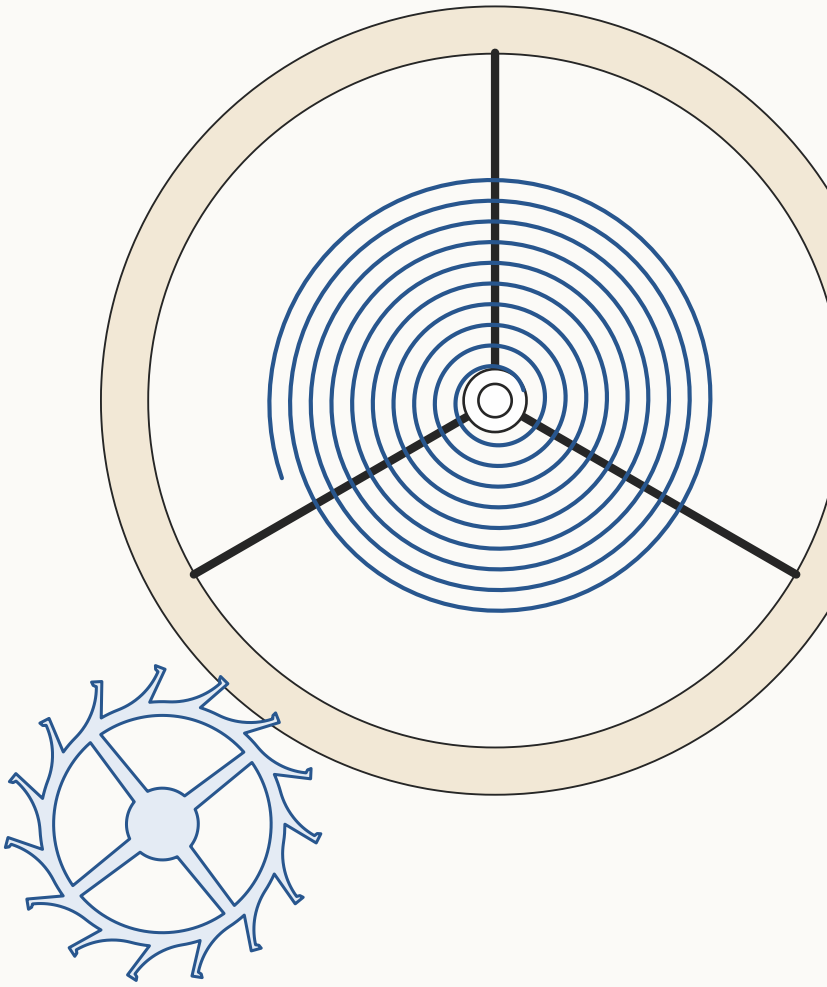
TRAINING · MASTER'S EXAMINATION · WORKSHOP

Watchmaking Formulary

Formulas, sketches and worked examples for watches and clocks – with mathematics, mechanics, manufacturing and statistics



Robin Israel
Master watchmaker · R.I.O. Uhrenwerkstätte Oppenheim
Chairman of the Hessian Watchmaking School

XL Edition 2026

How to read this formulary

Each formula has its own card: at the top its name, number and level, below it the formula, rearranged where that helps, and next to it the table of all quantities with their units. Then follow, in one sentence, the conditions under which the formula is valid, and a worked example with numbers and units. We assume that the reader can rearrange formulas; rearranged forms are given only where they are needed constantly in the workshop.

Levels

JOURNEYMAN — Basic knowledge for training and the journeyman's examination.

MASTER — Material of the master's examination.

ADVANCED — Background and design; not calculated in the examination, but useful for understanding what happens in the watch.

Notation

Fractions are always stacked, never written with a slash. Factors are separated by a multiplication dot, decimals use a decimal point, large numbers have thin spaces: $86\,400$.

Most formulas are **quantity equations**: one inserts number and unit and carries the units through the calculation. Any suitable unit is allowed, as long as it is converted cleanly. A few workshop formulas are **numerical-value equations**; they carry the label “Numerical-value equation” and are valid only with exactly the units given in their table of quantities.

Formulas with “≈” are approximations. How far they reach is stated under “Valid”.

Rate, oscillation and beat

The rate G is the deviation of a timepiece per day in seconds ($\frac{\mathrm{s}}{\mathrm{d}}$). Plus means the timepiece gains; minus means it loses.

An **oscillation** is there and back; its duration is the period T, and the frequency f counts full oscillations per second (Hz). A **beat** is a half-oscillation (vibration). The beat rate n_h counts half-oscillations, for watches per hour (A/h): $28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$ equal $4\ \mathrm{Hz}$.

In this formulary T is a time (only for the fits in Chapter 9 is it the tolerance). Temperatures are denoted ϑ, temperature changes $\Delta\vartheta$.

Angles

In trigonometric functions ($\sin$, $\cos$, $\tan$) an angle may be given in degrees; the calculator must then be set to degrees. If, however, an angle appears as a factor or as a power in a formula, it is inserted in radians: degrees times $\frac{\pi}{180}$. The table of quantities states which form is meant in each case.

Same symbol, different meaning

The most important symbols mean the same everywhere. Where a chapter needs a symbol in a different sense, for instance A for an area instead of the amplitude, this is stated in the chapter introduction and in the table of quantities of the formula. The table of quantities always applies.

© 2026 Robin Israel. Licensed under CC BY-NC-SA 4.0: this formulary may be shared and adapted free of charge, with attribution to the author, for non-commercial purposes and under the same terms. creativecommons.org/licenses/by-nc-sa/4.0

Contents

1 Fundamentals 5
- 1.1 Units and Conversions 5
- 1.2 Areas and Circumference 9
- 1.3 Solids and Mass 17
- 1.4 Triangles and Coordinates 22
- 1.5 Mean and Deviation 28

2 Mechanics 30
- 2.1 Motion 30
- 2.2 Force, Pressure and Inclined Plane 34
- 2.3 Lever and Torque 39
- 2.4 Work, Energy, Power and Efficiency 41
- 2.5 Friction 47
- 2.6 Moment of Inertia 50
- 2.7 Strength and Elasticity 53
- 2.8 Thermal Expansion 61

3 Power Source 63
- 3.1 Mainspring in the Barrel: Geometry 63
- 3.2 Torque and Energy of the Mainspring 69
- 3.3 Running Time 74
- 3.4 Weight Drive 77
- 3.5 Fusee and Chain 79
- 3.6 Power of the Drive 80
- 3.7 Winding and Hand Setting 82

4 Gear Train and Toothing 87
- 4.1 Gear Ratio 87
- 4.2 Calculating the Going Train 91
- 4.3 Motion Work 94
- 4.4 Tooth Geometry 96
- 4.5 Striking Work 108
- 4.6 Calendar and Moon Phase 110
- 4.7 Efficiency of the Gear Train 113

5 Oscillator: Balance and Balance Spring 115
- 5.1 Oscillation of the Balance 115
- 5.2 Balance Spring 118
- 5.3 Balance 124
- 5.4 Regulation from Nominal and Actual Values 128
- 5.5 Temperature 130
- 5.6 Energy and Quality Factor 133
- 5.7 Isochronism 135

6 Pendulum 137
- 6.1 Simple pendulum 137
- 6.2 Regulating by the pendulum length 140
- 6.3 Compound pendulum 144
- 6.4 Amplitude and circular error 146
- 6.5 Temperature and location 147

7 Escapements 151
7.1 Basic quantities of all escapements 151
7.2 Swiss lever escapement: geometry 154
7.3 Force and energy 162
7.4 The escapement on the timing machine 165
7.5 Clock escapements 165
7.6 Other escapements 167

8 Rate Testing and Measurement 169
8.1 Time error and rate 169
8.2 Timing machine 172
8.3 Positions and characteristic values 177
8.4 Chronometer testing 181
8.5 Water resistance and pressure 183
8.6 Magnetism and shock 185
8.7 Reading error and scatter 186

9 Manufacturing and Materials 188
9.1 Cutting data 188
9.2 Taper and slope 192
9.3 Threads 194
9.4 Fits and tolerances 197
9.5 Indexing 202
9.6 Bending and blank length 205
9.7 Precious metals 206
9.8 Hardness and heat treatment 210
9.9 Electroplating 212

10 Electrical Engineering, Quartz and Battery 215
10.1 Basic quantities of the circuit 215
10.2 Resistor circuits, battery under load 219
10.3 Capacitor and coil 222
10.4 Battery 226
10.5 Quartz crystal and frequency divider 228
10.6 Stepper motor 231
10.7 Measurements on the quartz watch 234

11 Time and Calendar 236
11.1 Day and year 236
11.2 Calendar 238
11.3 Equation of time 240
11.4 Local time and zone time 242
11.5 Moon phase 243
11.6 Chronograph scales 244

12 Quality and Statistics 248
12.1 Series of measurements 248
12.2 Machine and process capability 250
12.3 Defects and risk 252
12.4 Measuring instruments and measuring conditions 254

Symbols 256

Index 270

1 Fundamentals

This chapter collects the mathematical tools on which all other chapters draw: units, areas, solids and calculations in the triangle. The formulas are school mathematics. What is new is only where they appear in the watch and at the bench.

The symbols follow geometry: here A is an area, s a chord or width across flats, m a mass, D an outside diameter, α an angle and n a number of items. In later chapters some of these letters carry other meanings; the table of quantities next to each formula is always authoritative.

1.1 Units and Conversions

We calculate in the International System of Units (SI). Watchmaking has also kept units of its own: the ligne for movement sizes, seconds per day for the rate and vibrations (half-oscillations) per hour for the beat rate. Fineness and carat of precious metals are given in Chapter 9.

Prefixes for Decimal Multiples and Submultiples

Prefix	Symbol	Factor	Workshop example
giga	G	10^{9}	Young's modulus of steel: 210 GPa
mega	M	10^{6}	Tensile strength of a mainspring: 2000 MPa
kilo	k	10^{3}	Watch quartz: 32.768 kHz
deci	d	10^{-1}	1 dm = 100 mm
centi	c	10^{-2}	Moment of inertia of a balance: 10 mg·cm²
milli	m	10^{-3}	Pivot diameter: 0.1 mm
micro	µ	10^{-6}	Pivot side shake: 10 µm; current consumption of a quartz movement: 1 µA
nano	n	10^{-9}	Thickness of an anti-reflective coating: 100 nm

One hundredth of a millimetre is 10 µm, one thousandth of a millimetre is 1 µm.

Length Units of Watchmaking

Unit	Symbol	Conversion
Paris ligne	‴	2.2558 mm
Twelfth of a ligne (douzième)		0.188 mm
Paris inch (12 lignes)		27.07 mm
Inch	″ or in	25.4 mm (exact)
Micrometre	µm	0.001 mm

Today the ligne is used almost only to state the size of movements. The English inch appears in threads, clocks and machines.

Lignes to Millimetres

(1.1) JOURNEYMAN

$$d = N_\mathrm{L} \cdot 2.2558\,\mathrm{mm}$$

REARRANGED $$N_\mathrm{L} = \frac{d}{2.2558\,\mathrm{mm}}$$

d	diameter (or another length)	mm
N_L	number of lignes (size in lignes, pure number)	1

Movement sizes are still stated in Paris lignes, written with three strokes: 11½‴. One ligne is the twelfth part of the old Paris inch.

Valid: always; 1 ligne = 2.2558 mm.

Caution: The ligne size of a calibre is a size class, not a manufacturing dimension. The actual movement diameter often differs by a few tenths of a millimetre. For movement holders and cases, use the dimension from the technical data sheet.

EXAMPLE A movement is described as an 11½-ligne movement. How large is it in millimetres?

Given: $N_\mathrm{L} = 11.5$

$$\begin{aligned} d &= N_\mathrm{L} \cdot 2.2558\,\mathrm{mm} \\ &= 11.5 \cdot 2.2558\,\mathrm{mm} \\ &\approx 25.9\,\mathrm{mm} \end{aligned}$$

About 25.9 mm. This is the size class of most men's automatic wristwatches.

Minutes and Seconds as Decimal Value

(1.2) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$t = t_\mathrm{h} + \frac{t_\mathrm{min}}{60} + \frac{t_\mathrm{s}}{3600}$$

t	time (as decimal number in hours)	h
t_h	whole hours	h
t_min	minutes	min
t_s	seconds	s

For calculations we need times as decimal numbers. Angles are converted in the same way: degrees, minutes of arc and seconds of arc instead of hours, minutes and seconds, for example 12° 30′ = 12.5°.

Valid: numerical values in the units stated; one hour has 60 minutes, one minute 60 seconds.

EXAMPLE After full winding, a watch runs for 43 h 24 min 36 s. What is the running time as a decimal number?

Given: $t_\mathrm{h} = 43\,\mathrm{h}$; $t_\mathrm{min} = 24\,\mathrm{min}$; $t_\mathrm{s} = 36\,\mathrm{s}$

$$\begin{aligned} t &= t_\mathrm{h} + \frac{t_\mathrm{min}}{60} + \frac{t_\mathrm{s}}{3600} \\ &= 43 + \frac{24}{60} + \frac{36}{3600} \\ &= 43.41\,\mathrm{h} \end{aligned}$$

43.41 h. This value can be used for further calculation, for example when comparing with the specified running time.

Rate, Percent and ppm

Value	equals	Remark
1 day	24 h = 1440 min = 86 400 s	
1 s per day	11.57 ppm = 0.001 157 %	very good mechanical watch
1 ppm	0.0864 s per day	about 2.6 s per month, 31.5 s per year
1 ‰	86.4 s per day	
1 %	864 s per day	14.4 min per day

ppm means "parts per million", i.e. 10^{-6}. This unit is common for quartz watches.

Rate as Relative Deviation

(1.3) JOURNEYMAN

$$x = \frac{G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$$

REARRANGED $G = x \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$

x	relative deviation (as a number, in % or in ppm)	ppm
G	rate (+ means: the watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$

The rate in seconds per day is a ratio of two times. Dividing by the 86 400 seconds of a day gives the relative deviation. This makes mechanical watches, quartz watches and component tolerances comparable.

Valid: always; the sign is retained.

EXAMPLE A mechanical wristwatch gains 6 s per day. What is the relative deviation?

Given: $G = 6\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} x &= \frac{G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &= \frac{6\ \frac{\mathrm{s}}{\mathrm{d}}}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &\approx 69.4\,\mathrm{ppm} \end{aligned}$$

About 69 ppm or 0.007 %. The watch measures time to within seven hundred-thousandths.

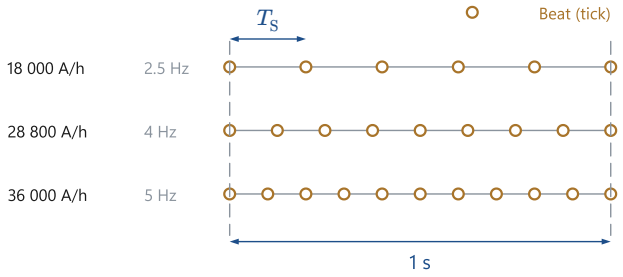
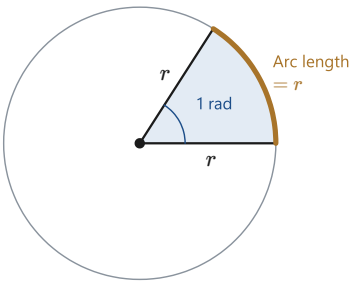


Fig. 1.1 Beats in one second: the higher the beat rate, the shorter the beat duration T_S



Full circle: 360° = 2π rad

Fig. 1.2 Radian measure: at an angle of 1 rad, about 57.3°, the arc is as long as the radius

Degrees to Radians

(1.4) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$\widehat{\alpha} = \frac{\pi \cdot \alpha}{180}$$

REARRANGED $\alpha = \frac{180 \cdot \widehat{\alpha}}{\pi}$

$\widehat{\alpha}$	angle (in radians)	rad
α	angle (in degrees)	$^\circ$

The radian measure states how long the arc is relative to the radius. The full circle is $2\pi\,\mathrm{rad}$, so $1\,\mathrm{rad} \approx 57.3^\circ$. Where an angle appears in a formula as a factor, it must be entered in radians.

Valid: numerical values: α in degrees as a decimal number, result in radians.

EXAMPLE A balance oscillates with an amplitude of 270°. What is this angle in radians?

Given: $\alpha = 270^\circ$

$$\begin{aligned}\widehat{\alpha} &= \frac{\pi \cdot \alpha}{180} \\ &= \frac{\pi \cdot 270}{180} \\ &\approx 4.71\,\mathrm{rad}\end{aligned}$$

About 4.71 rad, which is three quarters of a revolution.

Beat Rate and Frequency

(1.5) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot f$$

REARRANGED $f = \frac{n_\mathrm{h}}{2}$

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
f	frequency (full oscillations per second)	Hz

The frequency counts full oscillations, the beat rate counts vibrations (half-oscillations). Each full oscillation consists of two beats. For watches, the beat rate is stated in vibrations per hour (A/h, vph).

Valid: always; from beats per second to A/h multiply by 3600.

EXAMPLE A balance oscillates at 4 Hz. What is the beat rate?

Given: $f = 4\,\mathrm{Hz}$

$$\begin{aligned}n_\mathrm{h} &= 2 \cdot f \\ &= 2 \cdot 4\,\mathrm{Hz} \\ &= 8\,\frac{1}{\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &= 28\,800\,\tfrac{\mathrm{A}}{\mathrm{h}}\end{aligned}$$

28 800 vibrations per hour, i.e. 8 beats per second.

Beat Duration

(1.6) JOURNEYMAN

$$T_\mathrm{S} = \frac{1}{n_\mathrm{h}}$$

T_S	beat duration (duration of one vibration)	s
n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$

The beat duration is the time from one tick to the next. The period T of a full oscillation is twice as long.

Valid: always; enter n_h in beats per second if T_S is to come out in seconds.

EXAMPLE How long does one beat last at 28 800 A/h?

Given: $n_\mathrm{h} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$

$$\begin{aligned} T_\mathrm{S} &= \frac{1}{n_\mathrm{h}} \\ &= \frac{1}{28\,800\ \frac{\mathrm{A}}{\mathrm{h}}} \\ &= \frac{3600\,\mathrm{s}}{28\,800} \\ &= 0.125\,\mathrm{s} \end{aligned}$$

One eighth of a second. That is why the seconds hand makes eight small steps per second.

Common Beat Rates of Watches

Beat rate n_h in A/h	Frequency f in Hz	Beats per second	Beat duration T_S in s	Typical use
18 000	2.5	5	0.2	pocket watches, older wristwatches
19 800	2.75	5.5	0.1818	some older calibres
21 600	3	6	0.1667	many hand-wound and automatic movements
25 200	3.5	7	0.1429	some calibres
28 800	4	8	0.125	the most common beat rate today
36 000	5	10	0.1	high-beat movements

A stands for alternation, i.e. the vibration (half-oscillation).

1.2 Areas and Circumference

We need areas for cross-sections, pressure forces and as the base area of solids. In the workshop the diameter is measured, not the radius; the formulas are therefore written with d.

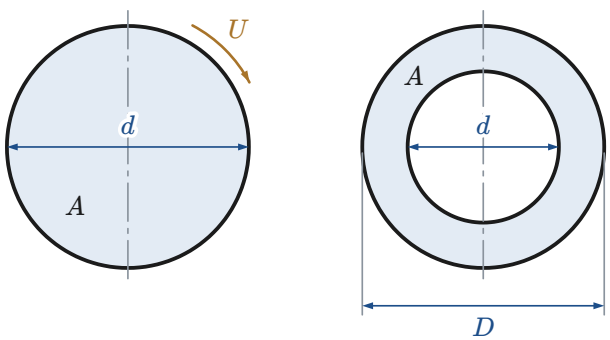


Fig. 1.3 Circle with diameter d, circumference U and area A; annulus with outside diameter D and inside diameter d

Circumference of a Circle

(1.7) JOURNEYMAN

$$U = \pi \cdot d$$

U	circumference	mm
d	diameter	mm

The circumference is π times the diameter, i.e. a little more than three times as long.

Valid: always.

EXAMPLE A balance rim has an outside diameter of 10 mm. What distance does a point on the rim travel in one full revolution?

Given: $d = 10\,\mathrm{mm}$

$$\begin{aligned} U &= \pi \cdot d \\ &= \pi \cdot 10\,\mathrm{mm} \\ &\approx 31.4\,\mathrm{mm} \end{aligned}$$

About 31.4 mm. At 270° amplitude, each vibration covers twice three quarters of this, i.e. about 47 mm.

Area of a Circle

(1.8) JOURNEYMAN

$$A = \frac{\pi \cdot d^2}{4}$$

A	area	mm^2
d	diameter	mm

REARRANGED $d = \sqrt{\frac{4 \cdot A}{\pi}}$

The area grows with the square of the diameter: double the diameter, four times the area.

Valid: always.

EXAMPLE A watch crystal has a diameter of 30 mm. What is its area?

Given: $d = 30\,\mathrm{mm}$

$$\begin{aligned} A &= \frac{\pi \cdot d^2}{4} \\ &= \frac{\pi \cdot (30\,\mathrm{mm})^2}{4} \\ &\approx 707\,\mathrm{mm}^2 \end{aligned}$$

About 707 mm². At a test pressure of 10 bar, i.e. $1\ \frac{\mathrm{N}}{\mathrm{mm}^2}$, about 707 N act on the crystal.

Annulus

(1.9) JOURNEYMAN

$$A = \frac{\pi \cdot \left(D^2 - d^2\right)}{4}$$

A	area	mm^2
D	diameter (outside)	mm
d	diameter (inside)	mm

The annulus is the large circular area minus the small one. This is how the cross-section of tubes, bushes, gaskets and rims is calculated.

Valid: both circles have the same centre.

EXAMPLE A balance rim measures 10 mm outside and 8.4 mm inside. What is its face area?

Given: $D = 10\,\mathrm{mm}$; $d = 8.4\,\mathrm{mm}$

$$\begin{aligned} A &= \frac{\pi \cdot \left(D^2 - d^2\right)}{4} \\ &= \frac{\pi \cdot \left((10\,\mathrm{mm})^2 - (8.4\,\mathrm{mm})^2\right)}{4} \\ &\approx 23.1\,\mathrm{mm}^2 \end{aligned}$$

About 23.1 mm². Multiplied by the rim height this gives the volume, see Section 1.3.

