the intermediate pinion	1	Ch. 1
z'_H	number of leaves of the escape pinion	1	Ch. 2
z'_K	number of teeth of the sliding pinion	1	Ch. 1
z'_K	number of leaves of the third pinion	1	Ch. 2
z'_M	number of teeth of the centre pinion	1	Ch. 1
z'_N	number of leaves of the pinion on the pin-wheel arbor	1	Ch. 2
z'_R	number of teeth of the rotor pinion	1	Ch. 1
z'_S	number of leaves of the fourth pinion	1	Ch. 2
z'_Sch	number of leaves of the pinion on the pin-wheel arbor that drives the count wheel	1	Ch. 2
z'_W	number of leaves of the minute pinion	1	Ch. 2
z_1	number of teeth of the first reduction wheel	1	Ch. 1
z_1	number of teeth of the first driving wheel	1	Ch. 2
z_2	number of teeth of the second reduction wheel	1	Ch. 1
z_2	number of teeth of the second driving wheel	1	Ch. 2

z_{24}	number of teeth of the 24-hour wheel	1	Ch. 2
z_3	number of teeth of the third driving wheel	1	Ch. 2
z_{A}	number of teeth of the driving wheel on the hour-wheel pipe	1	Ch. 2
z_{A}	number of teeth of the driving wheel	1	Ch. 2
z_{B}	number of teeth of the intermediate wheel	1	Ch. 1
z_{F}	number of teeth of the barrel	1	Ch. 1
z_{H}	number of teeth of the escape wheel	1	Ch. 2
z_{K}	number of teeth of the third wheel	1	Ch. 2
z_{M}	number of teeth of the centre wheel	1	Ch. 2
z_{N}	number of lifting pins	1	Ch. 2
z_{S}	number of teeth of the fourth wheel	1	Ch. 2
z_{Sch}	number of teeth of the count-wheel drive wheel	1	Ch. 2
z_{Sp}	number of teeth of the ratchet wheel	1	Ch. 1
z_{St}	number of teeth of the hour wheel	1	Ch. 2
z_{V}	number of teeth of the cannon pinion	1	Ch. 1
z_{V}	number of leaves of the cannon pinion	1	Ch. 2
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