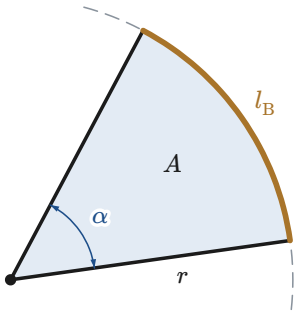


Fig. 1.4 Circular sector: central angle α, arc length l_B, radius r (half the diameter d)

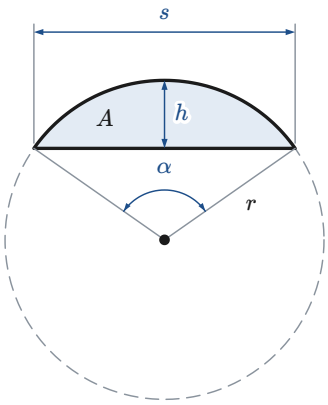


Fig. 1.5 Circular segment: chord s, height of arc h, central angle α, radius r (half the diameter d)

Arc Length

(1.10) JOURNEYMAN

$$l_\mathrm{B} = \frac{\pi \cdot d \cdot \alpha}{360^\circ}$$

l_B	arc length	mm
d	diameter	mm
α	central angle	$^\circ$

The arc is the part of the circumference that belongs to the angle α. With the angle in radians this is written more briefly: arc length equals radius times angle.

Valid: always; α in degrees.

EXAMPLE On a dial, the minute track lies on a circle of 30 mm diameter. How far apart are two minute marks? One minute corresponds to 6°.

Given: $d = 30\,\mathrm{mm}$; $\alpha = 6^\circ$

$$\begin{aligned} l_\mathrm{B} &= \frac{\pi \cdot d \cdot \alpha}{360^\circ} \\ &= \frac{\pi \cdot 30\,\mathrm{mm} \cdot 6^\circ}{360^\circ} \\ &\approx 1.57\,\mathrm{mm} \end{aligned}$$

About 1.57 mm from mark to mark.

Circular Sector

(1.11) JOURNEYMAN

$$A = \frac{\pi \cdot d^2 \cdot \alpha}{4 \cdot 360^\circ}$$

A	area	mm^2
d	diameter	mm
α	central angle	$^\circ$

The circular sector is the slice of cake. Its area is the angle's share of the full circle times the area of the circle.

Valid: always; α in degrees.

EXAMPLE The oscillating weight of an automatic movement is a circular sector with 28 mm diameter and an opening angle of 150°. What is its area?

Given: $d = 28\,\mathrm{mm}$; $\alpha = 150^\circ$

$$\begin{aligned} A &= \frac{\pi \cdot d^2 \cdot \alpha}{4 \cdot 360^\circ} \\ &= \frac{\pi \cdot (28\,\mathrm{mm})^2 \cdot 150^\circ}{4 \cdot 360^\circ} \\ &\approx 257\,\mathrm{mm}^2 \end{aligned}$$

About 257 mm², i.e. five twelfths of the full circular area.

Chord

(1.12) JOURNEYMAN

$$s = d \cdot \sin\left(\frac{\alpha}{2}\right)$$

s	chord (straight line joining two points on the circle)	mm
d	diameter	mm
α	central angle	$^\circ$

The chord is the straight distance between two points on the circle, i.e. the dimension shown by the calliper. The arc between them is always somewhat longer.

Valid: always; α is the angle between the two radii.

EXAMPLE An escape wheel has a diameter of 4.8 mm and 15 teeth, so the pitch is 24°. How far apart are two adjacent tooth tips?

Given: $d = 4.8\,\mathrm{mm}; \quad \alpha = 24^\circ$

$$\begin{aligned} s &= d \cdot \sin\left(\frac{\alpha}{2}\right) \\ &= 4.8\,\mathrm{mm} \cdot \sin\left(\frac{24^\circ}{2}\right) \\ &\approx 1.00\,\mathrm{mm} \end{aligned}$$

About 1.00 mm from tip to tip.

Height of Arc

(1.13) JOURNEYMAN

$$h = \frac{d}{2} \cdot \left(1 - \cos\left(\frac{\alpha}{2}\right)\right)$$

h	height of arc (greatest distance between chord and arc)	mm
d	diameter	mm
α	central angle	$^\circ$

The height of arc states how far the arc stands above the chord. When a flat is ground on an arbor, it is the depth that is removed.

Valid: α up to 180°.

EXAMPLE A flat is to be made on an arbor of 3 mm diameter. The flat is to span a central angle of 90°. How deep must material be removed?

Given: $d = 3\,\mathrm{mm}; \quad \alpha = 90^\circ$

$$\begin{aligned} h &= \frac{d}{2} \cdot \left(1 - \cos\left(\frac{\alpha}{2}\right)\right) \\ &= \frac{3\,\mathrm{mm}}{2} \cdot \left(1 - \cos\left(\frac{90^\circ}{2}\right)\right) \\ &\approx 0.44\,\mathrm{mm} \end{aligned}$$

About 0.44 mm. The flat will then be 2.12 mm wide, see formula (1.12).

Radius from Chord and Height of Arc

(1.14) MASTER

$$r = \frac{s^2}{8 \cdot h} + \frac{h}{2}$$

Chord and height of an arc are easy to measure, the radius is not. This formula gives the radius of curvature of a crystal or a domed dial.

Valid: always; follows from Pythagoras' theorem.

r	radius (of the arc)	mm
s	chord (straight line joining two points on the circle)	mm
h	height of arc (greatest distance between chord and arc)	mm

EXAMPLE A domed watch crystal has a diameter of 30 mm and is 1.9 mm higher in the centre than at the edge. What is its radius of curvature?

Given: $s = 30\,\mathrm{mm}$; $h = 1.9\,\mathrm{mm}$

$$\begin{aligned} r &= \frac{s^2}{8 \cdot h} + \frac{h}{2} \\ &= \frac{(30\,\mathrm{mm})^2}{8 \cdot 1.9\,\mathrm{mm}} + \frac{1.9\,\mathrm{mm}}{2} \\ &\approx 60.2\,\mathrm{mm} \end{aligned}$$

About 60 mm. Here the diameter of the crystal is the chord.

Circular Segment

(1.15) MASTER

$$A = \frac{d^2}{8} \cdot \left(\frac{\pi \cdot \alpha}{180^\circ} - \sin \alpha \right)$$

The circular segment is the area between chord and arc: the circular sector minus the triangle formed by the two radii and the chord.

Valid: α in degrees, up to 360°.

A	area	mm^2
d	diameter	mm
α	central angle	$^\circ$

EXAMPLE A segment with a central angle of 90° is removed from the arbor of 3 mm diameter. How much cross-section is lost?

Given: $d = 3\,\mathrm{mm}$; $\alpha = 90^\circ$

$$\begin{aligned} A &= \frac{d^2}{8} \cdot \left(\frac{\pi \cdot \alpha}{180^\circ} - \sin \alpha \right) \\ &= \frac{(3\,\mathrm{mm})^2}{8} \cdot \left(\frac{\pi \cdot 90^\circ}{180^\circ} - \sin\left(90^\circ\right) \right) \\ &\approx 0.64\,\mathrm{mm}^2 \end{aligned}$$

About 0.64 mm², i.e. 9 % of the full cross-section of 7.07 mm².

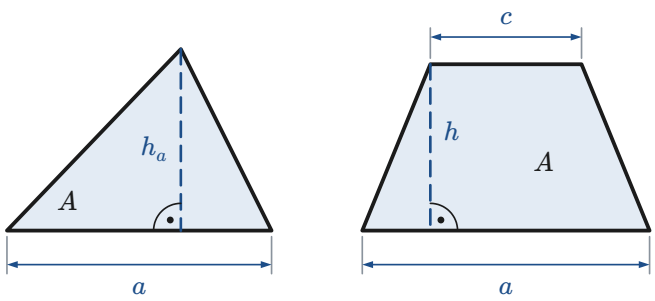


Fig. 1.6 Triangle with base a and height h_a; trapezium with parallel sides a and c and height h

Triangle

(1.16) JOURNEYMAN

$$A = \frac{a \cdot h_a}{2}$$

A	area	mm^2
a	side (base)	mm
h_a	height (perpendicular to the base)	mm

Every triangle is half a rectangle formed by base and height. Any side may serve as the base; the height must be perpendicular to it.

Valid: always.

EXAMPLE A triangular hour marker is 2.4 mm wide at the base and 5 mm high. What is its area?

Given: $a = 2.4\,\mathrm{mm}$; $h_a = 5\,\mathrm{mm}$

$$\begin{aligned} A &= \frac{a \cdot h_a}{2} \\ &= \frac{2.4\,\mathrm{mm} \cdot 5\,\mathrm{mm}}{2} \\ &= 6.0\,\mathrm{mm}^2 \end{aligned}$$

6 mm². This is, for example, the area to be covered with luminous compound.

Trapezium

(1.17) JOURNEYMAN

$$A = \frac{a + c}{2} \cdot h$$

A	area	mm^2
a	side (first of the two parallel sides)	mm
c	side (second of the two parallel sides)	mm
h	height (distance between the parallel sides)	mm

The trapezium has the area of a rectangle with the mean width. If both parallel sides are equal, side times height remains: the area of rectangle and parallelogram.

Valid: two sides are parallel.

EXAMPLE Over a length of 14 mm, a hand tapers from 3 mm to 1.2 mm width. What is its area?

Given: $a = 3\,\mathrm{mm}$; $c = 1.2\,\mathrm{mm}$; $h = 14\,\mathrm{mm}$

$$\begin{aligned} A &= \frac{a + c}{2} \cdot h \\ &= \frac{3\,\mathrm{mm} + 1.2\,\mathrm{mm}}{2} \cdot 14\,\mathrm{mm} \\ &= 29.4\,\mathrm{mm}^2 \end{aligned}$$

29.4 mm². With thickness and density this gives the mass of the hand, see Section 1.3.

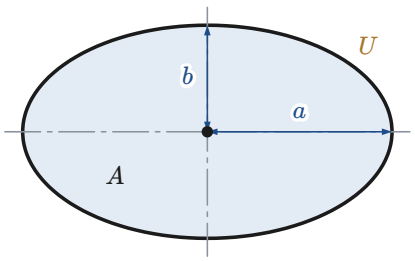


Fig. 1.7 Ellipse with semi-axes a and b

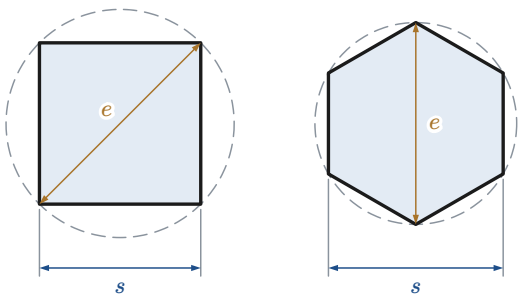


Fig. 1.8 Square and hexagon: width across flats s, across-corners dimension e

Ellipse: Area

(1.18) JOURNEYMAN

$$A = \pi \cdot a \cdot b$$

The ellipse is a stretched circle. If both semi-axes are equal, it becomes the area of a circle.

Valid: always; a and b are half the outside dimensions.

A	area	mm^2
a	semi-axis (major, half the length)	mm
b	semi-axis (minor, half the width)	mm

EXAMPLE An oval watch crystal measures 22 mm × 18 mm. What is its area?

Given: $a = 11\,\mathrm{mm}; \quad b = 9\,\mathrm{mm}$

$$\begin{aligned} A &= \pi \cdot a \cdot b \\ &= \pi \cdot 11\,\mathrm{mm} \cdot 9\,\mathrm{mm} \\ &\approx 311\,\mathrm{mm}^2 \end{aligned}$$

About 311 mm². The semi-axes are half the outside dimensions, i.e. 11 mm and 9 mm.

Ellipse: Circumference

(1.19) APPROXIMATION ADVANCED

$$U \approx \pi \cdot \left(3 \cdot (a+b) - \sqrt{(3 \cdot a + b) \cdot (a + 3 \cdot b)}\right)$$

There is no simple closed formula for the circumference of an ellipse. This approximation is sufficient for any workshop task, for example for the length of a gasket or a crystal ring.

Valid: approximation; the error stays below 0.01 % as long as the ellipse is at most three times as long as it is wide.

U	circumference	mm
a	semi-axis (major)	mm
b	semi-axis (minor)	mm

EXAMPLE How long is the gasket for the oval crystal of 22 mm × 18 mm?

Given: $a = 11\,\mathrm{mm}; \quad b = 9\,\mathrm{mm}$

$$\begin{aligned} U &\approx \pi \cdot \left(3 \cdot (a+b) - \sqrt{(3 \cdot a + b) \cdot (a + 3 \cdot b)}\right) \\ &= \pi \cdot \left(3 \cdot (11\,\mathrm{mm} + 9\,\mathrm{mm}) - \sqrt{(3 \cdot 11\,\mathrm{mm} + 9\,\mathrm{mm}) \cdot (11\,\mathrm{mm} + 3 \cdot 9\,\mathrm{mm})}\right) \\ &\approx 63.0\,\mathrm{mm} \end{aligned}$$

About 63 mm. A circle with the mean diameter of 20 mm would have a circumference of 62.8 mm.

Square: Across Corners

(1.20) JOURNEYMAN

$$e = \sqrt{2} \cdot s$$

REARRANGED $s = \dfrac{e}{\sqrt{2}}$

e	across-corners dimension (measured over the corners)	mm
s	width across flats (measured over the flats)	mm

The across-corners dimension is the diagonal of the square. It states the minimum size of round stock from which a square can be filed or milled. The area of the square is s^2.

Valid: sharp-edged square. In practice the corners are often chamfered, then slightly less is sufficient.

EXAMPLE The winding arbor of a clock is to receive a square with 4 mm across flats. What is the minimum diameter of the arbor?

Given: $s = 4\,\mathrm{mm}$

$$\begin{aligned} e &= \sqrt{2} \cdot s \\ &= \sqrt{2} \cdot 4\,\mathrm{mm} \\ &\approx 5.66\,\mathrm{mm} \end{aligned}$$

At least 5.66 mm if the corners are to be fully formed.

Hexagon: Across Corners

(1.21) JOURNEYMAN

$$e = \frac{2 \cdot s}{\sqrt{3}}$$

REARRANGED $s = \dfrac{\sqrt{3} \cdot e}{2}$

e	across-corners dimension (measured over the corners)	mm
s	width across flats (measured over the flats)	mm

For a hexagon, the across-corners dimension is about 15.5 % larger than the width across flats. The area of the hexagon is $0.866 \cdot s^2$.

Valid: regular, sharp-edged hexagon.

EXAMPLE A hexagon nut has a width across flats of 5.5 mm. What is the minimum diameter of the round stock?

Given: $s = 5.5\,\mathrm{mm}$

$$\begin{aligned} e &= \frac{2 \cdot s}{\sqrt{3}} \\ &= \frac{2 \cdot 5.5\,\mathrm{mm}}{\sqrt{3}} \\ &\approx 6.35\,\mathrm{mm} \end{aligned}$$

At least 6.35 mm.

Key point: Double the diameter means double the circumference, but four times the area.

1.3 Solids and Mass

The volume and the density of the material give the mass of a part. We need this for balance rims, pendulum bobs, weights and for the material required in precious metals.

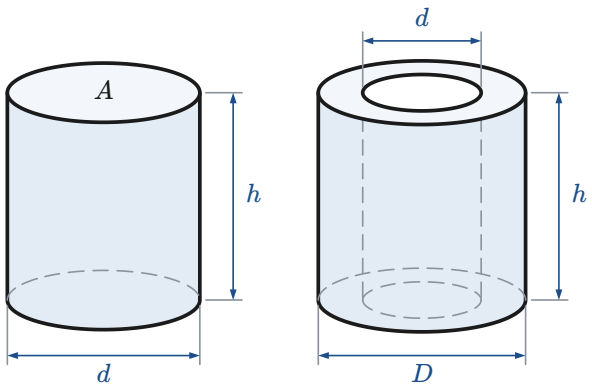


Fig. 1.9 Cylinder and hollow cylinder; base area A times height h gives the volume

Prism: Solid with Constant Cross-Section

(1.22) JOURNEYMAN

$$V = A \cdot h$$

V	volume	mm^3
A	area (base area, cross-section of the solid)	mm^2
h	height (perpendicular to the base area)	mm

Every solid whose cross-section remains the same over its whole height has the volume base area times height: bars, sheets, profiles. The base area comes from Section 1.2.

Valid: cross-section the same everywhere, height perpendicular to the base area.

EXAMPLE A piece 20 mm long is parted off a hexagon bar with 5 mm across flats (cross-section 21.65 mm²). What is its volume?

Given: $A = 21.65\,\mathrm{mm}^2$; $h = 20\,\mathrm{mm}$

$$\begin{aligned} V &= A \cdot h \\ &= 21.65\,\mathrm{mm}^2 \cdot 20\,\mathrm{mm} \\ &= 433\,\mathrm{mm}^3 \end{aligned}$$

433 mm³, i.e. 0.433 cm³.

Cylinder

(1.23) JOURNEYMAN

$$V = \frac{\pi \cdot d^2}{4} \cdot h$$

V	volume	mm^3
d	diameter	mm
h	height (perpendicular to the base area)	mm

The cylinder is the prism with a circular base: arbors, pins, blanks, discs.

Valid: right circular cylinder.

EXAMPLE A brass blank has a diameter of 30 mm and is 2 mm thick. What is its volume?

Given: $d = 30\,\mathrm{mm}$; $h = 2\,\mathrm{mm}$

$$\begin{aligned} V &= \frac{\pi \cdot d^2}{4} \cdot h \\ &= \frac{\pi \cdot (30\,\mathrm{mm})^2}{4} \cdot 2\,\mathrm{mm} \\ &\approx 1414\,\mathrm{mm}^3 \end{aligned}$$

About 1414 mm³ or 1.414 cm³.

Hollow Cylinder

(1.24) JOURNEYMAN

$$V = \frac{\pi \cdot \left(D^2 - d^2\right)}{4} \cdot h$$

Annulus times height: tubes, bushes, rings and the rim of a balance.

Valid: right hollow cylinder, bore in the centre.

V	volume	mm^3
D	diameter (outside)	mm
d	diameter (inside)	mm
h	height (perpendicular to the base area)	mm

EXAMPLE A balance rim measures 10 mm outside, 8.4 mm inside and is 0.8 mm high. What is its volume?

Given: $D = 10\,\mathrm{mm}$; $d = 8.4\,\mathrm{mm}$; $h = 0.8\,\mathrm{mm}$

$$\begin{aligned} V &= \frac{\pi \cdot \left(D^2 - d^2\right)}{4} \cdot h \\ &= \frac{\pi \cdot \left((10\,\mathrm{mm})^2 - (8.4\,\mathrm{mm})^2\right)}{4} \cdot 0.8\,\mathrm{mm} \\ &\approx 18.5\,\mathrm{mm}^3 \end{aligned}$$

About 18.5 mm³. Made of beryllium bronze, this rim weighs about 0.15 g.

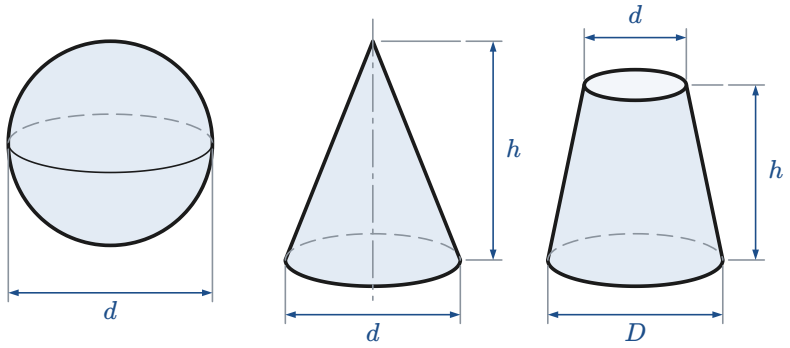


Fig. 1.10 Sphere, cone and truncated cone

Sphere

(1.25) JOURNEYMAN

$$V = \frac{\pi \cdot d^3}{6}$$

The sphere fills a little more than half of the cube into which it just fits. Its surface area is $\pi \cdot d^2$.

Valid: always.

V	volume	mm^3
d	diameter	mm

EXAMPLE A steel ball has a diameter of 2 mm. What is its volume?

Given: $d = 2\,\mathrm{mm}$

$$\begin{aligned} V &= \frac{\pi \cdot d^3}{6} \\ &= \frac{\pi \cdot (2\,\mathrm{mm})^3}{6} \\ &\approx 4.19\,\mathrm{mm}^3 \end{aligned}$$

About 4.19 mm³; the ball therefore weighs about 33 mg.

Cone

(1.26) JOURNEYMAN

$$V = \frac{\pi \cdot d^2 \cdot h}{12}$$

V	volume	mm^3
d	diameter (of the base)	mm
h	height (perpendicular to the base area)	mm

The cone has one third of the volume of the cylinder with the same base area and height. The same applies to the pyramid compared with the prism.

Valid: height measured perpendicular to the base area.

EXAMPLE A 60° centre-punch point is 3 mm in diameter at the base and 2.6 mm high. What is the volume of the point?

Given: $d = 3\,\mathrm{mm}$; $h = 2.6\,\mathrm{mm}$

$$\begin{aligned} V &= \frac{\pi \cdot d^2 \cdot h}{12} \\ &= \frac{\pi \cdot (3\,\mathrm{mm})^2 \cdot 2.6\,\mathrm{mm}}{12} \\ &\approx 6.13\,\mathrm{mm}^3 \end{aligned}$$

About 6.13 mm³. The cylinder with the same dimensions would have three times this.

Truncated Cone

(1.27) JOURNEYMAN

$$V = \frac{\pi \cdot h \cdot \left(D^2 + D \cdot d + d^2\right)}{12}$$

V	volume	mm^3
h	height (perpendicular to the base area)	mm
D	diameter (at the large end)	mm
d	diameter (at the small end)	mm

The truncated cone is a cone without its tip: tapered pins, reamers, conical case parts. With $d = 0$ it becomes the cone, with $d = D$ the cylinder.

Valid: right truncated cone, top face parallel to the base.

EXAMPLE A tapered brass plug is 10 mm long and tapers from 6 mm to 4 mm diameter. What is its volume?

Given: $h = 10\,\mathrm{mm}$; $D = 6\,\mathrm{mm}$; $d = 4\,\mathrm{mm}$

$$\begin{aligned} V &= \frac{\pi \cdot h \cdot \left(D^2 + D \cdot d + d^2\right)}{12} \\ &= \frac{\pi \cdot 10\,\mathrm{mm} \cdot \left((6\,\mathrm{mm})^2 + 6\,\mathrm{mm} \cdot 4\,\mathrm{mm} + (4\,\mathrm{mm})^2\right)}{12} \\ &\approx 199\,\mathrm{mm}^3 \end{aligned}$$

About 199 mm³. Calculating as a cylinder with the mean diameter of 5 mm would give 196 mm³, i.e. slightly too little.

Mass from Volume and Density

(1.28) JOURNEYMAN

$$m = \varrho \cdot V$$

REARRANGED $\varrho = \dfrac{m}{V}$

m	mass	g
ϱ	density (of the material)	$\frac{\mathrm{g}}{\mathrm{cm}^3}$
V	volume	cm^3

The density states how much mass is contained in one cubic centimetre. Rearranged, the formula serves to identify a material by weighing and measuring.

Valid: solid part without cavities, uniform material. $1\,\mathrm{cm}^3 = 1000\,\mathrm{mm}^3$.

EXAMPLE The brass blank from formula (1.23) has a volume of 1414 mm³, i.e. 1.414 cm³. How heavy is it at a density of $8.5\ \frac{\mathrm{g}}{\mathrm{cm}^3}$?

Given: $\varrho = 8.5\ \frac{\mathrm{g}}{\mathrm{cm}^3}$; $\quad V = 1.414\,\mathrm{cm}^3$

$$\begin{aligned} m &= \varrho \cdot V \\ &= 8.5\ \tfrac{\mathrm{g}}{\mathrm{cm}^3} \cdot 1.414\,\mathrm{cm}^3 \\ &\approx 12.0\,\mathrm{g} \end{aligned}$$

About 12 g. Made of 750 gold, the same blank would weigh about 22 g.

Density of Watchmaking Materials (Guide Values at 20 °C)

Material	ϱ in $\frac{\mathrm{g}}{\mathrm{cm}^3}$
aluminium	2.7
titanium	4.5
steel, unalloyed	7.85
stainless steel (1.4404, 316L)	8.0
beryllium bronze (CuBe2, balance rims)	8.3
brass (CuZn39Pb3)	8.5
nickel silver	8.6 … 8.7
silver 925	10.4
gold 585	13 … 14
gold 750	15 … 16
fine gold	19.3
platinum 950	20 … 21.5
sapphire and ruby (corundum)	4.0
mineral glass	2.5
acrylic glass	1.19

For alloys the density depends on the exact composition; the manufacturer's data sheet is authoritative. $1\ \frac{\mathrm{g}}{\mathrm{cm}^3}$ is the same as $1\ \frac{\mathrm{mg}}{\mathrm{mm}^3}$.

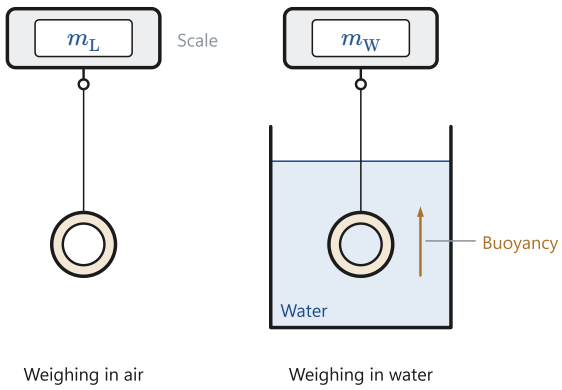


Fig. 1.11 Density from two weighings: in air the balance shows m_L, with the part in water only m_W. The difference is the mass of the displaced water

Density by Weighing in Air and in Water

(1.29) JOURNEYMAN

$$\varrho = \frac{m_L}{m_L - m_W} \cdot \varrho_W$$

An immersed part becomes lighter by the weight of the water it displaces (Archimedes' principle). The difference between the two weighings therefore gives the volume without having to measure the part. This makes it possible to check whether the density of a ring or case matches the stamped fineness.

Valid: solid part of one material, without cavities, stones or inlays; fully immersed, without air bubbles and without touching the vessel; thin suspension wire. Buoyancy in air is neglected; the error is about 0.1 %.

ϱ	density (of the part)	$\frac{g}{cm^3}$
m_L	mass (reading of the balance, part in air)	g
m_W	reading of the balance when the part hangs fully immersed in water	g
ϱ_W	density of water ($0.998\ \frac{g}{cm^3}$ at 20 °C)	$\frac{g}{cm^3}$

Caution: Density is an indication, not proof. Different alloys can have the same density, and tungsten is almost exactly as dense as fine gold. Solder joints, springs and cavities falsify the result.

EXAMPLE A wedding ring weighs 8.40 g in air. Hanging on a thin wire fully immersed in water at 20 °C, the balance shows 7.86 g. What is the density of the ring?

Given: $m_L = 8.40\,g$; $m_W = 7.86\,g$; $\varrho_W = 0.998\ \frac{g}{cm^3}$

$$\varrho = \frac{m_L}{m_L - m_W} \cdot \varrho_W$$
$$= \frac{8.40\,g}{8.40\,g - 7.86\,g} \cdot 0.998\ \frac{g}{cm^3}$$
$$\approx 15.5\ \frac{g}{cm^3}$$

About $15.5\ \frac{g}{cm^3}$. According to the table this matches 750 gold, not 585 gold.

1.4 Triangles and Coordinates

With the right-angled triangle we can solve almost every dimensional problem: distances from coordinates, slopes, angles on the dividing apparatus. The angles of a triangle always add up to 180°. For the trigonometric functions the calculator must be set to degrees (DEG).

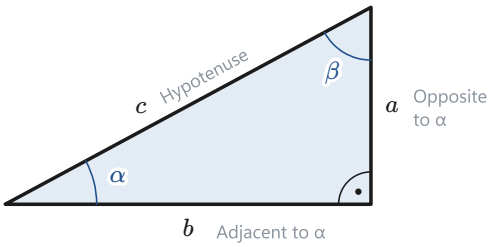


Fig. 1.12 Right-angled triangle: the hypotenuse c lies opposite the right angle

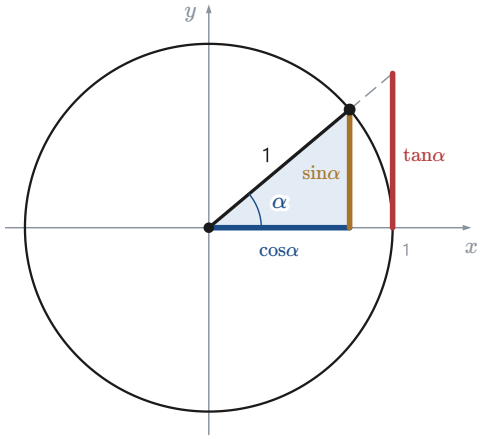


Fig. 1.13 Unit circle (radius 1): sine, cosine and tangent as line segments

Pythagoras' Theorem

(1.30) JOURNEYMAN

$$c = \sqrt{a^2 + b^2}$$

REARRANGED $a = \sqrt{c^2 - b^2}$

c	hypotenuse (longest side, opposite the right angle)	mm
a	leg (cathetus)	mm
b	leg (cathetus)	mm

In a right-angled triangle, the square on the hypotenuse equals the sum of the squares on the two other sides.

Valid: only in a right-angled triangle.

EXAMPLE A rectangular plate of 15.3 mm × 17.2 mm is to be made from a round blank. What is the minimum diameter of the blank?

Given: $a = 15.3\,\mathrm{mm}; \quad b = 17.2\,\mathrm{mm}$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{(15.3\,\mathrm{mm})^2 + (17.2\,\mathrm{mm})^2} \\ &\approx 23.0\,\mathrm{mm} \end{aligned}$$

At least 23 mm, because the diagonal of the rectangle is the diameter of the smallest circle around the plate.

Sine

(1.31) JOURNEYMAN

$$a = c \cdot \sin\alpha$$

REARRANGED $\alpha = \arcsin\left(\frac{a}{c}\right)$

a	leg (opposite side, lies opposite the angle)	mm
c	hypotenuse	mm
α	angle	°

The sine is the ratio of opposite side to hypotenuse. The rearranged form gives the angle from two sides; the key for this is usually labelled $\sin^{-1}$.

Valid: only in a right-angled triangle.

EXAMPLE A sine bar with 100 mm roller spacing is to be set to 15°. How high must the gauge block stack be?

Given: $c = 100\,\mathrm{mm}; \quad \alpha = 15^\circ$

$$\begin{aligned} a &= c \cdot \sin\alpha \\ &= 100\,\mathrm{mm} \cdot \sin(15^\circ) \\ &\approx 25.88\,\mathrm{mm} \end{aligned}$$

Gauge blocks totalling 25.88 mm go under one roller.

Cosine

(1.32) JOURNEYMAN

$$b = c \cdot \cos \alpha$$

REARRANGED $\alpha = \arccos\left(\frac{b}{c}\right)$

b	leg (adjacent side, lies next to the angle)	mm
c	hypotenuse	mm
α	angle	$^\circ$

The cosine is the ratio of adjacent side to hypotenuse. It states how much of an inclined distance remains effective in one direction.

Valid: only in a right-angled triangle.

EXAMPLE A lever 20 mm long is inclined at 30° to the horizontal. How long is it measured horizontally?

Given: $c = 20\,\mathrm{mm}$; $\alpha = 30^\circ$

$$\begin{aligned} b &= c \cdot \cos \alpha \\ &= 20\,\mathrm{mm} \cdot \cos(30^\circ) \\ &\approx 17.32\,\mathrm{mm} \end{aligned}$$

About 17.32 mm. Perpendicular to this it is 20 mm times sin 30°, i.e. exactly 10 mm.

Tangent

(1.33) JOURNEYMAN

$$a = b \cdot \tan \alpha$$

REARRANGED $\alpha = \arctan\left(\frac{a}{b}\right)$

a	leg (opposite side)	mm
b	leg (adjacent side)	mm
α	angle	$^\circ$

The tangent is the ratio of the two legs, i.e. the gradient of a slope. It is used to calculate chamfers, countersinks and tapers; more on tapers in Chapter 9.

Valid: only in a right-angled triangle; α less than 90°.

EXAMPLE Over a length of 12 mm, a taper becomes 0.5 mm smaller in radius. To what angle is the top slide set?

Given: $a = 0.5\,\mathrm{mm}$; $b = 12\,\mathrm{mm}$

$$\begin{aligned} \alpha &= \arctan\left(\frac{a}{b}\right) \\ &= \arctan\left(\frac{0.5\,\mathrm{mm}}{12\,\mathrm{mm}}\right) \\ &\approx 2.39^\circ \end{aligned}$$

To about 2.39°. This is half the taper angle.

Key point: Sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, tangent is opposite over adjacent.

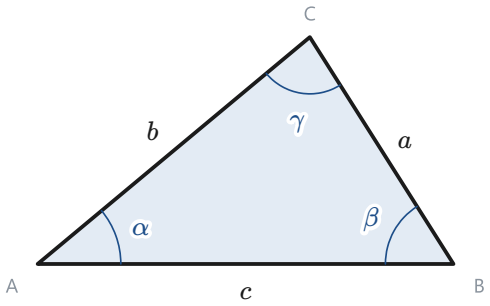


Fig. 1.14 General triangle: side a lies opposite angle α, b opposite angle β, c opposite angle γ

Law of Sines

(1.34) MASTER

$$a = \frac{b \cdot \sin\alpha}{\sin\beta}$$

a	side (opposite the angle α)	mm
b	side (opposite the angle β)	mm
α	angle (opposite the side a)	°
β	angle (opposite the side b)	°

In every triangle the sides are in the same ratio as the sines of the opposite angles. The law of sines applies when one side and two angles are known.

Valid: in every triangle. If an angle is sought from two sides, two solutions are possible (acute or obtuse).

EXAMPLE In a triangle formed by three bearing points, the side b = 6.0 mm and the angles α = 40° and β = 75° are known. How long is the side a?

Given: $b = 6.0\,\mathrm{mm}; \quad \alpha = 40°; \quad \beta = 75°$

$$\begin{aligned} a &= \frac{b \cdot \sin\alpha}{\sin\beta} \\ &= \frac{6.0\,\mathrm{mm} \cdot \sin(40°)}{\sin(75°)} \\ &\approx 3.99\,\mathrm{mm} \end{aligned}$$

About 3.99 mm. The third angle is 180° − 40° − 75° = 65°.

Law of Cosines

(1.35) MASTER

$$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$$

REARRANGED $\gamma = \arccos\left(\frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}\right)$

c	side (opposite the angle γ)	mm
a	side (adjacent to the angle γ)	mm
b	side (adjacent to the angle γ)	mm
γ	angle (between the sides a and b)	°

The law of cosines is Pythagoras' theorem for any triangle. It applies when two sides and the angle between them are known or, rearranged, when an angle is sought from three sides.

Valid: in every triangle; for $\gamma = 90°$ the last term vanishes.

EXAMPLE From the third wheel, the centre wheel is 7.2 mm away and the fourth wheel 5.4 mm. The two connecting lines enclose 110°. How far apart are the centre wheel and the fourth wheel?

Given: $a = 7.2\,\mathrm{mm}; \quad b = 5.4\,\mathrm{mm}; \quad \gamma = 110°$

$$\begin{aligned} c &= \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma} \\ &= \sqrt{(7.2\,\mathrm{mm})^2 + (5.4\,\mathrm{mm})^2 - 2 \cdot 7.2\,\mathrm{mm} \cdot 5.4\,\mathrm{mm} \cdot \cos(110°)} \\ &\approx 10.37\,\mathrm{mm} \end{aligned}$$

About 10.37 mm from arbor to arbor.

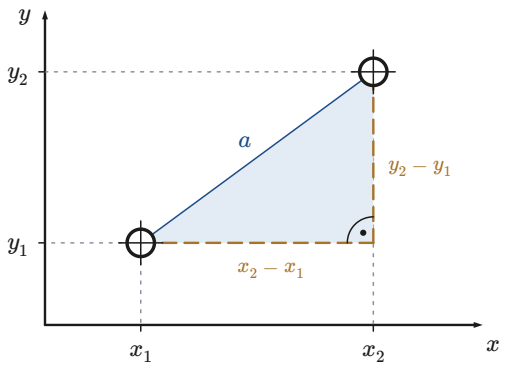


Fig. 1.15 Centre distance a as the hypotenuse over the coordinate differences

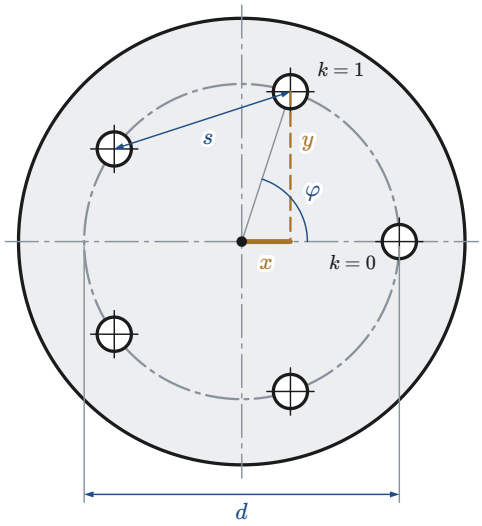


Fig. 1.16 Hole circle with $n = 5$ holes: pitch angle φ, chord s, coordinates x and y of hole $k = 1$

Centre Distance from Coordinates

(1.36) JOURNEYMAN

$$a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

On the jig borer and in the CAD drawing, bearing holes are defined by x and y. Their distance is the hypotenuse over the two coordinate differences.

Valid: rectangular coordinate system; both points referred to the same origin.

a	centre distance (depthing distance)	mm
x_1	coordinate (x direction, first hole)	mm
y_1	coordinate (y direction, first hole)	mm
x_2	coordinate (x direction, second hole)	mm
y_2	coordinate (y direction, second hole)	mm

EXAMPLE Two bearing holes are located at (2.50 mm | 1.20 mm) and (7.30 mm | 5.60 mm). What is the centre distance?

Given: $x_1 = 2.50\,\mathrm{mm}$; $y_1 = 1.20\,\mathrm{mm}$; $x_2 = 7.30\,\mathrm{mm}$; $y_2 = 5.60\,\mathrm{mm}$

$$a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(7.30\,\mathrm{mm} - 2.50\,\mathrm{mm})^2 + (5.60\,\mathrm{mm} - 1.20\,\mathrm{mm})^2}$$
$$\approx 6.51\,\mathrm{mm}$$

About 6.51 mm. This value is compared with the depthing distance of the wheel pair, see Chapter 4.

Hole Circle: Distance between Adjacent Holes

(1.37) JOURNEYMAN

$$s = d \cdot \sin\left(\frac{180^\circ}{n}\right)$$

s	chord (from hole centre to hole centre)	mm
d	diameter (of the hole circle)	mm
n	number (of evenly spaced holes)	1

With the chord, a hole circle can be marked out with dividers and checked with the calliper after drilling. The pitch angle between two holes is 360° divided by n.

Valid: even spacing.

EXAMPLE Five holes lie on a hole circle of 24 mm diameter. What is the distance between adjacent holes?

Given: $d = 24\,\mathrm{mm}$; $n = 5$

$$\begin{aligned} s &= d \cdot \sin\left(\frac{180^\circ}{n}\right) \\ &= 24\,\mathrm{mm} \cdot \sin\left(\frac{180^\circ}{5}\right) \\ &\approx 14.11\,\mathrm{mm} \end{aligned}$$

About 14.11 mm from centre to centre; the pitch angle is 72°.

Coordinates on the Pitch Circle: x

(1.38) JOURNEYMAN

$$x = \frac{d}{2} \cdot \cos\varphi$$

x	coordinate (x direction, from the centre of the circle)	mm
d	diameter (of the pitch circle)	mm
φ	angle (from the x axis, anticlockwise)	°

For drilling by coordinates, each point of the pitch circle is resolved into x and y. Hole number k lies at $\varphi = k \cdot 360^\circ$ divided by n, counted from $k = 0$ on the x axis.

Valid: origin at the centre of the circle. The sign follows automatically: to the left of the centre x becomes negative.

EXAMPLE Hole circle with 24 mm diameter and 5 holes: where does hole $k = 1$ lie in the x direction?

Given: $d = 24\,\mathrm{mm}$; $\varphi = 72^\circ$

$$\begin{aligned} x &= \frac{d}{2} \cdot \cos\varphi \\ &= \frac{24\,\mathrm{mm}}{2} \cdot \cos(72^\circ) \\ &\approx 3.71\,\mathrm{mm} \end{aligned}$$

3.71 mm to the right of the centre.

Coordinates on the Pitch Circle: y

(1.39) JOURNEYMAN

$$y = \frac{d}{2} \cdot \sin \varphi$$

The second coordinate of the same point. As a check, x and y must give the radius again by Pythagoras' theorem.

Valid: as for formula (1.38); below the centre y becomes negative.

y	coordinate (y direction, from the centre of the circle)	mm
d	diameter (of the pitch circle)	mm
φ	angle (from the x axis, anticlockwise)	$^\circ$

EXAMPLE Where does the same hole lie in the y direction?

Given: $d = 24\,\mathrm{mm}$; $\varphi = 72^\circ$

$$\begin{aligned} y &= \frac{d}{2} \cdot \sin \varphi \\ &= \frac{24\,\mathrm{mm}}{2} \cdot \sin(72^\circ) \\ &\approx 11.41\,\mathrm{mm} \end{aligned}$$

11.41 mm above the centre. Check: from 3.71 mm and 11.41 mm, Pythagoras' theorem gives the radius 12.00 mm.

1.5 Mean and Deviation

Two small tools for every series of measurements. Range, standard deviation and the evaluation of complete series of measurements are given in Section 12.1, rate measurement in Chapter 8.

Arithmetic Mean

(1.40) JOURNEYMAN

$$\bar{x} = \frac{\sum x_i}{n}$$

The mean evens out random fluctuations of individual measurements. It says nothing about how far apart the individual values lie.

Valid: all values in the same unit; observe the sign.

$\bar{x}$	mean value (unit as for the individual values)	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum x_i$	sum of all measured values (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$
n	number (of measured values)	1

EXAMPLE In five positions a watch shows rates of +4, +7, −2, +5 and +6 seconds per day. The sum is +20 seconds per day. What is the mean rate?

Given: $\sum x_i = 20\,\frac{\mathrm{s}}{\mathrm{d}}$; $n = 5$

$$\begin{aligned} \bar{x} &= \frac{\sum x_i}{n} \\ &= \frac{20\,\frac{\mathrm{s}}{\mathrm{d}}}{5} \\ &= 4.0\,\frac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

On average +4 s per day. The largest difference between two positions is 9 s per day.

Deviation in Percent

(1.41) JOURNEYMAN

$$p = \frac{x_{\text{ist}} - x_{\text{soll}}}{x_{\text{soll}}}$$

The deviation is always referred to the nominal value. As a number times 100 it gives percent, times 1000 per mille.

Valid: nominal value not zero; both values in the same unit.

p	relative deviation (+ means: actual value too large)	$\%$
x_{ist}	actual value (measured)	mm
x_{soll}	nominal value (unit as for the actual value)	mm

EXAMPLE An arbor is to have a diameter of 2.50 mm; 2.47 mm is measured. What is the deviation?

Given: $x_{\text{ist}} = 2.47\,\text{mm}$; $x_{\text{soll}} = 2.50\,\text{mm}$

$$\begin{aligned} p &= \frac{x_{\text{ist}} - x_{\text{soll}}}{x_{\text{soll}}} \\ &= \frac{2.47\,\text{mm} - 2.50\,\text{mm}}{2.50\,\text{mm}} \\ &= -1.2\,\% \end{aligned}$$

The arbor is 1.2 % too thin.

2 Mechanics

A mechanical watch is applied mechanics: a spring or a weight supplies force, wheels and levers transmit it, friction consumes part of it, and in the end an inert mass keeps the beat. This chapter collects the formulas on which all following chapters are built.

The examples calculate with quantities from the workshop, from the balance of a few milligrams to the driving weight of the longcase clock. Details of the drive, gear train, balance and pendulum are given in Chapters 3 to 6. Some symbols here follow the custom of mechanics: A is an area, s usually a distance, m a mass, P a power, G the shear modulus and α also an angle of inclination. The table of quantities next to each formula is always authoritative.

2.1 Motion

Parts of a watch move in a straight line, like a falling weight, or they rotate, like wheels and balance. There is a speed for both kinds of motion; the rotational speed links them.

Speed

(2.1) JOURNEYMAN

$$v = \frac{s}{t}$$

v	speed	$\frac{\mathrm{m}}{\mathrm{s}}$
s	distance (travelled)	m
t	time (required for it)	s

Speed is distance per unit time.

Valid: uniform motion. If the speed changes along the way, the formula gives the mean value.

EXAMPLE The weight of a longcase clock falls 1.2 m in 8 days. How fast does it fall?

Given: $s = 1.2\,\mathrm{m}; \quad t = 8\,\mathrm{d}$

$$\begin{aligned} v &= \frac{s}{t} \\ &= \frac{1.2\,\mathrm{m}}{8\,\mathrm{d}} \\ &= \frac{1200\,\mathrm{mm}}{192\,\mathrm{h}} \\ &= 6.25\ \tfrac{\mathrm{mm}}{\mathrm{h}} \end{aligned}$$

The weight falls 6.25 mm per hour, i.e. 150 mm per day. The position of the weight therefore shows when the clock must be wound again.

Acceleration

(2.2) JOURNEYMAN

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is the change in speed per unit time. Shocks are very large accelerations over a very short time.

Valid: uniform acceleration; otherwise mean value over Δt.

a	acceleration (when braking: deceleration)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
Δv	change in speed	$\frac{\mathrm{m}}{\mathrm{s}}$
Δt	time interval (in which the speed changes)	s

EXAMPLE A wristwatch falls from a height of 1 m and hits the floor at $4.4\ \frac{\mathrm{m}}{\mathrm{s}}$. The floor brings it to rest in about 1 ms. What is the deceleration?

Given: $\Delta v = 4.4\ \frac{\mathrm{m}}{\mathrm{s}}; \quad \Delta t = 1\,\mathrm{ms}$

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ &= \frac{4.4\ \frac{\mathrm{m}}{\mathrm{s}}}{1\,\mathrm{ms}} \\ &= 4400\ \frac{\mathrm{m}}{\mathrm{s}^2} \end{aligned}$$

This is about 450 times the gravitational acceleration. The harder the floor, the shorter the braking time and the greater the deceleration.

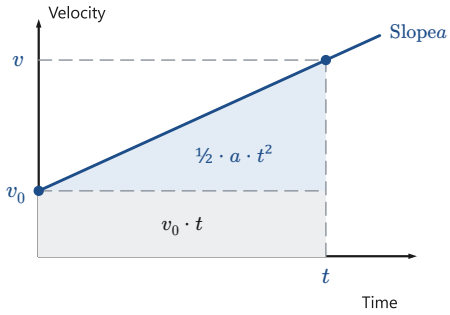


Fig. 2.1 Uniformly accelerated motion: the speed increases uniformly from v_0 to v. The area under the line is the distance s

Speed under Uniform Acceleration

(2.3) MASTER

$$v = v_0 + a \cdot t$$

Under uniform acceleration, the speed increases by the same amount every second. In free fall, the acceleration is the gravitational acceleration g.

Valid: acceleration constant in magnitude and direction, motion along a straight line. Free fall without air resistance: $a = g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

v	speed (at the end)	$\frac{\mathrm{m}}{\mathrm{s}}$
v_0	initial speed	$\frac{\mathrm{m}}{\mathrm{s}}$
a	acceleration (negative when braking)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
t	time (since the start of acceleration)	s

EXAMPLE A wristwatch slides off the workbench and falls for 0.45 s until it hits the floor. How fast is it then?

Given: $v_0 = 0\ \frac{\mathrm{m}}{\mathrm{s}}; \quad a = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}; \quad t = 0.45\,\mathrm{s}$

$$\begin{aligned} v &= v_0 + a \cdot t \\ &= 0\ \tfrac{\mathrm{m}}{\mathrm{s}} + 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 0.45\,\mathrm{s} \\ &\approx 4.4\ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

About $4.4\ \frac{\mathrm{m}}{\mathrm{s}}$, i.e. 16 km per hour. The example for formula (2.2) uses this speed.

Distance under Uniform Acceleration

(2.4) MASTER

$$s = v_0 \cdot t + \frac{a \cdot t^2}{2}$$

The distance has two parts: the distance at the initial speed and the increase due to the acceleration. In the diagram of speed against time these are a rectangle and a triangle.

Valid: as for formula (2.3).

s	distance (travelled in this time)	m
v_0	initial speed	$\frac{\mathrm{m}}{\mathrm{s}}$
t	time (since the start of acceleration)	s
a	acceleration (negative when braking)	$\frac{\mathrm{m}}{\mathrm{s}^2}$

EXAMPLE How far does the watch from the previous example fall in 0.45 s?

Given: $v_0 = 0\,\frac{\mathrm{m}}{\mathrm{s}}$; $t = 0.45\,\mathrm{s}$; $a = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} s &= v_0 \cdot t + \frac{a \cdot t^2}{2} \\ &= 0\,\tfrac{\mathrm{m}}{\mathrm{s}} \cdot 0.45\,\mathrm{s} + \frac{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot (0.45\,\mathrm{s})^2}{2} \\ &\approx 0.99\,\mathrm{m} \end{aligned}$$

About 1 m. The shock test for shock-resistant watches is modelled on a fall from this height onto a wooden floor.

Speed and Distance without Time

(2.5) MASTER

$$v^2 = v_0^2 + 2 \cdot a \cdot s$$

REARRANGED $v = \sqrt{v_0^2 + 2 \cdot a \cdot s}$ $s = \frac{v^2 - v_0^2}{2 \cdot a}$

The formula links speed and distance without the time having to be known. Starting from rest it becomes $v = \sqrt{2 \cdot a \cdot s}$, so for free fall from the height h it is $v = \sqrt{2 \cdot g \cdot h}$.

Valid: as for formula (2.3); when braking, a is negative.

v	speed (at the end)	$\frac{\mathrm{m}}{\mathrm{s}}$
v_0	initial speed	$\frac{\mathrm{m}}{\mathrm{s}}$
a	acceleration (negative when braking)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
s	distance (over which acceleration or braking takes place)	m

Good to know: Braking distance to standstill: with $v = 0$, $s = \frac{v_0^2}{2 \cdot \lvert a \rvert}$. Double the speed means four times the braking distance. The stopping distance is the braking distance plus the distance travelled unbraked during the reaction time.

EXAMPLE Shortly after winding, the line of a longcase clock breaks. The weight falls 1.3 m. How fast does it hit the bottom of the case?

Given: $v_0 = 0\,\frac{\mathrm{m}}{\mathrm{s}}$; $a = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$; $s = 1.3\,\mathrm{m}$

$$\begin{aligned} v &= \sqrt{v_0^2 + 2 \cdot a \cdot s} \\ &= \sqrt{\left(0\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)^2 + 2 \cdot 9.81\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 1.3\,\mathrm{m}} \\ &\approx 5.05\,\tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

About $5\,\frac{\mathrm{m}}{\mathrm{s}}$, i.e. 18 km per hour. A weight of 4 kg strikes with 51 J; this is why worn gut lines and cables should be replaced in good time.

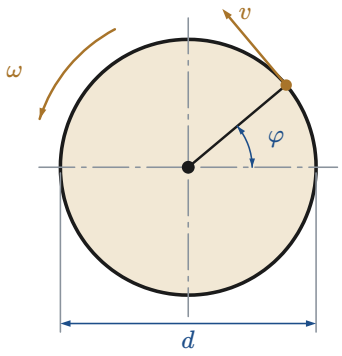


Fig. 2.2 Rotating wheel: angle of rotation φ, angular velocity ω and peripheral speed v

Peripheral Speed

(2.6) JOURNEYMAN

$$v = \pi \cdot d \cdot n$$

In each revolution a point on the circumference travels once around the circumference $\pi \cdot d$. The same formula is used in Chapter 9 to calculate the cutting speed.

Valid: d and n belong to the same part. For the result in metres per second, enter the diameter in metres and the rotational speed per second.

v	peripheral speed (speed of a point on the circumference)	$\frac{\mathrm{m}}{\mathrm{s}}$
d	diameter	mm
n	rotational speed (revolutions per unit time)	$\frac{1}{\mathrm{min}}$

EXAMPLE A polishing wheel of 100 mm diameter runs at 2800 revolutions per minute. How fast does its circumference move?

Given: $d = 100\,\mathrm{mm}; \quad n = 2800\ \frac{1}{\mathrm{min}}$

$$\begin{aligned} v &= \pi \cdot d \cdot n \\ &= \pi \cdot 100\,\mathrm{mm} \cdot 2800\ \tfrac{1}{\mathrm{min}} \\ &= \pi \cdot 0{,}1\,\mathrm{m} \cdot \frac{2800}{60\,\mathrm{s}} \\ &\approx 14.7\ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

The circumference moves at about $14.7\ \frac{\mathrm{m}}{\mathrm{s}}$, i.e. 53 km per hour.

Angular Velocity

(2.7) MASTER

$$\omega = \frac{\varphi}{t}$$

ω	angular velocity	$\frac{\text{rad}}{\text{s}}$
φ	angle of rotation (in radians, one revolution = 2π)	rad
t	time (required for it)	s

The angular velocity is the angle swept per unit time. All points of a wheel have the same angular velocity, but different peripheral speeds depending on their distance from the axis.

Valid: uniform rotation; angle in radians: degrees times $\frac{\pi}{180}$.

Good to know: Peripheral speed and angular velocity are related by the radius: $v = \omega \cdot r$.

EXAMPLE The fourth wheel makes one complete revolution (360°) in 60 s. What is its angular velocity?

Given: $\varphi = 360°; \quad t = 60\,\text{s}$

$$\begin{aligned}\omega &= \frac{\varphi}{t} \\ &= \frac{360°}{60\,\text{s}} \\ &= \frac{2\pi}{60\,\text{s}} \\ &\approx 0.105\ \frac{\text{rad}}{\text{s}}\end{aligned}$$

About $0.105\ \frac{\text{rad}}{\text{s}}$. The centre wheel is 60 times slower.

Angular Velocity from Rotational Speed

(2.8) MASTER

$$\omega = 2 \cdot \pi \cdot n$$

REARRANGED $\quad n = \frac{\omega}{2 \cdot \pi}$

ω	angular velocity	$\frac{\text{rad}}{\text{s}}$
n	rotational speed (revolutions per unit time)	$\frac{1}{\text{min}}$

Each revolution sweeps the angle 2π. The angular velocity is needed for the power and kinetic energy of rotating parts (Section 2.4).

Valid: enter the rotational speed per second if ω is to come out in $\frac{\text{rad}}{\text{s}}$.

EXAMPLE An arbor runs on the watchmaker's lathe at 3000 revolutions per minute, i.e. 50 revolutions per second.

Given: $n = 3000\ \frac{1}{\text{min}}$

$$\begin{aligned}\omega &= 2 \cdot \pi \cdot n \\ &= 2 \cdot \pi \cdot 3000\ \frac{1}{\text{min}} \\ &= 2 \cdot \pi \cdot 50\ \frac{1}{\text{s}} \\ &\approx 314\ \frac{\text{rad}}{\text{s}}\end{aligned}$$

The arbor rotates at $314\ \frac{\text{rad}}{\text{s}}$.

2.2 Force, Pressure and Inclined Plane

Forces have a magnitude and a direction. If a force acts obliquely on a surface, only part of it acts in the desired direction. Resolving and combining forces is the basis for escapement, thread and wedge.

Force and Acceleration

(2.9) JOURNEYMAN

$$F = m \cdot a$$

A force accelerates a mass. The unit newton follows from this formula: 1 N accelerates 1 kg at $1\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

F	force	N
m	mass	kg
a	acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$

EXAMPLE In the impact from the acceleration example, the balance (mass 60 mg) is also decelerated at $4400\ \frac{\mathrm{m}}{\mathrm{s}^2}$. What force acts on it?

Given: $m = 60\,\mathrm{mg}$; $a = 4400\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} F &= m \cdot a \\ &= 60\,\mathrm{mg} \cdot 4400\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \\ &\approx 0.26\,\mathrm{N} \end{aligned}$$

About 0.26 N, 450 times its weight. Without shock protection, two pivots of about 0.1 mm diameter would have to take this force.

Weight Force

(2.10) JOURNEYMAN

$$F_\mathrm{G} = m \cdot g$$

The weight force is the force with which the earth attracts a mass. It always acts vertically downwards.

F_G	weight force	N
m	mass	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$ in Central Europe)	$\frac{\mathrm{m}}{\mathrm{s}^2}$

EXAMPLE What force does a driving weight of 3.5 kg exert on the line?

Given: $m = 3.5\,\mathrm{kg}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} F_\mathrm{G} &= m \cdot g \\ &= 3.5\,\mathrm{kg} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \\ &\approx 34.3\,\mathrm{N} \end{aligned}$$

The weight pulls with 34.3 N. As a rule of thumb: 1 kg weighs about 10 N, 100 g weigh about 1 N.

Pressure

(2.11) JOURNEYMAN

$$p = \frac{F}{A}$$

REARRANGED $F = p \cdot A$

p	pressure	bar
F	force	N
A	area (on which the force acts perpendicularly)	$\mathrm{mm^2}$

Pressure is force per unit area. Conversion: $1\,\mathrm{bar} = 0.1\,\frac{\mathrm{N}}{\mathrm{mm^2}} = 100\,000\,\mathrm{Pa}$.

Valid: force evenly distributed over the area.

Good to know: 10 bar corresponds to the water pressure at a depth of about 100 m.

EXAMPLE A watch is tested at 10 bar overpressure. The crystal has a diameter of 30 mm, i.e. an area of 707 mm². With what force does the water press on the crystal?

Given: $p = 10\,\mathrm{bar}$; $A = 707\,\mathrm{mm^2}$

$$\begin{aligned} F &= p \cdot A \\ &= 10\,\mathrm{bar} \cdot 707\,\mathrm{mm^2} \\ &= 1\,\frac{\mathrm{N}}{\mathrm{mm^2}} \cdot 707\,\mathrm{mm^2} \\ &= 707\,\mathrm{N} \end{aligned}$$

707 N, as much as the weight of 72 kg. This is why pressure-resistant watches need thick crystals and stiff casebacks.

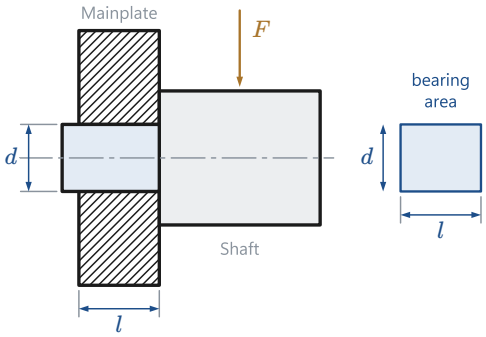


Fig. 2.3 Bearing pressure: the pivot bears on the projected area $d \cdot l$

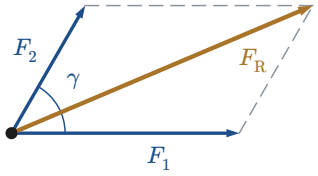


Fig. 2.4 Parallelogram of forces: F_R replaces F_1 and F_2

Bearing Pressure in a Pivot Bearing

(2.12) MASTER

$$p = \frac{F}{d \cdot l}$$

Bearing pressure is the pressure between two solid bodies. For a pivot, the projected area $d \cdot l$ is used, i.e. the rectangle that the pivot casts as a shadow in the bearing.

Valid: mean value. At the edges and with a tilted pivot the pressure is locally higher.

p	bearing pressure	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
F	force (bearing load transverse to the axis)	N
d	diameter (of the pivot)	mm
l	length (load-bearing length of the bearing, plate thickness)	mm

EXAMPLE The pivot of a weight barrel carries 20 N. It has a diameter of 4 mm and runs in a brass plate 3 mm thick.

Given: $F = 20\,\mathrm{N}$; $d = 4\,\mathrm{mm}$; $l = 3\,\mathrm{mm}$

$$\begin{aligned} p &= \frac{F}{d \cdot l} \\ &= \frac{20\,\mathrm{N}}{4\,\mathrm{mm} \cdot 3\,\mathrm{mm}} \\ &\approx 1.67\,\frac{\mathrm{N}}{\mathrm{mm}^2} \end{aligned}$$

About $1.7\,\frac{\mathrm{N}}{\mathrm{mm}^2}$. A thicker plate or a longer bush reduces the pressure, a thinner pivot increases it.

Combining Two Forces

(2.13) MASTER

$$F_\mathrm{R} = \sqrt{F_1^2 + F_2^2 + 2 \cdot F_1 \cdot F_2 \cdot \cos\gamma}$$

Two forces acting at the same point can be replaced by a single one: the diagonal of the parallelogram of forces. If the forces are perpendicular to each other, Pythagoras' theorem remains: $F_\mathrm{R} = \sqrt{F_1^2 + F_2^2}$.

Valid: both forces act at the same point, or their lines of action intersect.

F_R	resultant (equivalent force)	N
F_1	first force	N
F_2	second force	N
γ	angle (between the two directions of force)	$^\circ$

EXAMPLE The tooth forces from two meshes act on a wheel: 0.8 N and 0.5 N, their directions enclosing 60°. What is the force on the bearings?

Given: $F_1 = 0.8\,\mathrm{N}$; $F_2 = 0.5\,\mathrm{N}$; $\gamma = 60^\circ$

$$\begin{aligned} F_\mathrm{R} &= \sqrt{F_1^2 + F_2^2 + 2 \cdot F_1 \cdot F_2 \cdot \cos\gamma} \\ &= \sqrt{(0.8\,\mathrm{N})^2 + (0.5\,\mathrm{N})^2 + 2 \cdot 0.8\,\mathrm{N} \cdot 0.5\,\mathrm{N} \cdot \cos(60^\circ)} \\ &= \sqrt{1{,}29\,\mathrm{N}^2} \\ &\approx 1.14\,\mathrm{N} \end{aligned}$$

The bearings carry 1.14 N in total. If both forces acted in the same direction it would be 1.3 N, in opposite directions only 0.3 N.

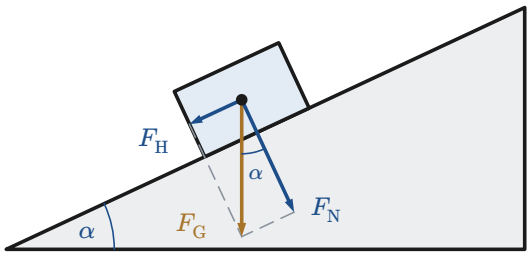


Fig. 2.5 Inclined plane: the weight force resolves into downhill force and normal force

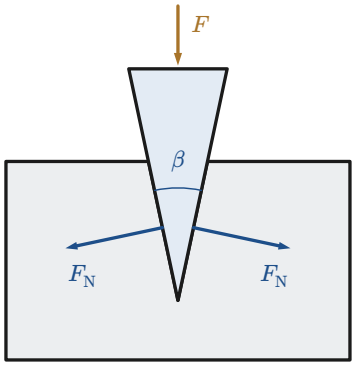


Fig. 2.6 Wedge with the wedge angle β

Inclined Plane: Downhill Force

(2.14) JOURNEYMAN

$$F_H = F_G \cdot \sin \alpha$$

On the inclined plane a force is resolved into two components: one parallel to the plane and one perpendicular to it. This is how calculations are made wherever a force acts obliquely on a surface: on impulse faces of the escapement, thread flanks, wedges and inclined faces of levers.

Valid: The resolution always applies. Whether the body actually slides is decided by friction (Section 2.5).

F_H	downhill force (parallel to the plane, downwards)	N
F_G	weight force	N
α	angle of inclination (of the plane to the horizontal)	°

EXAMPLE A part with a weight of 4.9 N (0.5 kg) lies on a plate inclined at 15°. With what force does it tend to slide off?

Given: $F_G = 4.9\,\mathrm{N}; \quad \alpha = 15°$

$$\begin{aligned} F_H &= F_G \cdot \sin \alpha \\ &= 4.9\,\mathrm{N} \cdot \sin(15°) \\ &\approx 1.27\,\mathrm{N} \end{aligned}$$

Only 1.27 N pulls the part down the slope, a little more than a quarter of its weight.

Inclined Plane: Normal Force

(2.15) JOURNEYMAN

$$F_N = F_G \cdot \cos \alpha$$

The normal force presses the body onto its support. The steeper the plane, the smaller it becomes.

F_N	normal force (perpendicular to the plane)	N
F_G	weight force	N
α	angle of inclination (of the plane to the horizontal)	°

EXAMPLE With what force does the part from the previous example press on the plate?

Given: $F_G = 4.9\,\mathrm{N}; \quad \alpha = 15°$

$$\begin{aligned} F_N &= F_G \cdot \cos \alpha \\ &= 4.9\,\mathrm{N} \cdot \cos(15°) \\ &\approx 4.73\,\mathrm{N} \end{aligned}$$

With 4.73 N. This normal force determines the friction (Section 2.5).

Wedge

(2.16) MASTER

$$F_{\mathrm{N}} = \frac{F}{2 \cdot \sin\left(\frac{\beta}{2}\right)}$$

F_{N}	normal force (perpendicular to each flank)	N
F	force (driving the wedge in)	N
β	wedge angle (between the two flanks)	$^\circ$

The wedge is a double inclined plane. It converts a small force over a long distance into two large forces over a short distance.

Valid: symmetrical wedge, without friction. Friction at the flanks reduces the flank forces considerably (Section 2.5).

EXAMPLE A case knife with a wedge angle of 20° is pressed with 30 N into the joint between caseback and case.

Given: $F = 30\,\mathrm{N}; \quad \beta = 20^\circ$

$$\begin{aligned} F_{\mathrm{N}} &= \frac{F}{2 \cdot \sin\left(\frac{\beta}{2}\right)} \\ &= \frac{30\,\mathrm{N}}{2 \cdot \sin\left(\frac{20^\circ}{2}\right)} \\ &\approx 86.4\,\mathrm{N} \end{aligned}$$

Each flank presses with about 86 N, almost three times the hand force. The more slender the wedge, the greater the force and the smaller the stroke.

Key point: The flatter the inclined plane and the more slender the wedge, the greater the force ratio and the longer the distance.

2.3 Lever and Torque

Almost every moving part of the watch is a lever: pallet fork, rockers, clicks, striking-work levers and every gear wheel. What matters is never the force alone, but force times lever arm.

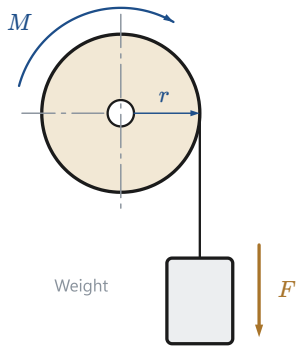


Fig. 2.7 Torque: force F on the lever arm r

Torque

(2.17) JOURNEYMAN

$$M = F \cdot r$$

Torque is the turning effect of a force: force times lever arm.

Valid: The lever arm is perpendicular to the direction of the force. If the force acts obliquely, only this perpendicular distance counts.

M	torque	$\mathrm{N{\cdot}mm}$
F	force	N
r	lever arm (perpendicular distance of the line of force from the axis of rotation)	mm

Good to know: Conversion: $1\,\mathrm{N} \cdot \mathrm{m} = 1000\,\mathrm{N} \cdot \mathrm{mm}$. Older documents give spring torques in pond times centimetre (often written "g·cm"): $1\,\mathrm{p} \cdot \mathrm{cm}$ is about $0.098\,\mathrm{N} \cdot \mathrm{mm}$.

EXAMPLE The driving weight pulls with 34.3 N on the line of a barrel with 20 mm radius. What torque drives the movement?

Given: $F = 34.3\,\mathrm{N}; \quad r = 20\,\mathrm{mm}$

$$\begin{aligned} M &= F \cdot r \\ &= 34.3\,\mathrm{N} \cdot 20\,\mathrm{mm} \\ &= 686\,\mathrm{N{\cdot}mm} \end{aligned}$$

686 N·mm, i.e. just under 0.7 N·m. A thicker barrel gives more torque but makes the weight fall faster.

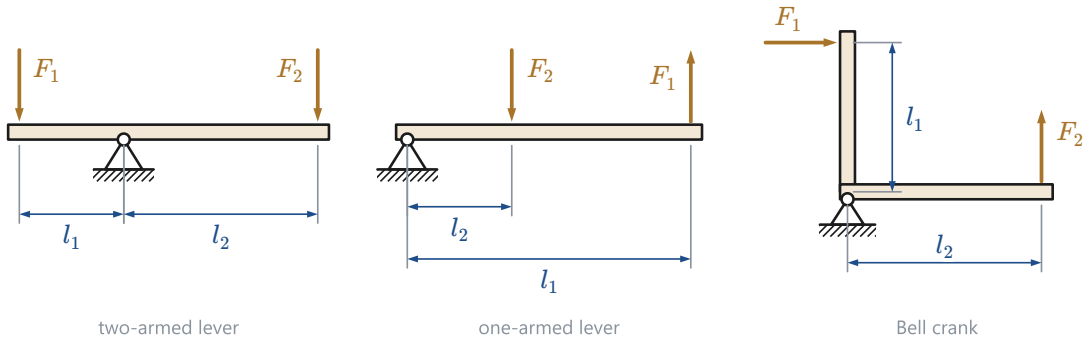


Fig. 2.8 Types of lever. The lever arms are always measured from the fulcrum and perpendicular to the force

Law of the Lever

(2.18) JOURNEYMAN

$$F_1 \cdot l_1 = F_2 \cdot l_2$$

REARRANGED $F_2 = \dfrac{F_1 \cdot l_1}{l_2}$

A lever is in equilibrium when both torques are equal and turn in opposite directions. This applies to every type of lever. If several forces act, the sum of the anticlockwise moments is compared with the sum of the clockwise moments.

Valid: lever arms measured perpendicular to the respective direction of force; without friction at the fulcrum.

F_1	first force (on the first lever arm)	N
l_1	first lever arm	mm
F_2	second force (on the second lever arm)	N
l_2	second lever arm	mm

EXAMPLE A pusher acts with 2 N on a bell-crank lever at a distance of 12 mm from the fulcrum. The short arm is 3 mm long. With what force does it press on the next part?

Given: $F_1 = 2\,\mathrm{N}$; $l_1 = 12\,\mathrm{mm}$; $l_2 = 3\,\mathrm{mm}$

$$\begin{aligned} F_2 &= \frac{F_1 \cdot l_1}{l_2} \\ &= \frac{2\,\mathrm{N} \cdot 12\,\mathrm{mm}}{3\,\mathrm{mm}} \\ &= 8\,\mathrm{N} \end{aligned}$$

With 8 N. What the lever gains in force it loses in distance: the short arm moves only a quarter as far.

Key point: Force times force arm equals load times load arm.

The same applies in the gear train: each wheel is a rotating lever. How the torque changes from wheel to pinion is given in Chapter 4.

2.4 Work, Energy, Power and Efficiency

When winding, work is put into the watch and stored as energy, in the tensioned spring or in the raised weight. The movement releases it over days as a tiny power. Energy is never lost in the process, but friction turns it into heat, and then it is no longer available to the oscillator.

Work

(2.19) JOURNEYMAN

$$W = F \cdot s$$

Work is force times distance. Unit: $1\,\mathrm{J} = 1\,\mathrm{N} \cdot \mathrm{m} = 1\,\mathrm{W} \cdot \mathrm{s}$.

Valid: constant force in the direction of the distance.

W	work	J
F	force	N
s	distance (in the direction of the force)	m

EXAMPLE When winding a chain-driven clock, the chain is pulled 0.9 m with 25 N.

Given: $F = 25\,\mathrm{N}$; $s = 0.9\,\mathrm{m}$

$$\begin{aligned} W &= F \cdot s \\ &= 25\,\mathrm{N} \cdot 0.9\,\mathrm{m} \\ &= 22.5\,\mathrm{J} \end{aligned}$$

The hand does 22.5 J of work. Most of it is then stored as potential energy in the raised weight.

Work in Rotation

(2.20) MASTER

$$W = M \cdot \varphi$$

For rotating parts, torque takes the place of force and the angle of rotation takes the place of distance.

Valid: constant torque; otherwise calculate with the mean torque.

W	work	J
M	torque	$\mathrm{N{\cdot}m}$
φ	angle of rotation (in radians, one revolution = 2π)	rad

EXAMPLE The mainspring of a wristwatch delivers 8 N·mm on average while the barrel turns 7 times. How much work does it deliver?

Given: $M = 8\,\mathrm{N{\cdot}mm}; \quad \varphi = 7\,\mathrm{U}$

$$\begin{aligned} W &= M \cdot \varphi \\ &= 8\,\mathrm{N{\cdot}mm} \cdot 7\,\mathrm{U} \\ &= 8\,\mathrm{N{\cdot}mm} \cdot 7 \cdot 2\pi \\ &\approx 352\,\mathrm{mJ} \end{aligned}$$

About 0.35 J. This is the watch's entire energy reserve for almost two days.

Potential Energy

(2.21) JOURNEYMAN

$$E_{\mathrm{pot}} = m \cdot g \cdot h$$

A raised weight stores the lifting work. As it falls, it delivers this work to the movement.

E_{pot}	potential energy	J
m	mass	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$ in Central Europe)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	height (above the lowest position)	m

EXAMPLE The weight of a longcase clock (3.5 kg) is raised by 1.2 m.

Given: $m = 3.5\,\mathrm{kg}; \quad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}; \quad h = 1.2\,\mathrm{m}$

$$\begin{aligned} E_{\mathrm{pot}} &= m \cdot g \cdot h \\ &= 3.5\,\mathrm{kg} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 1.2\,\mathrm{m} \\ &\approx 41.2\,\mathrm{J} \end{aligned}$$

The wound-up weight stores 41.2 J, a good hundred times as much as the mainspring of a wristwatch.

Kinetic Energy

(2.22) MASTER

$$E_{\text{kin}} = \frac{m \cdot v^2}{2}$$

Speed counts squared: twice as fast means four times the energy.

E_{kin}	kinetic energy	J
m	mass	kg
v	speed	$\frac{\text{m}}{\text{s}}$

Good to know: When a body falls from the height h, potential energy becomes kinetic energy. This gives the impact speed $v = \sqrt{2 \cdot g \cdot h}$. From a height of 1 m this is $4.43\ \frac{\text{m}}{\text{s}}$; the hammer of the standardized shock test also strikes the watch at this speed.

EXAMPLE A wristwatch of 80 g strikes at $4.43\ \frac{\text{m}}{\text{s}}$ after a free fall of 1 m.

Given: $m = 80\,\text{g}$; $v = 4.43\ \frac{\text{m}}{\text{s}}$

$$\begin{aligned} E_{\text{kin}} &= \frac{m \cdot v^2}{2} \\ &= \frac{80\,\text{g} \cdot \left(4.43\ \frac{\text{m}}{\text{s}}\right)^2}{2} \\ &\approx 0.785\,\text{J} \end{aligned}$$

On impact, 0.785 J must be absorbed. This is exactly the potential energy of the watch at a height of 1 m.

Rotational Energy

(2.23) MASTER

$$E_{\text{rot}} = \frac{J \cdot \omega^2}{2}$$

For rotating parts, the moment of inertia takes the place of mass and the angular velocity takes the place of speed.

Valid: J in $\text{kg} \cdot \text{m}^2$ and ω in $\frac{\text{rad}}{\text{s}}$ give joules.

E_{rot}	rotational energy	J
J	moment of inertia (about the axis of rotation, Section 2.6)	$\text{kg}\cdot\text{m}^2$
ω	angular velocity	$\frac{\text{rad}}{\text{s}}$

EXAMPLE A balance with $10\,\text{mg} \cdot \text{cm}^2$ oscillates at 4 Hz with 270° amplitude. At the zero crossing it rotates at $118.4\ \frac{\text{rad}}{\text{s}}$.

Given: $J = 10\,\text{mg}\cdot\text{cm}^2$; $\omega = 118.4\ \frac{\text{rad}}{\text{s}}$

$$\begin{aligned} E_{\text{rot}} &= \frac{J \cdot \omega^2}{2} \\ &= \frac{10\,\text{mg}\cdot\text{cm}^2 \cdot \left(118.4\ \frac{\text{rad}}{\text{s}}\right)^2}{2} \\ &= \frac{10^{-9}\,\text{kg}\cdot\text{m}^2 \cdot \left(118{,}4\ \frac{1}{\text{s}}\right)^2}{2} \\ &\approx 7.01\,\mu\text{J} \end{aligned}$$

The oscillating balance contains about 7 µJ. In each oscillation a small part of this is lost through friction and replaced by the escapement.

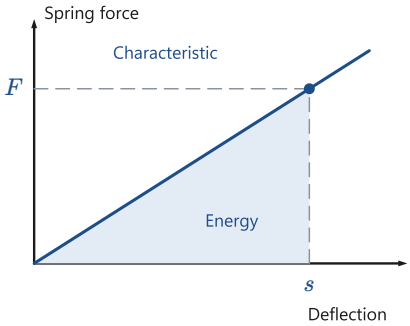


Fig. 2.9 Characteristic of a spring: the area under the characteristic is the stored energy

Spring Force

(2.24) JOURNEYMAN

$$F = c \cdot s$$

The force of a spring increases in proportion to its deflection. The spring rate states how much force is needed per millimetre of deflection.

Valid: elastic range and linear characteristic, for example coil springs and slightly bent leaf springs.

F	force	N
c	spring rate (spring constant; also R in standards)	$\frac{\mathrm{N}}{\mathrm{mm}}$
s	spring deflection (from the relaxed position)	mm

EXAMPLE A pusher spring with a spring rate of 0.8 N per millimetre is compressed by 2 mm.

Given: $c = 0.8\ \frac{\mathrm{N}}{\mathrm{mm}}$; $s = 2\,\mathrm{mm}$

$$\begin{aligned} F &= c \cdot s \\ &= 0.8\ \tfrac{\mathrm{N}}{\mathrm{mm}} \cdot 2\,\mathrm{mm} \\ &= 1.6\,\mathrm{N} \end{aligned}$$

The spring pushes back with 1.6 N.

Energy of a Tensioned Spring

(2.25) MASTER

$$E_{\mathrm{F}} = \frac{c \cdot s^2}{2}$$

During tensioning the force increases from zero to the final value; on average, half the final force acts. The energy is the triangular area under the characteristic.

Valid: as for the spring force; $1\,\mathrm{N} \cdot \mathrm{mm} = 1\,\mathrm{mJ}$.

E_{F}	energy of the tensioned spring	mJ
c	spring rate	$\frac{\mathrm{N}}{\mathrm{mm}}$
s	spring deflection (from the relaxed position)	mm

EXAMPLE How much energy does the pusher spring from the previous example store?

Given: $c = 0.8\ \frac{\mathrm{N}}{\mathrm{mm}}$; $s = 2\,\mathrm{mm}$

$$\begin{aligned} E_{\mathrm{F}} &= \frac{c \cdot s^2}{2} \\ &= \frac{0.8\ \frac{\mathrm{N}}{\mathrm{mm}} \cdot (2\,\mathrm{mm})^2}{2} \\ &= 1.6\,\mathrm{mJ} \end{aligned}$$

1.6 mJ. With twice the deflection it would be four times as much.

Energy of a Torsion Spring

(2.26) MASTER

$$E_{\mathrm{F}} = \frac{D \cdot \varphi^2}{2}$$

For balance springs and other torsion springs with a linear characteristic $M = D \cdot \varphi$ (Chapter 5).

Valid: elastic range; angle in radians.

E_{F}	energy of the tensioned spring	J
D	restoring torque coefficient (torque per angle)	$\frac{\mathrm{N \cdot m}}{\mathrm{rad}}$
φ	angle of twist (from the rest position, in radians)	rad

EXAMPLE The balance spring of the balance from the rotational-energy example has a restoring torque coefficient of 0.632 µN·m per radian. How much energy does it store at the turning point at 270° amplitude, i.e. 4.71 rad?

Given: $D = 0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$; $\varphi = 4.71\,\mathrm{rad}$

$$\begin{aligned} E_{\mathrm{F}} &= \frac{D \cdot \varphi^2}{2} \\ &= \frac{0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}} \cdot (4.71\,\mathrm{rad})^2}{2} \\ &= \frac{0.632\,\mu\mathrm{N \cdot m} \cdot 4.71^2}{2} \\ &\approx 7.01\,\mu\mathrm{J} \end{aligned}$$

About 7 µJ, as much as the kinetic energy of the balance at the zero crossing. With every vibration the energy swings back and forth between balance spring and balance.

Power

(2.27) JOURNEYMAN

$$P = \frac{W}{t}$$

Power is work per unit time. Unit: 1 W = 1 J per second.

P	power	W
W	work	J
t	time (in which the work is done)	s

EXAMPLE The weight of the longcase clock delivers its 41.2 J evenly over 8 days. What power drives the clock?

Given: $W = 41.2\,\mathrm{J}$; $t = 8\,\mathrm{d}$

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{41.2\,\mathrm{J}}{8\,\mathrm{d}} \\ &= \frac{41{,}2\,\mathrm{J}}{691\,200\,\mathrm{s}} \\ &\approx 59.6\,\mu\mathrm{W} \end{aligned}$$

Just under 60 µW, i.e. 60 millionths of a watt. A wristwatch manages with a few microwatts.

Power in Rotation

(2.28) MASTER

$$P = M \cdot \omega$$

Power is torque times angular velocity. Written with the rotational speed, $P = 2\pi \cdot M \cdot n$, with the rotational speed in revolutions per second.

P	power	W
M	torque	$\mathrm{N{\cdot}m}$
ω	angular velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$

EXAMPLE A barrel delivers 8 N·mm and turns once in 6 hours, i.e. $0.000\,291\ \frac{\mathrm{rad}}{\mathrm{s}}$.

Given: $M = 8\,\mathrm{N{\cdot}mm}; \quad \omega = 0.000\,291\ \frac{\mathrm{rad}}{\mathrm{s}}$

$$\begin{aligned} P &= M \cdot \omega \\ &= 8\,\mathrm{N{\cdot}mm} \cdot 0.000\,291\ \tfrac{\mathrm{rad}}{\mathrm{s}} \\ &\approx 2.33\,\mu\mathrm{W} \end{aligned}$$

The barrel supplies 2.33 µW. A fast wheel at the end of the gear train transmits the same power, apart from losses, with small torque and high rotational speed.

Efficiency

(2.29) JOURNEYMAN

$$\eta = \frac{P_{\mathrm{ab}}}{P_{\mathrm{zu}}}$$

Efficiency states what share of the input arrives as useful output. Instead of powers, work or energies can also be compared.

η	efficiency (always less than 1)	1
P_{ab}	output (useful) power	W
P_{zu}	input power	W

EXAMPLE The barrel delivers 2.33 µW into the gear train; 1.63 µW arrive at the escape wheel.

Given: $P_{\mathrm{ab}} = 1.63\,\mu\mathrm{W}; \quad P_{\mathrm{zu}} = 2.33\,\mu\mathrm{W}$

$$\begin{aligned} \eta &= \frac{P_{\mathrm{ab}}}{P_{\mathrm{zu}}} \\ &= \frac{1.63\,\mu\mathrm{W}}{2.33\,\mu\mathrm{W}} \\ &\approx 0.70 \end{aligned}$$

The gear train has an efficiency of 0.70 or 70 %. The remaining 30 % turn into heat in the bearings and meshes.

Overall Efficiency

(2.30) MASTER

$$\eta_{\mathrm{ges}} = \eta_1 \cdot \eta_2 \cdot \eta_3$$

If several stages are connected in series, the individual efficiencies are multiplied. This applies to any number of stages.

Valid: stages connected in series, each receiving only what the previous one delivers.

η_{ges}	overall efficiency	1
η_1	efficiency of the first stage	1
η_2	efficiency of the second stage	1
η_3	efficiency of the third stage	1

EXAMPLE Assume that the barrel passes on 85 % of the spring energy, the gear train 70 % and the escapement 35 %. What share reaches the balance?

Given: $\eta_1 = 0.85; \quad \eta_2 = 0.70; \quad \eta_3 = 0.35$

$$\begin{aligned} \eta_{\mathrm{ges}} &= \eta_1 \cdot \eta_2 \cdot \eta_3 \\ &= 0.85 \cdot 0.70 \cdot 0.35 \\ &\approx 0.21 \end{aligned}$$

Only about one fifth of the spring energy arrives at the balance. The figures are orders of magnitude; they show why clean bearings and correct depthings yield so much amplitude.

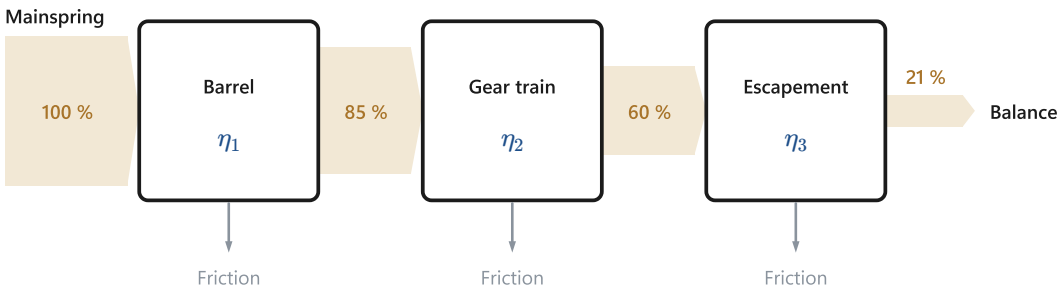


Fig. 2.10 Energy flow from the mainspring to the balance with the efficiencies from the example for formula (2.30)

2.5 Friction

Friction brakes every bearing and every mesh, but it also holds screws, hands and friction settings in place. Calculations use the coefficient of friction of the two materials sliding on each other.

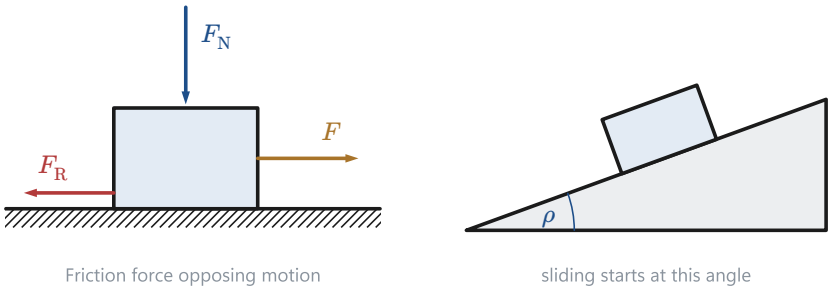


Fig. 2.11 Sliding friction (left) and angle of friction ρ (right)

Sliding Friction

(2.31) JOURNEYMAN

$$F_R = \mu \cdot F_N$$

The friction force depends on the normal force and on the material pairing, not on the size of the contact area.

Valid: dry and thinly oiled surfaces. If a continuous oil film carries the load, friction also depends on speed and viscosity of the oil.

F_R	friction force (acts against the motion)	N
μ	coefficient of friction (sliding friction, depends on the material pairing)	1
F_N	normal force (perpendicular to the contact surface)	N

EXAMPLE A steel spring presses with 2 N on a sliding steel lever. The point is oiled; the calculation uses a coefficient of friction of 0.15.

Given: $\mu = 0.15$; $F_N = 2\,\mathrm{N}$

$$\begin{aligned} F_R &= \mu \cdot F_N \\ &= 0.15 \cdot 2\,\mathrm{N} \\ &= 0.3\,\mathrm{N} \end{aligned}$$

Friction brakes with 0.3 N.

Static Friction

(2.32) JOURNEYMAN

$$F_{R0} = \mu_0 \cdot F_N$$

F_{R0}	maximum static friction force (breakaway force)	N
μ_0	coefficient of static friction	1
F_N	normal force	N

As long as a body is at rest, static friction holds it in place. It is as large as the applied force, but at most $\mu_0 \cdot F_N$. Sticking is somewhat stronger than sliding: a part needs more force to break away than to keep sliding.

EXAMPLE A screw presses a bridge onto the mainplate with 150 N. The coefficient of static friction for brass on brass is cautiously taken as 0.2.

Given: $\mu_0 = 0.2$; $F_N = 150\,\mathrm{N}$

$$\begin{aligned} F_{R0} &= \mu_0 \cdot F_N \\ &= 0.2 \cdot 150\,\mathrm{N} \\ &= 30\,\mathrm{N} \end{aligned}$$

Up to a transverse force of 30 N, friction alone holds the bridge in place. The steady pins additionally secure its exact position.

Coefficients of Sliding Friction μ (Guide Values)

Pairing	dry	oiled
hardened steel on ruby or sapphire	0.12 … 0.25	0.05 … 0.15
hardened steel on brass	0.18 … 0.30	0.08 … 0.20
hardened steel on bronze	0.15 … 0.30	0.07 … 0.12
hardened steel on hardened steel	0.15 … 0.35	0.05 … 0.12
steel on plastic (POM)	0.20 … 0.35	0.07 … 0.12
steel on PTFE	0.05 … 0.15	–

The values vary widely with surface finish, oil, load and speed. The coefficient of static friction is somewhat higher. Usual design values for oiled pivot bearings: 0.15 for steel in ruby, 0.2 for steel in brass.

Angle of Friction

(2.33) MASTER

$$\rho = \arctan \mu$$

REARRANGED $\mu = \tan \rho$

ρ	angle of friction	°
μ	coefficient of friction	1

The angle of friction is the angle of inclination of an inclined plane at which a body just begins to slide. There the downhill force equals the friction force. If the plane is flatter, the body stays where it is: self-locking.

Valid: for the start of sliding use the coefficient of static friction μ_0, for continued sliding use μ.

EXAMPLE What is the angle of friction for a coefficient of friction of 0.15?

Given: $\mu = 0.15$

$$\begin{aligned} \rho &= \arctan \mu \\ &= \arctan 0.15 \\ &\approx 8.53^\circ \end{aligned}$$

8.53°. With this coefficient of friction, a surface inclined at less than about 8.5° holds a part in place by itself.

Inclined Plane, Wedge and Thread with Friction

(2.34) ADVANCED

$$F = F_{\mathrm{Q}} \cdot \tan(\alpha + \rho)$$

If a force pushes a body up an inclined plane, friction acts like a slope steeper by ρ. A thread is an inclined plane wrapped around a bolt, a wedge is a movable one.

Valid: motion against the load (tightening, driving in). For V-threads the effective friction is somewhat greater because of the inclined flanks. If a wedge also slides on a second surface, its friction must be added.

F	force (pushing force parallel to the base, for a thread: tangential force)	N
F_{Q}	load perpendicular to the base (for a thread: axial force of the screw)	N
α	angle of inclination (for a thread: helix angle)	°
ρ	angle of friction	°

Good to know: In the direction of the load (loosening), $F = F_{\mathrm{Q}} \cdot \tan(\alpha - \rho)$. If α is smaller than ρ, this force becomes negative: the joint holds by itself and must be actively released. Fastening screws and wedges are therefore self-locking.

EXAMPLE A thread has a helix angle of 3°, the angle of friction is 8.53°. The screw is to clamp with 100 N. What tangential force is required in the thread when tightening?

Given: $F_{\mathrm{Q}} = 100\,\mathrm{N}$; $\alpha = 3°$; $\rho = 8.53°$

$$\begin{aligned} F &= F_{\mathrm{Q}} \cdot \tan(\alpha + \rho) \\ &= 100\,\mathrm{N} \cdot \tan(3° + 8.53°) \\ &= 100\,\mathrm{N} \cdot \tan 11{,}53° \\ &\approx 20.4\,\mathrm{N} \end{aligned}$$

20.4 N. Without friction it would be only 5.2 N: most of the tightening force goes into friction, and the friction under the screw head comes on top.

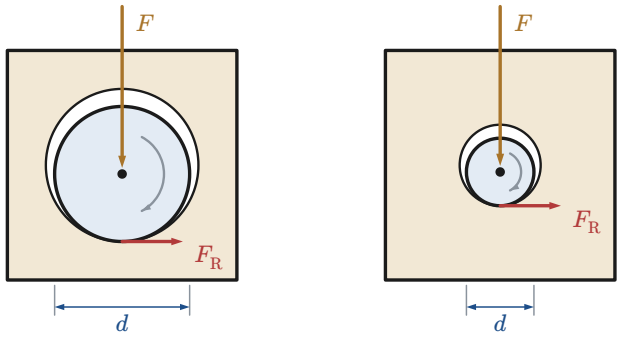


Fig. 2.12 Pivot friction: under the same load, the friction force on the thin pivot has the smaller lever arm

Friction Torque in a Pivot Bearing

(2.35) MASTER

$$M_{\mathrm{R}} = \frac{\mu \cdot F \cdot d}{2}$$

The friction force acts at the circumference of the pivot; its lever arm is the pivot radius. A thin pivot therefore brakes less than a thick one.

Valid: pivot with bearing clearance, in line contact; coefficient of friction as a design value.

M_{R}	friction torque (brakes the rotation)	$\mathrm{N{\cdot}mm}$
μ	coefficient of friction (pivot in bearing)	1
F	force (bearing load transverse to the axis)	N
d	diameter (of the pivot)	mm

Good to know: This is why the pivots become ever thinner towards the end of the gear train: there the torque is small and every friction torque counts. When the watch lies flat, the balance runs on the end face of a pivot; the lever arm of friction is then even smaller and the amplitude larger than in the vertical positions.

EXAMPLE A pivot of 1.2 mm diameter carries 3 N and runs oiled in brass (design value of the coefficient of friction 0.2).

Given: $\mu = 0.2$; $F = 3\,\mathrm{N}$; $d = 1.2\,\mathrm{mm}$

$$\begin{aligned} M_{\mathrm{R}} &= \frac{\mu \cdot F \cdot d}{2} \\ &= \frac{0.2 \cdot 3\,\mathrm{N} \cdot 1.2\,\mathrm{mm}}{2} \\ &= 0.36\,\mathrm{N{\cdot}mm} \end{aligned}$$

Friction brakes with 0.36 N·mm. With a pivot diameter of 0.6 mm it would be only half as much under the same load.

Key point: Low friction requires hard, smooth surfaces, little normal force and a small lever arm.

2.6 Moment of Inertia

What mass is for linear motion, the moment of inertia is for rotation: the resistance to acceleration and braking. It depends on how far the mass is from the axis of rotation. In watches it is stated in $\mathrm{mg \cdot cm^2}$. How the moment of inertia of the balance determines the period is given in Chapter 5.

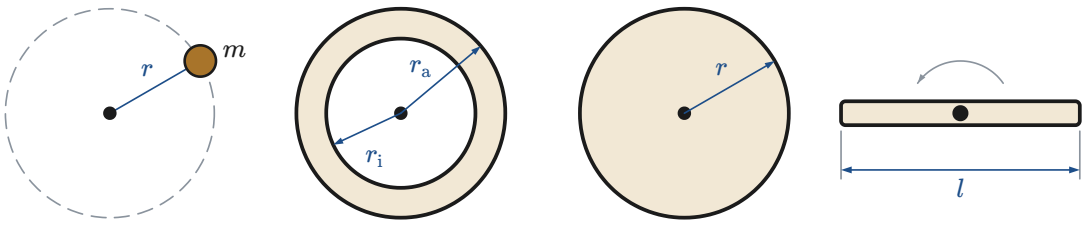


Fig. 2.13 Bodies whose moment of inertia is easy to calculate; the axis of rotation is perpendicular to the drawing plane

Point Mass and Thin Ring

(2.36) MASTER

$$J = m \cdot r^2$$

J	moment of inertia (about the axis of rotation)	$\mathrm{mg \cdot cm^2}$
m	mass	mg
r	distance (of the mass from the axis of rotation)	mm

Every particle of mass counts with the square of its distance from the axis. The formula also applies to a thin ring, because there all the mass is at the same distance.

Valid: dimensions of the mass small compared with its distance from the axis; for the ring, wall thickness small compared with the radius.

Good to know: Conversion: $1\,\mathrm{mg \cdot cm^2} = 10^{-10}\,\mathrm{kg \cdot m^2}$ and $1\,\mathrm{g \cdot cm^2} = 10^{-7}\,\mathrm{kg \cdot m^2}$. Wristwatch balances lie at about 2 to $25\,\mathrm{mg \cdot cm^2}$.

EXAMPLE A timing screw of 5 mg sits 4.5 mm from the balance axis.

Given: $m = 5\,\mathrm{mg}$; $r = 4.5\,\mathrm{mm}$

$$\begin{aligned} J &= m \cdot r^2 \\ &= 5\,\mathrm{mg} \cdot (4.5\,\mathrm{mm})^2 \\ &= 5\,\mathrm{mg} \cdot (0{,}45\,\mathrm{cm})^2 \\ &\approx 1.01\,\mathrm{mg \cdot cm^2} \end{aligned}$$

The screw contributes about $1\,\mathrm{mg \cdot cm^2}$. For a balance of $10\,\mathrm{mg \cdot cm^2}$ this is 10 %; this is why even small changes to the screws have a marked effect on the rate.

Ring and Hollow Cylinder

(2.37) MASTER

$$J = \frac{m \cdot \left(r_\mathrm{a}^2 + r_\mathrm{i}^2\right)}{2}$$

J	moment of inertia (about the central axis)	$\mathrm{mg \cdot cm^2}$
m	mass (of the ring)	mg
r_a	outer radius	mm
r_i	inner radius	mm

The rim of the balance is a hollow cylinder. Because its entire mass lies far out, it provides almost the whole moment of inertia of the balance.

Valid: material of uniform density, axis of rotation is the central axis.

EXAMPLE A balance rim has an outside diameter of 9 mm, an inside diameter of 8 mm and weighs 55 mg.

Given: $m = 55\,\mathrm{mg}$; $r_\mathrm{a} = 4.5\,\mathrm{mm}$; $r_\mathrm{i} = 4\,\mathrm{mm}$

$$\begin{aligned} J &= \frac{m \cdot \left(r_\mathrm{a}^2 + r_\mathrm{i}^2\right)}{2} \\ &= \frac{55\,\mathrm{mg} \cdot \left((4.5\,\mathrm{mm})^2 + (4\,\mathrm{mm})^2\right)}{2} \\ &= \frac{55\,\mathrm{mg} \cdot (0{,}2025 + 0{,}16)\ \mathrm{cm^2}}{2} \\ &\approx 9.97\,\mathrm{mg \cdot cm^2} \end{aligned}$$

The rim alone has $9.97\,\mathrm{mg \cdot cm^2}$. The approximation as a thin ring with the mean radius of 4.25 mm gives $9.93\,\mathrm{mg \cdot cm^2}$, i.e. almost the same.

Solid Disc and Cylinder

(2.38) MASTER

$$J = \frac{m \cdot r^2}{2}$$

J	moment of inertia (about the central axis)	$\mathrm{kg \cdot m^2}$
m	mass	kg
r	radius	m

A disc has only half the moment of inertia of a thin ring of the same mass and diameter, because a large part of its mass lies close to the axis.

Valid: material of uniform density, axis of rotation is the central axis; the thickness plays no part.

EXAMPLE A disc-shaped pendulum bob has a mass of 1.2 kg and a radius of 60 mm. What is its moment of inertia about its own centre?

Given: $m = 1.2\,\mathrm{kg}$; $r = 60\,\mathrm{mm}$

$$\begin{aligned} J &= \frac{m \cdot r^2}{2} \\ &= \frac{1.2\,\mathrm{kg} \cdot (60\,\mathrm{mm})^2}{2} \\ &= 0.002\,16\,\mathrm{kg \cdot m^2} \end{aligned}$$

The pendulum is calculated with this value in Section 6.3.

Thin Rod about Its Centre

(2.39) MASTER

$$J = \frac{m \cdot l^2}{12}$$

J	moment of inertia (about the transverse axis through the centre of the rod)	$\mathrm{mg \cdot cm^2}$
m	mass (of the rod)	mg
l	length (total length of the rod)	mm

For arms, bars and hands rotating transversely to their length.

Valid: rod thin compared with its length, uniform cross-section.

Good to know: If the rod rotates about its end, like a hand without counterweight, the moment of inertia is four times as large: $J = \frac{1}{3} \cdot m \cdot l^2$.

EXAMPLE The arm of a two-armed balance is 8 mm long and weighs 6 mg.

Given: $m = 6\,\mathrm{mg}$; $l = 8\,\mathrm{mm}$

$$\begin{aligned} J &= \frac{m \cdot l^2}{12} \\ &= \frac{6\,\mathrm{mg} \cdot (8\,\mathrm{mm})^2}{12} \\ &= 0.32\,\mathrm{mg \cdot cm^2} \end{aligned}$$

The arm contributes only $0.32\,\mathrm{mg \cdot cm^2}$, about 3 % of what the rim from the previous example provides.

Parallel Axis Theorem (Steiner)

(2.40) MASTER

$$J_A = J_S + m \cdot a^2$$

If the axis of rotation does not pass through the centre of gravity, the term mass times distance squared must be added. For composite parts, the contributions of all components are added. The application to the pendulum is given in Section 6.3.

Valid: both axes parallel; one of them passes through the centre of gravity.

J_A	moment of inertia about the axis of rotation	$\mathrm{mg \cdot cm^2}$
J_S	moment of inertia about the parallel axis through the centre of gravity	$\mathrm{mg \cdot cm^2}$
m	mass (of the component)	mg
a	distance (between the two axes)	mm

EXAMPLE A minute hand (20 mg, 12 mm long, uniform) rotates about its end. About its centre it has $J_S = \frac{1}{12} \cdot m \cdot l^2 = 2.4\,\mathrm{mg \cdot cm^2}$; the centre of gravity lies 6 mm from the axis of rotation.

Given: $J_S = 2.4\,\mathrm{mg{\cdot}cm^2}$; $m = 20\,\mathrm{mg}$; $a = 6\,\mathrm{mm}$

$$\begin{aligned} J_A &= J_S + m \cdot a^2 \\ &= 2.4\,\mathrm{mg{\cdot}cm^2} + 20\,\mathrm{mg} \cdot (6\,\mathrm{mm})^2 \\ &= 2{,}4\,\mathrm{mg{\cdot}cm^2} + 7{,}2\,\mathrm{mg{\cdot}cm^2} \\ &= 9.6\,\mathrm{mg{\cdot}cm^2} \end{aligned}$$

$9.6\,\mathrm{mg \cdot cm^2}$, four times the value about the centre. A minute hand can therefore be as inert as a whole balance.

Key point: Mass on the outside makes for inertia: the distance from the axis counts squared.

2.7 Strength and Elasticity

Every loaded part deforms. If the stress stays small enough, it springs back completely; all the springs of the watch depend on this. If the stress becomes too large, the part bends permanently or breaks.

Tensile and Compressive Stress

(2.41) JOURNEYMAN

$$\sigma = \frac{F}{A}$$

Stress is force per unit cross-sectional area. It states how heavily the material is loaded, independently of the size of the part. Conversion: $1\,\frac{\mathrm{N}}{\mathrm{mm^2}} = 1\,\mathrm{MPa}$.

Valid: force in the direction of the rod axis, evenly distributed over the cross-section.

σ	stress	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
F	force (tension or compression)	N
A	area (cross-section transverse to the force)	$\mathrm{mm^2}$

EXAMPLE The driving weight (34.3 N) hangs on a steel wire of 1 mm diameter, i.e. 0.785 mm² cross-section.

Given: $F = 34.3\,\mathrm{N}$; $A = 0.785\,\mathrm{mm^2}$

$$\begin{aligned} \sigma &= \frac{F}{A} \\ &= \frac{34.3\,\mathrm{N}}{0.785\,\mathrm{mm^2}} \\ &\approx 43.7\,\frac{\mathrm{N}}{\mathrm{mm^2}} \end{aligned}$$

The stress in the wire is $43.7\,\frac{\mathrm{N}}{\mathrm{mm^2}}$, far below the strength of steel.

Strain

(2.42) JOURNEYMAN

$$\varepsilon = \frac{\Delta l}{l_0}$$

Strain is a dimensionless ratio, often stated in percent.

ε	strain (change in length per length)	1
Δl	change in length (+ means: longer)	mm
l_0	original length (without load)	mm

EXAMPLE A wire 1200 mm long becomes 0.25 mm longer under load.

Given: $\Delta l = 0.25\,\mathrm{mm}$; $l_0 = 1200\,\mathrm{mm}$

$$\begin{aligned}\varepsilon &= \frac{\Delta l}{l_0}\\ &= \frac{0.25\,\mathrm{mm}}{1200\,\mathrm{mm}}\\ &\approx 0.0208\,\%\end{aligned}$$

The wire stretches by 0.0208 %, i.e. by about two ten-thousandths of its length.

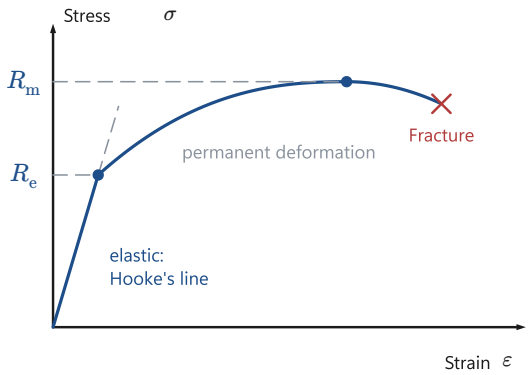


Fig. 2.14 Stress–strain diagram of a ductile metal (schematic): elastic up to R_e, maximum stress R_m

Hooke's Law

(2.43) JOURNEYMAN

$$\sigma = E \cdot \varepsilon$$

REARRANGED $\varepsilon = \dfrac{\sigma}{E}$

In the elastic range, stress increases in proportion to strain. The modulus of elasticity is the stiffness of the material: steel is twice as stiff as brass.

Valid: only up to the yield strength R_e. Beyond it the part deforms permanently. The tensile strength R_m is the highest stress the material can withstand.

σ	stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
E	modulus of elasticity (of the material)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
ε	strain	1

EXAMPLE How much does the steel wire from the stress example stretch ($\sigma = 43.7\,\frac{\mathrm{N}}{\mathrm{mm}^2}$, $E = 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$)?

Given: $\sigma = 43.7\,\frac{\mathrm{N}}{\mathrm{mm}^2}$; $E = 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$

$$\begin{aligned}\varepsilon &= \frac{\sigma}{E}\\ &= \frac{43.7\,\frac{\mathrm{N}}{\mathrm{mm}^2}}{210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}}\\ &\approx 0.0208\,\%\end{aligned}$$

By 0.0208 %. A wire 1.2 m long therefore becomes 0.25 mm longer and springs back completely after unloading.

Permissible Stress and Safety Factor

(2.44) MASTER

$$\sigma_{\text{zul}} = \frac{R_{\text{e}}}{\nu}$$

σ_{zul}	permissible stress	$\frac{\text{N}}{\text{mm}^2}$
R_{e}	yield strength (for hard materials the 0.2 % proof stress $R_{\text{p0.2}}$)	$\frac{\text{N}}{\text{mm}^2}$
ν	safety factor (usually 1.2 to 2)	1

A part must not deform permanently in service. The stress is therefore kept a safety margin below the yield strength. If the part must merely not break, the tensile strength is used accordingly.

Valid: static load. Under constantly alternating load the material fatigues and withstands considerably less.

EXAMPLE A spring steel has a proof stress of $1500\ \frac{\text{N}}{\text{mm}^2}$. The calculation uses a safety factor of 1.5.

Given: $R_{\text{e}} = 1500\ \frac{\text{N}}{\text{mm}^2}$; $\nu = 1.5$

$$\begin{aligned}\sigma_{\text{zul}} &= \frac{R_{\text{e}}}{\nu} \\ &= \frac{1500\ \frac{\text{N}}{\text{mm}^2}}{1.5} \\ &= 1000\ \frac{\text{N}}{\text{mm}^2}\end{aligned}$$

The spring may be loaded up to $1000\ \frac{\text{N}}{\text{mm}^2}$.

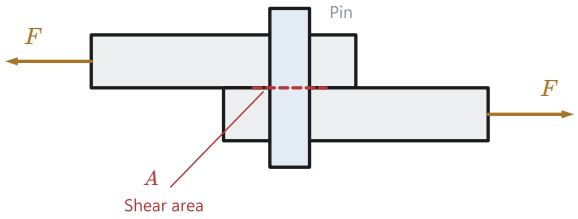


Fig. 2.15 Shearing of a pin in the joint plane

Shear

(2.45) MASTER

$$\tau = \frac{F}{A}$$

τ	shear stress	$\frac{\text{N}}{\text{mm}^2}$
F	force (transverse to the pin)	N
A	area (shear area: cross-section in the joint plane)	mm^2

With pins, rivets and screws under transverse force, the cross-section in the joint plane is sheared. The stress then lies in the area, not perpendicular to it.

Valid: one shear plane (single-shear joint). If the pin sits in a fork, two shear planes carry the load: take A twice.

Good to know: Guide value for steel: the shear strength is about 80 % of the tensile strength.

EXAMPLE A steel pin of 0.8 mm diameter (0.503 mm² cross-section) joins two parts and is loaded transversely with 60 N.

Given: $F = 60\,\text{N}$; $A = 0.503\,\text{mm}^2$

$$\begin{aligned}\tau &= \frac{F}{A} \\ &= \frac{60\,\text{N}}{0.503\,\text{mm}^2} \\ &\approx 119\ \frac{\text{N}}{\text{mm}^2}\end{aligned}$$

$119\ \frac{\text{N}}{\text{mm}^2}$. A soft steel pin withstands about three times this.

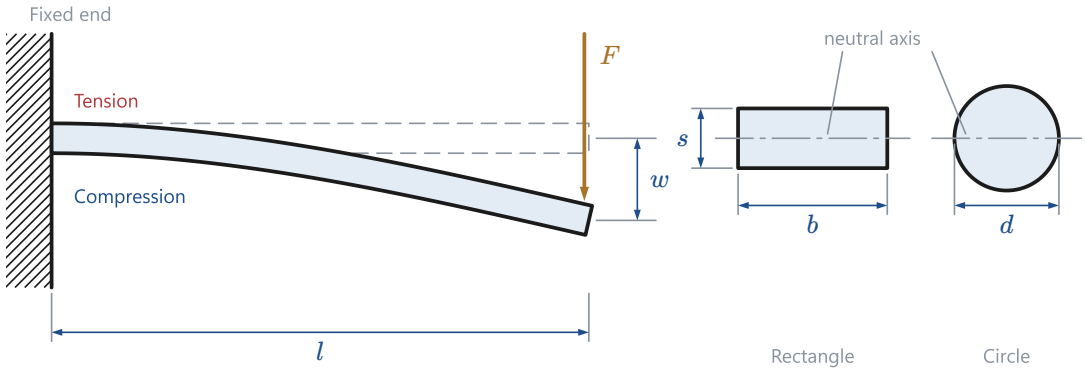


Fig. 2.16 Cantilever under the force F (left); cross-sections with neutral axis (right)

Bending Stress

(2.46) MASTER

$$\sigma_\mathrm{b} = \frac{M_\mathrm{b}}{W_\mathrm{b}}$$

In bending, one side is stretched and the other compressed; between them lies the stress-free neutral axis. The greatest stress is at the outer edge. For a cantilever with the force F at its end, the bending moment is greatest at the clamping point: $M_\mathrm{b} = F \cdot l$. That is where a spring bends or breaks first.

Valid: elastic range, straight bar, cross-section symmetrical about the neutral axis.

σ_b	maximum bending stress (in the outer fibre)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
M_b	bending moment (at the point considered)	$\mathrm{N{\cdot}mm}$
W_b	section modulus (for bending)	mm^3

EXAMPLE A leaf spring (1 mm high, 0.2 mm thick) is 10 mm long and is loaded at its end with 0.42 N. At the clamping point the bending moment is 0.42 N · 10 mm = 4.2 N·mm; the section modulus is 0.00667 mm³.

Given: $M_\mathrm{b} = 4.2\,\mathrm{N{\cdot}mm}$; $\quad W_\mathrm{b} = 0.006\,67\,\mathrm{mm}^3$

$$\begin{aligned}\sigma_\mathrm{b} &= \frac{M_\mathrm{b}}{W_\mathrm{b}} \\ &= \frac{4.2\,\mathrm{N{\cdot}mm}}{0.006\,67\,\mathrm{mm}^3} \\ &\approx 630\ \tfrac{\mathrm{N}}{\mathrm{mm}^2}\end{aligned}$$

$630\ \frac{\mathrm{N}}{\mathrm{mm}^2}$. Hardened spring steel can take this; soft brass would long since have bent permanently.

Section Modulus: Rectangle

(2.47) MASTER

$$W_\mathrm{b} = \frac{b \cdot s^2}{6}$$

The section modulus states how much bending moment a cross-section carries at a given stress.

Valid: bending about the flat side, as with every leaf spring, mainspring and balance spring.

W_b	section modulus (for bending)	mm^3
b	blade height (width of the rectangle transverse to the direction of bending)	mm
s	blade thickness (thickness in the direction of bending)	mm

EXAMPLE Leaf spring with a height of 1 mm and a thickness of 0.2 mm.

Given: $b = 1\,\mathrm{mm}; \quad s = 0.2\,\mathrm{mm}$

$$\begin{aligned} W_\mathrm{b} &= \frac{b \cdot s^2}{6} \\ &= \frac{1\,\mathrm{mm} \cdot (0.2\,\mathrm{mm})^2}{6} \\ &\approx 0.006\,67\,\mathrm{mm}^3 \end{aligned}$$

0.00667 mm³. With a thickness of 0.3 mm it would be 2.25 times as much: thickness counts squared.

Section Modulus: Circle

(2.48) MASTER

$$W_\mathrm{b} = \frac{\pi \cdot d^3}{32}$$

For arbors, pivots, pins and round spring wires.

W_b	section modulus (for bending)	mm^3
d	diameter	mm

EXAMPLE Round wire or pivot with a diameter of 1 mm.

Given: $d = 1\,\mathrm{mm}$

$$\begin{aligned} W_\mathrm{b} &= \frac{\pi \cdot d^3}{32} \\ &= \frac{\pi \cdot (1\,\mathrm{mm})^3}{32} \\ &\approx 0.0982\,\mathrm{mm}^3 \end{aligned}$$

0.0982 mm³. A pivot of 0.5 mm diameter has only one eighth of this: the diameter counts to the third power. This is why thin pivots break so easily.

Second Moment of Area: Rectangle

(2.49) MASTER

$$I = \frac{b \cdot s^3}{12}$$

The second moment of area describes how stiff a cross-section is against bending. The thickness enters to the third power: 10 % more thickness makes a spring one third stiffer. This is why mainsprings and balance springs are rolled to within thousandths of a millimetre.

I	second moment of area (area moment of inertia)	mm^4
b	blade height (width of the rectangle transverse to the direction of bending)	mm
s	blade thickness (thickness in the direction of bending)	mm

Valid: bending about the flat side.

Caution: The second moment of area I (unit mm⁴) describes a cross-section. It has nothing to do with the moment of inertia J of a rotating mass from Section 2.6.

EXAMPLE Leaf spring with a height of 1 mm and a thickness of 0.2 mm.

Given: $b = 1\,\mathrm{mm}; \quad s = 0.2\,\mathrm{mm}$

$$\begin{aligned} I &= \frac{b \cdot s^3}{12} \\ &= \frac{1\,\mathrm{mm} \cdot (0.2\,\mathrm{mm})^3}{12} \\ &\approx 0.000\,667\,\mathrm{mm}^4 \end{aligned}$$

0.000667 mm⁴. A spring only 0.1 mm thick would have one eighth of this at the same height.

Second Moment of Area: Circle

(2.50) MASTER

$$I = \frac{\pi \cdot d^4}{64}$$

The diameter counts to the fourth power.

I	second moment of area	mm^4
d	diameter	mm

EXAMPLE Round wire with a diameter of 1 mm.

Given: $d = 1\,\mathrm{mm}$

$$\begin{aligned} I &= \frac{\pi \cdot d^4}{64} \\ &= \frac{\pi \cdot (1\,\mathrm{mm})^4}{64} \\ &\approx 0.0491\,\mathrm{mm}^4 \end{aligned}$$

0.0491 mm⁴. At half the diameter only one sixteenth remains.

Deflection of a Cantilever

(2.51) MASTER

$$w = \frac{F \cdot l^3}{3 \cdot E \cdot I}$$

This is the basic formula of all leaf springs: click springs, jumper springs, pusher springs. The length counts to the third power; a spring twice as long is eight times as soft.

Valid: elastic range, constant cross-section, deflection small compared with the length.

w	deflection at the free end (often f in reference tables)	mm
F	force (at the free end, transverse to the bar)	N
l	length (free, from the clamping point to the force)	mm
E	modulus of elasticity (of the material)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
I	second moment of area	mm^4

EXAMPLE The steel leaf spring ($E = 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$) with $I = 0.000\,667\,\mathrm{mm}^4$ is 10 mm long and is loaded at its end with 0.42 N.

Given: $F = 0.42\,\mathrm{N}$; $l = 10\,\mathrm{mm}$; $E = 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$; $I = 0.000\,667\,\mathrm{mm}^4$

$$\begin{aligned} w &= \frac{F \cdot l^3}{3 \cdot E \cdot I} \\ &= \frac{0.42\,\mathrm{N} \cdot (10\,\mathrm{mm})^3}{3 \cdot 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2} \cdot 0.000\,667\,\mathrm{mm}^4} \\ &\approx 1.00\,\mathrm{mm} \end{aligned}$$

The end of the spring yields by 1 mm.

Spring Rate of a Leaf Spring

(2.52) ADVANCED

$$c = \frac{E \cdot b \cdot s^3}{4 \cdot l^3}$$

The formula combines deflection and the second moment of area of the rectangle. It shows what makes a spring softer: most strongly less thickness and more length, less height only in simple proportion.

Valid: cantilevered leaf spring with constant rectangular cross-section, small deflection.

c	spring rate (at the free end)	$\frac{\mathrm{N}}{\mathrm{mm}}$
E	modulus of elasticity (of the material)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
b	blade height	mm
s	blade thickness	mm
l	length (free, from the clamping point to the force)	mm

Good to know: The bending stress at the clamping point follows from the deflection: $\sigma_\mathrm{b} = \frac{3 \cdot E \cdot s \cdot w}{2 \cdot l^2}$. A thinner or longer spring tolerates more deflection at the same stress.

EXAMPLE Steel leaf spring, 1 mm high, 0.2 mm thick, 10 mm long.

Given: $E = 210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$; $b = 1\,\mathrm{mm}$; $s = 0.2\,\mathrm{mm}$; $l = 10\,\mathrm{mm}$

$$\begin{aligned} c &= \frac{E \cdot b \cdot s^3}{4 \cdot l^3} \\ &= \frac{210\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2} \cdot 1\,\mathrm{mm} \cdot (0.2\,\mathrm{mm})^3}{4 \cdot (10\,\mathrm{mm})^3} \\ &= 0.42\,\frac{\mathrm{N}}{\mathrm{mm}} \end{aligned}$$

0.42 N per millimetre of deflection. If the spring is thinned to 0.15 mm, only 42 % of this remains.

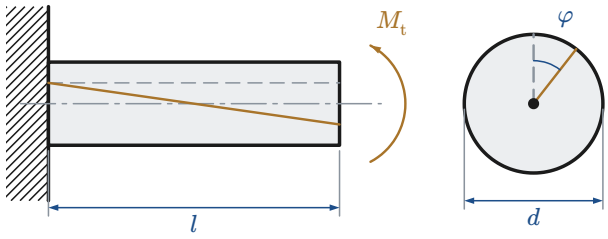


Fig. 2.17 Torsion: the moment M_t twists the free end through the angle φ

Torsion: Stress in a Round Shaft

(2.53) ADVANCED

$$\tau_t = \frac{16 \cdot M_t}{\pi \cdot d^3}$$

A torque about the longitudinal axis twists the shaft. The stress is greatest at the circumference and zero at the centre.

Valid: solid round cross-section, elastic range.

τ_t	maximum torsional stress (at the circumference)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
M_t	torsional moment (torque about the longitudinal axis)	$\mathrm{N{\cdot}mm}$
d	diameter	mm

EXAMPLE A winding stem transmits 10 N·mm through a shoulder of 1 mm diameter.

Given: $M_t = 10\,\mathrm{N{\cdot}mm}$; $d = 1\,\mathrm{mm}$

$$\begin{aligned}\tau_t &= \frac{16 \cdot M_t}{\pi \cdot d^3} \\ &= \frac{16 \cdot 10\,\mathrm{N{\cdot}mm}}{\pi \cdot (1\,\mathrm{mm})^3} \\ &\approx 50.9\,\tfrac{\mathrm{N}}{\mathrm{mm}^2}\end{aligned}$$

About $51\,\frac{\mathrm{N}}{\mathrm{mm}^2}$, no problem for hardened steel. At the square and at grooves, however, the stress is locally much higher; that is where winding stems break.

Torsion: Angle of Twist

(2.54) ADVANCED

$$\varphi = \frac{32 \cdot M_t \cdot l}{\pi \cdot G \cdot d^4}$$

A round wire acts as a torsion spring. The diameter counts to the fourth power: a wire only 10 % thinner twists by half as much again.

Valid: solid round cross-section, elastic range. For flat strips, such as torsion pendulum clocks use as torsion springs, other coefficients apply.

φ	angle of twist (between the ends, in radians)	rad
M_t	torsional moment	$\mathrm{N{\cdot}mm}$
l	length (twisted section)	mm
G	shear modulus (of the material; here not the rate)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
d	diameter	mm

EXAMPLE The shoulder of the winding stem (1 mm diameter) is 10 mm long, steel with $G = 80\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$, torque 10 N·mm.

Given: $M_t = 10\,\mathrm{N{\cdot}mm}$; $l = 10\,\mathrm{mm}$; $G = 80\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2}$; $d = 1\,\mathrm{mm}$

$$\begin{aligned}\varphi &= \frac{32 \cdot M_t \cdot l}{\pi \cdot G \cdot d^4} \\ &= \frac{32 \cdot 10\,\mathrm{N{\cdot}mm} \cdot 10\,\mathrm{mm}}{\pi \cdot 80\,000\,\frac{\mathrm{N}}{\mathrm{mm}^2} \cdot (1\,\mathrm{mm})^4} \\ &= 0{,}0127\,\mathrm{rad} \\ &\approx 0.73^\circ\end{aligned}$$

The stem twists by 0.73°, which cannot be felt when winding.

Modulus of Elasticity, Shear Modulus and Tensile Strength of Watch Materials (Guide Values at 20 °C)

Material	E in 1000 $\frac{\mathrm{N}}{\mathrm{mm^2}}$	G in 1000 $\frac{\mathrm{N}}{\mathrm{mm^2}}$	R_m in $\frac{\mathrm{N}}{\mathrm{mm^2}}$
steel, unalloyed, soft	210	80	350 … 700
spring steel, hardened and tempered	206 … 210	80	1500 … 2200
cobalt-nickel-chromium spring alloy (Nivaflex)	200 … 225	75 … 85	2400 … 3200
stainless steel 1.4435 (316L)	200	77	500 … 700
brass (CuZn37, CuZn39Pb3), hard	95 … 110	36 … 40	400 … 550
nickel silver (CuNi18Zn20), hard	125 … 135	47 … 50	500 … 700
beryllium bronze (CuBe2), precipitation-hardened	125 … 135	48 … 50	1100 … 1400
titanium grade 5	114	42	900 … 1000
silicon, single-crystal	130 … 190	50 … 80	brittle
ruby, sapphire	350 … 450	150 … 175	brittle

The modulus of elasticity hardly depends on the state of hardness, the strength on the other hand strongly does. Silicon has different stiffness depending on the crystal direction. Brittle materials break without permanent deformation; their strength depends mainly on the surface. For most metals, $G \approx 0.38 \cdot E$.

2.8 Thermal Expansion

Heat makes parts larger. In the watch this changes fits, the length of the pendulum and the dimensions of balance and balance spring.

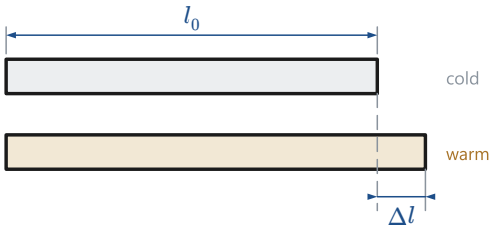


Fig. 2.18 Linear expansion on heating

Linear Expansion

(2.55) JOURNEYMAN

$$\Delta l = \alpha \cdot l_0 \cdot \Delta\vartheta$$

Almost all materials expand when heated, by the same fraction in every direction. Bores and inside diameters also become larger.

Valid: temperature changes up to about 100 K; above that α changes noticeably. A temperature difference in °C has the same numerical value as in K.

Δl	change in length (+ means: longer)	mm
α	coefficient of linear expansion (of the material)	$\frac{1}{\mathrm{K}}$
l_0	original length (also a diameter)	mm
$\Delta\vartheta$	change in temperature (+ means: warmer)	K

EXAMPLE A brass ring ($\alpha = 19 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) with an inside diameter of 30 mm is heated by 100 K. By how much does the diameter grow?

Given: $\alpha = 19 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$; $\quad l_0 = 30\,\mathrm{mm}$; $\quad \Delta\vartheta = 100\,\mathrm{K}$

$$\begin{aligned}\Delta l &= \alpha \cdot l_0 \cdot \Delta\vartheta \\ &= 19 \cdot 10^{-6}\ \tfrac{1}{\mathrm{K}} \cdot 30\,\mathrm{mm} \cdot 100\,\mathrm{K} \\ &= 0.057\,\mathrm{mm}\end{aligned}$$

By 0.057 mm, i.e. just under 6 hundredths. That is enough to shrink a ring on: push it on warm, and on cooling it sits tight.

Coefficients of Linear Expansion (Guide Values at 20 °C)

Material	α in $10^{-6}\,\frac{1}{\mathrm{K}}$
zinc	30
aluminium	23
silver	19
brass	18 … 21
nickel silver	17 … 18
beryllium bronze (CuBe2)	17
stainless steel 1.4435 (316L)	16
gold	14
steel, unalloyed	11 … 13
platinum	9
titanium	8.5 … 9
mineral glass	8 … 9
sapphire, ruby	5 … 7
silicon	2.6
Invar (FeNi36)	1.2 … 2
fused quartz	0.5

The values vary with alloy and heat treatment. How the expansion of the pendulum rod affects the rate is given in Section 6.5.

Volume Expansion

(2.56) MASTER

$$\Delta V = \gamma \cdot V_0 \cdot \Delta\vartheta$$

For solids the coefficient of volume expansion is three times the coefficient of linear expansion: $\gamma \approx 3 \cdot \alpha$. Liquids expand much more, oils by about $700 \cdot 10^{-6}\,\frac{1}{\mathrm{K}}$.

Valid: small changes in temperature.

ΔV	change in volume (+ means: larger)	cm^3
γ	coefficient of volume expansion	$\frac{1}{\mathrm{K}}$
V_0	original volume	cm^3
$\Delta\vartheta$	change in temperature (+ means: warmer)	K

EXAMPLE The jar of a mercury pendulum contains 100 cm³ of mercury ($\gamma = 181 \cdot 10^{-6}\,\frac{1}{\mathrm{K}}$). The temperature rises by 10 K.

Given: $\gamma = 181 \cdot 10^{-6}\,\frac{1}{\mathrm{K}}$; $V_0 = 100\,\mathrm{cm}^3$; $\Delta\vartheta = 10\,\mathrm{K}$

$$\begin{aligned}\Delta V &= \gamma \cdot V_0 \cdot \Delta\vartheta \\ &= 181 \cdot 10^{-6}\,\tfrac{1}{\mathrm{K}} \cdot 100\,\mathrm{cm}^3 \cdot 10\,\mathrm{K} \\ &= 0.181\,\mathrm{cm}^3\end{aligned}$$

The mercury expands by 0.181 cm³ and rises in the jar. Its centre of gravity moves upwards and compensates for the lengthening of the pendulum rod.

3 Power Source

The power source supplies the energy consumed by gear train, escapement and oscillator. In watches and in many clocks it is stored by a mainspring in the barrel, in the pendulum clock often by a weight. The spring is compact, but its torque decreases as it runs down. The weight needs a height of fall, but in return it always pulls with the same force.

Calculations use workshop units: dimensions in mm, forces in N, torques in $\mathrm{N} \cdot \mathrm{mm}$. How the torque reaches the escapement through the gear train is given in Chapter 4. Departing from the usual assignment, in this chapter P stands for power and m for the mass of the weight; N counts coils, turns and revolutions.

3.1 Mainspring in the Barrel: Geometry

The mainspring lies as a pack in the barrel: run down against the wall, wound around the barrel arbor. For the calculation we assume the coils lie tightly against each other. The spring then covers the same area in both states, namely length times blade thickness. All formulas in this section follow from this.

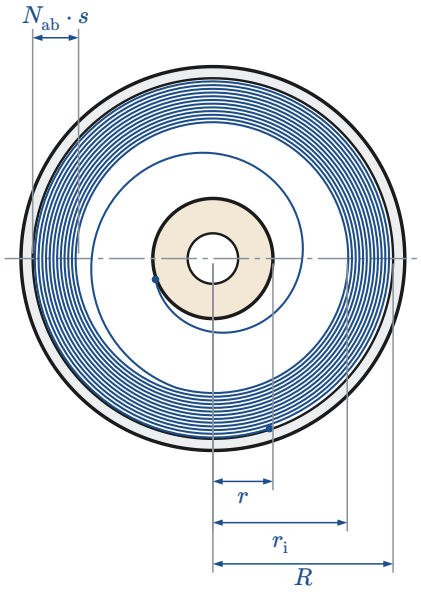


Fig. 3.1 Run down: the pack lies against the wall and reaches inwards to r_{i}

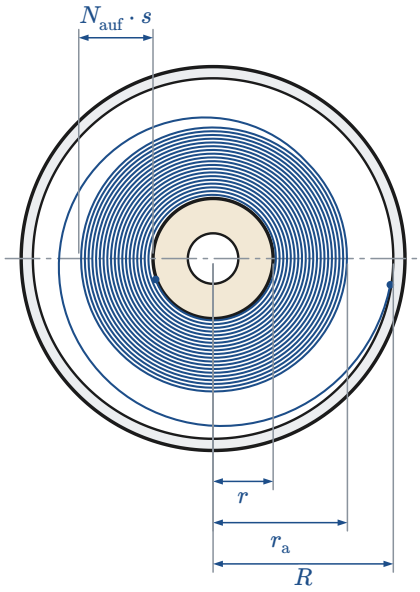


Fig. 3.2 Wound: the pack lies around the arbor and reaches outwards to r_{a}

Radius of the Barrel Arbor (Guide Value)

(3.1) APPROXIMATION JOURNEYMAN

$$r \approx \frac{R}{3}$$

r	radius (of the barrel arbor)	mm
R	inner radius (of the barrel)	mm

The barrel arbor takes up about one third of the inner radius. A thinner arbor would give more turns, but bends the innermost coil too sharply.

Valid: guide value for watches and clocks.

Good to know: Second condition: depending on the spring material, the arbor radius should be at least about 10 to 16 times the blade thickness (guide value). A thinner arbor bends the spring too sharply; it fatigues at the arbor and breaks there.

EXAMPLE A barrel has an inside diameter of 11.4 mm, i.e. a radius of 5.7 mm. What should the radius of the barrel arbor be?

Given: $R = 5.7\,\text{mm}$

$$\begin{aligned} r &\approx \frac{R}{3} \\ &= \frac{5.7\,\text{mm}}{3} \\ &= 1.9\,\text{mm} \end{aligned}$$

The arbor gets a radius of about 1.9 mm, i.e. a diameter of 3.8 mm. With a blade of 0.12 mm this is just under 16 times the blade thickness.

Coils of the Wound Spring

(3.2) MASTER

$$N_{\text{auf}} = \frac{\sqrt{r^2 + \frac{l \cdot s}{\pi}} - r}{s}$$

N_{auf}	coils of the wound spring (around the arbor)	1
r	radius (of the barrel arbor)	mm
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm

The square root is the outer radius r_{a} of the wound pack. It follows from equal areas, $l \cdot s = \pi \cdot (r_{\text{a}}^2 - r^2)$. The width of the pack $r_{\text{a}} - r$ divided by the blade thickness gives the coils.

Valid: coils lie tightly together; spring ends, hooks and bridle not included.

EXAMPLE A mainspring 420 mm long with a blade thickness of 0.12 mm is wound around an arbor of 1.9 mm radius. How many coils does the pack have?

Given: $r = 1.9\,\text{mm}; \quad l = 420\,\text{mm}; \quad s = 0.12\,\text{mm}$

$$\begin{aligned} N_{\text{auf}} &= \frac{\sqrt{r^2 + \frac{l \cdot s}{\pi}} - r}{s} \\ &= \frac{\sqrt{(1.9\,\text{mm})^2 + \frac{420\,\text{mm} \cdot 0.12\,\text{mm}}{\pi}} - 1.9\,\text{mm}}{0.12\,\text{mm}} \\ &\approx 21.1 \end{aligned}$$

About 21 coils. The pack extends to a radius of 4.43 mm.

Coils of the Run-Down Spring

(3.3) MASTER

$$N_{\text{ab}} = \frac{R - \sqrt{R^2 - \dfrac{l \cdot s}{\pi}}}{s}$$

N_{ab}	coils of the run-down spring (against the wall)	1
R	inner radius (of the barrel)	mm
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm

Here the square root is the inner radius r_{i} of the pack against the wall, again from equal areas: $l \cdot s = \pi \cdot (R^2 - r_{\text{i}}^2)$.

Valid: coils lie tightly against the wall. The spring must fit into the barrel: $l \cdot s$ smaller than $\pi \cdot (R^2 - r^2)$.

EXAMPLE The same spring (420 mm long, 0.12 mm thick) lies run down against the wall of a barrel with an inner radius of 5.7 mm.

Given: $R = 5.7\,\text{mm}$; $l = 420\,\text{mm}$; $s = 0.12\,\text{mm}$

$$\begin{aligned} N_{\text{ab}} &= \frac{R - \sqrt{R^2 - \dfrac{l \cdot s}{\pi}}}{s} \\ &= \frac{5.7\,\text{mm} - \sqrt{(5.7\,\text{mm})^2 - \dfrac{420\,\text{mm} \cdot 0.12\,\text{mm}}{\pi}}}{0.12\,\text{mm}} \\ &\approx 13.7 \end{aligned}$$

Just under 14 coils. On the inside the pack ends at a radius of 4.06 mm.

Turns of the Barrel

(3.4) JOURNEYMAN

$$N = N_{\text{auf}} - N_{\text{ab}}$$

N	turns (of the barrel)	1
N_{auf}	coils of the wound spring (around the arbor)	1
N_{ab}	coils of the run-down spring (against the wall)	1

As the spring runs down, the coils move from the arbor to the wall. The barrel turns through as many turns as the wound spring has more coils than the run-down one.

Valid: coils counted in both states inside the barrel. The value is the upper limit: the spring ends do not take part, and in practice up to one turn is lost.

EXAMPLE The spring from the two previous examples has 21.1 coils wound and 13.7 coils run down.

Given: $N_{\text{auf}} = 21.1$; $N_{\text{ab}} = 13.7$

$$\begin{aligned} N &= N_{\text{auf}} - N_{\text{ab}} \\ &= 21.1 - 13.7 \\ &= 7.4 \end{aligned}$$

By calculation the barrel makes 7.4 turns.

Optimum Spring Length

(3.5) MASTER

$$l = \frac{\pi \cdot \left(R^2 - r^2\right)}{2 \cdot s}$$

l	length (of the mainspring)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm
s	blade thickness (thickness of the spring)	mm

The most turns are given by the spring that covers exactly half of the free area between arbor and wall. A shorter spring has too few coils, a longer one leaves them no room to unwind.

Valid: coils lying tightly; allowance for attaching the ends not included.

Good to know: The maximum is flat (Fig. 3.3). In the example, a spring 20 % shorter or longer costs only about 3 % of the turns. The length therefore need not be exact to the millimetre, whereas the blade thickness matters a great deal (Section 3.2).

EXAMPLE Which length is optimum for a blade of 0.12 mm in a barrel with an inner radius of 5.7 mm and an arbor radius of 1.9 mm?

Given: $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$

$$\begin{aligned} l &= \frac{\pi \cdot \left(R^2 - r^2\right)}{2 \cdot s} \\ &= \frac{\pi \cdot \left((5.7\,\mathrm{mm})^2 - (1.9\,\mathrm{mm})^2\right)}{2 \cdot 0.12\,\mathrm{mm}} \\ &\approx 378\,\mathrm{mm} \end{aligned}$$

About 378 mm. The 420 mm spring from the previous examples is therefore somewhat too long, but hardly loses any turns as a result.

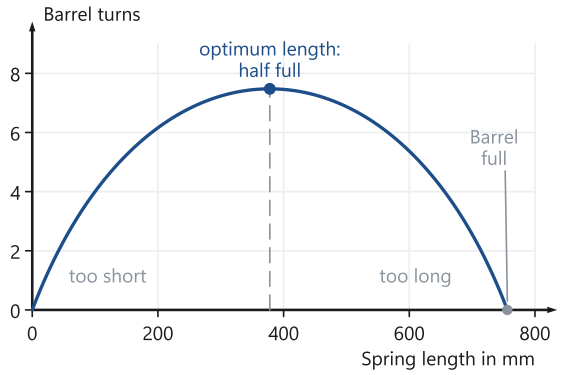


Fig. 3.3 Turns as a function of spring length, calculated for $R = 5.7\,\mathrm{mm}$, $r = 1.9\,\mathrm{mm}$, $s = 0.12\,\mathrm{mm}$

Filling Ratio of the Barrel

(3.6) MASTER

$$k_\mathrm{F} = \frac{l \cdot s}{\pi \cdot (R^2 - r^2)}$$

The filling ratio shows whether an existing or ordered spring suits the barrel. The optimum is 0.5.

Valid: coils lying tightly.

k_F	filling ratio: share of the free area covered by the spring	1
l	length (of the mainspring)	mm
s	blade thickness (thickness of the spring)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm

EXAMPLE How much does the spring 420 mm long with a blade thickness of 0.12 mm fill the barrel (inner radius 5.7 mm, arbor radius 1.9 mm)?

Given: $l = 420\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$; $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$

$$\begin{aligned} k_\mathrm{F} &= \frac{l \cdot s}{\pi \cdot (R^2 - r^2)} \\ &= \frac{420\,\mathrm{mm} \cdot 0.12\,\mathrm{mm}}{\pi \cdot \left((5.7\,\mathrm{mm})^2 - (1.9\,\mathrm{mm})^2\right)} \\ &\approx 0.56 \end{aligned}$$

The spring covers 56 % of the free area, slightly more than the optimum half.

Dividing Circle at Optimum Length

(3.7) MASTER

$$\rho = \sqrt{\frac{R^2 + r^2}{2}}$$

At optimum length, the wound pack extends exactly as far outwards as the run-down pack extends inwards. This dividing circle splits the free area into two equal halves.

Valid: spring of optimum length, filling ratio 0.5.

ρ	radius (of the dividing circle)	mm
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm

Good to know: Check on the open barrel: if the run-down spring reaches further in than the dividing circle, it is too long. If it stays further out, it is too short.

EXAMPLE Where is the dividing circle in the barrel with an inner radius of 5.7 mm and an arbor radius of 1.9 mm?

Given: $R = 5.7\,\mathrm{mm}$; $r = 1.9\,\mathrm{mm}$

$$\begin{aligned} \rho &= \sqrt{\frac{R^2 + r^2}{2}} \\ &= \sqrt{\frac{(5.7\,\mathrm{mm})^2 + (1.9\,\mathrm{mm})^2}{2}} \\ &\approx 4.25\,\mathrm{mm} \end{aligned}$$

At a radius of 4.25 mm. The run-down pack is then 1.45 mm wide, the wound pack 2.35 mm.

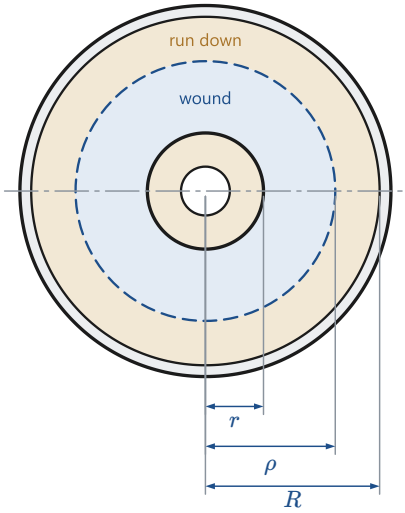


Fig. 3.4 The dividing circle ρ halves the free area: the wound spring lies inside, the run-down spring outside

Maximum Number of Turns

(3.8) MASTER

$$N_{\max} = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{s}$$

REARRANGED $$s = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{N_{\max}}$$

This many turns are possible with the optimum spring length. Rearranged, the formula gives the blade thickness for a required number of turns.

Valid: spring of optimum length (filling ratio 0.5), coils lying tightly. Theoretical upper limit as for formula (3.4).

$N_{\max}$	maximum possible turns of the barrel	1
R	inner radius (of the barrel)	mm
r	radius (of the barrel arbor)	mm
s	blade thickness (thickness of the spring)	mm

EXAMPLE What is the maximum number of turns the barrel (inner radius 5.7 mm, arbor radius 1.9 mm) can achieve with a blade of 0.12 mm?

Given: $R = 5.7\,\text{mm}$; $r = 1.9\,\text{mm}$; $s = 0.12\,\text{mm}$

$$N_{\max} = \frac{\sqrt{2 \cdot (R^2 + r^2)} - R - r}{s}$$

$$= \frac{\sqrt{2 \cdot \left((5.7\,\text{mm})^2 + (1.9\,\text{mm})^2\right)} - 5.7\,\text{mm} - 1.9\,\text{mm}}{0.12\,\text{mm}}$$

$$\approx 7.5$$

About 7.5 turns. With a blade of 0.10 mm it would be 9, with 0.14 mm only 6.4.

Guide Values for Arbor Radius $r = \frac{1}{3}\,R$ and Optimum Spring Length

Quantity	Value
radius of the dividing circle	$\rho = 0.745 \cdot R$
width of the run-down pack	$0.255 \cdot R$, about one quarter of the inner radius
width of the wound pack	$0.412 \cdot R$
optimum spring length	$l = 1.40 \cdot \frac{R^2}{s}$
maximum number of turns	$N_{\max} = 0.157 \cdot \frac{R}{s}$

Calculated from formulas (3.5), (3.7) and (3.8). The values apply only if the arbor measures one third of the inner radius.

Spring Length from the Coils of the Run-Down Spring

(3.9) JOURNEYMAN

$$l = \pi \cdot (R + r_\mathrm{i}) \cdot N_\mathrm{ab}$$

l	length (of the mainspring)	mm
R	inner radius (of the barrel)	mm
r_i	inner radius of the run-down spring pack	mm
N_ab	coils of the run-down spring (against the wall)	1

The length is the mean circumference of the pack times the number of coils. This allows the spring length to be determined without removing the spring. The blade thickness follows from the width of the pack: $s = (R - r_\mathrm{i}) : N_\mathrm{ab}$.

Valid: coils lie tightly against the wall. The piece from the pack to the arbor is not included; the actual spring is longer by about one inner coil.

EXAMPLE In a barrel with an inner radius of 5.7 mm, the run-down spring lies against the wall with 13.75 coils. The inner edge of the pack has a radius of 4.05 mm.

Given: $R = 5.7\,\mathrm{mm}$; $r_\mathrm{i} = 4.05\,\mathrm{mm}$; $N_\mathrm{ab} = 13.75$

$$\begin{aligned} l &= \pi \cdot (R + r_\mathrm{i}) \cdot N_\mathrm{ab} \\ &= \pi \cdot (5.7\,\mathrm{mm} + 4.05\,\mathrm{mm}) \cdot 13.75 \\ &\approx 421\,\mathrm{mm} \end{aligned}$$

The pack contains about 421 mm of spring. The blade thickness is 1.65 mm divided by 13.75, i.e. 0.12 mm.

Key point: The spring should fill half of the free area of the barrel. More turns are then only possible with a thinner blade.

3.2 Torque and Energy of the Mainspring

The wound spring tends to bend back and in doing so turns the barrel. Its torque depends mainly on the blade thickness, the stored energy on the volume of the spring and on the material.

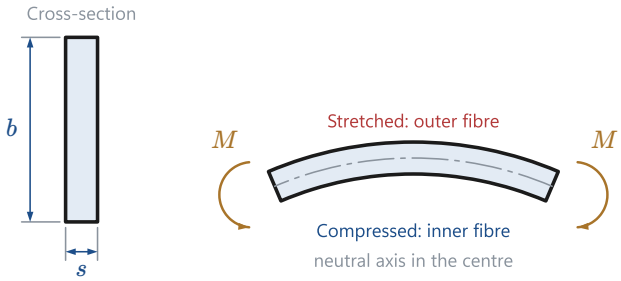


Fig. 3.5 Blade cross-section with height b and thickness s (thickness exaggerated); in bending the outside is stretched, the inside compressed

Torque of the Mainspring

(3.10) MASTER

$$M = \frac{E \cdot b \cdot s^3 \cdot \varphi}{12 \cdot l}$$

The torque increases with the angle of twist, with the blade height and with the third power of the blade thickness. A longer spring is softer. One revolution corresponds to $\varphi = 2\pi$.

Valid: elastic range, rectangular cross-section, coils do not touch or rub against each other. The angle is counted from the relaxed shape of the spring outside the barrel.

M	torque (of the spring)	$\mathrm{N \cdot mm}$
E	modulus of elasticity (of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm
φ	angle of twist (relative to the relaxed shape, in radians)	rad
l	length (of the mainspring)	mm

Caution: In the barrel, coils lie against each other at the arbor and at the wall. They do not work and they rub. The actual characteristic is therefore not a straight line (Fig. 3.6). The formula shows what the torque depends on; the torque of a particular spring is measured.

EXAMPLE A mainspring ($E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}}$, height 1.3 mm, thickness 0.12 mm, length 420 mm) is tensioned by 12 revolutions relative to its relaxed shape, i.e. 75.4 rad.

Given: $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}}$; $b = 1.3\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$; $\varphi = 75.4\,\mathrm{rad}$; $l = 420\,\mathrm{mm}$

$$M = \frac{E \cdot b \cdot s^3 \cdot \varphi}{12 \cdot l}$$
$$= \frac{200\,000\ \frac{\mathrm{N}}{\mathrm{mm^2}} \cdot 1.3\,\mathrm{mm} \cdot (0.12\,\mathrm{mm})^3 \cdot 75.4\,\mathrm{rad}}{12 \cdot 420\,\mathrm{mm}}$$
$$\approx 6.72\,\mathrm{N \cdot mm}$$

About 6.7 $\mathrm{N \cdot mm}$, a usual order of magnitude for the barrel of a man's wristwatch.

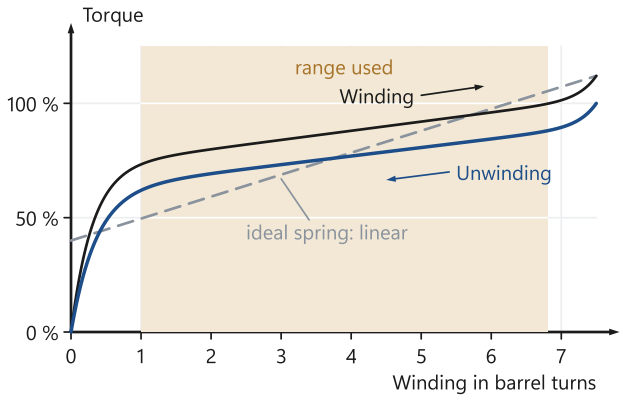


Fig. 3.6 Torque in the barrel during winding and running down (schematic, not a measured curve); 100 % is the torque at which the run-down begins from full wind

Fig. 3.6 shows the curve in the barrel. At the very top and the very bottom the torque changes sharply; in between it falls only slowly. The flat part is used. During winding the curve lies higher than during running down; the difference is the friction between the coils. Reverse-curve (S-shaped) springs (Fig. 3.8) are bent against their later winding direction in the relaxed state. They are stressed more evenly over their whole length and deliver a more even torque than springs with a simple spiral shape.

Maximum Torque from the Permissible Bending Stress

(3.11) MASTER

$$M_{\max} = \frac{\sigma_{\text{zul}} \cdot b \cdot s^2}{6}$$

$M_{\max}$	maximum torque transmissible elastically	$\mathrm{N \cdot mm}$
σ_{zul}	permissible bending stress (elastic limit of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm

The blade cannot withstand more torque elastically: the outer fibre reaches the permissible stress. The fraction $\frac{b \cdot s^2}{6}$ is the section modulus of the rectangle (Chapter 2).

Valid: rectangular cross-section, pure bending. Beyond the elastic limit the spring deforms permanently; it sets.

EXAMPLE What torque does a blade of 1.3 mm height and 0.12 mm thickness transmit at a permissible bending stress of $2500\ \frac{\mathrm{N}}{\mathrm{mm^2}}$?

Given: $\sigma_{\text{zul}} = 2500\ \frac{\mathrm{N}}{\mathrm{mm^2}}$; $b = 1.3\,\mathrm{mm}$; $s = 0.12\,\mathrm{mm}$

$$\begin{aligned} M_{\max} &= \frac{\sigma_{\text{zul}} \cdot b \cdot s^2}{6} \\ &= \frac{2500\ \frac{\mathrm{N}}{\mathrm{mm^2}} \cdot 1.3\,\mathrm{mm} \cdot (0.12\,\mathrm{mm})^2}{6} \\ &= 7.8\,\mathrm{N \cdot mm} \end{aligned}$$

At most 7.8 $\mathrm{N \cdot mm}$. The previous example, at 6.72 $\mathrm{N \cdot mm}$, lies below this.

Spring Materials for Mainsprings (Guide Values)

Material	E in $\frac{\mathrm{N}}{\mathrm{mm^2}}$	σ_{zul} in $\frac{\mathrm{N}}{\mathrm{mm^2}}$
spring steel (carbon steel, hardened and tempered)	206 000 … 210 000	1 500 … 2 000
cobalt-nickel-chromium alloy (for example Nivaflex)	200 000 … 225 000	2 200 … 2 800

Rough guide values for estimates. The permissible stress depends strongly on alloy, degree of rolling and heat treatment. The spring manufacturer's data are authoritative. Modulus of elasticity and tensile strength of further materials are given in Section 2.7.

Influence of Blade Thickness

(3.12) JOURNEYMAN

$$M_2 = M_1 \cdot \left(\frac{s_2}{s_1}\right)^3$$

M_2	torque of the replacement spring	$\mathrm{N \cdot mm}$
M_1	torque of the original spring	$\mathrm{N \cdot mm}$
s_2	blade thickness of the replacement spring	mm
s_1	blade thickness of the original spring	mm

The torque changes with the third power of the blade thickness. Even one hundredth of a millimetre makes a clear difference.

Valid: same blade height, same length, same material, same angle of twist.

Caution: A thicker blade fills the barrel more and achieves fewer turns. With a different blade thickness, therefore also recalculate length and turns according to Section 3.1.

EXAMPLE Instead of a spring with a blade thickness of 0.12 mm and 6.0 $\mathrm{N \cdot mm}$, one with 0.125 mm is fitted. What will the torque be?

Given: $M_1 = 6.0\,\mathrm{N \cdot mm}$; $s_2 = 0.125\,\mathrm{mm}$; $s_1 = 0.12\,\mathrm{mm}$

$$\begin{aligned} M_2 &= M_1 \cdot \left(\frac{s_2}{s_1}\right)^3 \\ &= 6.0\,\mathrm{N \cdot mm} \cdot \left(\frac{0.125\,\mathrm{mm}}{0.12\,\mathrm{mm}}\right)^3 \\ &\approx 6.8\,\mathrm{N \cdot mm} \end{aligned}$$

About 6.8 $\mathrm{N \cdot mm}$. The blade, only 0.005 mm thicker, gives 13 % more torque.

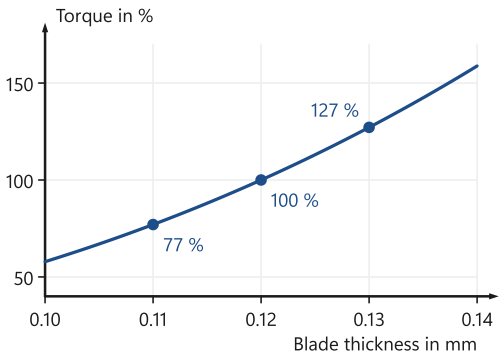


Fig. 3.7 The torque increases with the third power of the blade thickness (100 % at $s = 0.12\,\mathrm{mm}$)

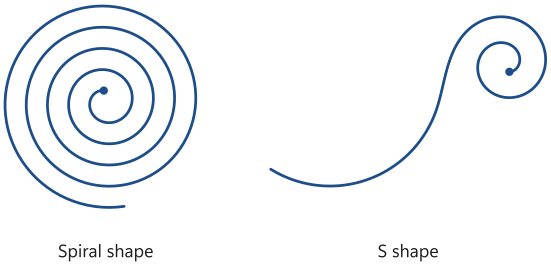


Fig. 3.8 Relaxed mainsprings: simple spiral shape and reverse-curve (S) shape

Replacement Spring with the Same Effect

(3.13) JOURNEYMAN

$$b_1 \cdot s_1^3 = b_2 \cdot s_2^3$$

REARRANGED $s_2 = s_1 \cdot \sqrt[3]{\dfrac{b_1}{b_2}}$

If the original spring is not available, a slightly thicker blade compensates for a lower height and vice versa. The torque remains the same if the product of height and the third power of thickness remains the same.

Valid: same length, same material. The replacement spring must not be higher than the barrel allows.

b_1	blade height of the original spring	mm
s_1	blade thickness of the original spring	mm
b_2	blade height of the replacement spring	mm
s_2	blade thickness of the replacement spring	mm

EXAMPLE The original spring measures 1.50 mm × 0.12 mm. Only a spring with a height of 1.40 mm is available. Which blade thickness gives the same torque?

Given: $s_1 = 0.12\,\mathrm{mm}$; $b_1 = 1.50\,\mathrm{mm}$; $b_2 = 1.40\,\mathrm{mm}$

$$\begin{aligned} s_2 &= s_1 \cdot \sqrt[3]{\frac{b_1}{b_2}} \\ &= 0.12\,\mathrm{mm} \cdot \sqrt[3]{\frac{1.50\,\mathrm{mm}}{1.40\,\mathrm{mm}}} \\ &\approx 0.1228\,\mathrm{mm} \end{aligned}$$

By calculation just under 0.123 mm. The next thickness on the list is chosen, here 0.125 mm, accepting slightly more torque.

Stored Energy of the Ideal Spring

(3.14) MASTER

$$W = \frac{M \cdot \varphi}{2}$$

Because the torque rises evenly from the relaxed state, on average half the final torque counts. $1\,\mathrm{N \cdot mm} = 1\,\mathrm{mJ}$.

Valid: ideal spring with a linear characteristic according to formula (3.10).

W	work (energy stored in the spring, tensioning work)	mJ
M	torque (of the spring)	$\mathrm{N{\cdot}mm}$
φ	angle of twist (relative to the relaxed shape, in radians)	rad

EXAMPLE The spring from the example for formula (3.10) has a torque of $6.72\,\mathrm{N \cdot mm}$ at 75.4 rad.

Given: $M = 6.72\,\mathrm{N{\cdot}mm}; \quad \varphi = 75.4\,\mathrm{rad}$

$$\begin{aligned} W &= \frac{M \cdot \varphi}{2} \\ &= \frac{6.72\,\mathrm{N{\cdot}mm} \cdot 75.4\,\mathrm{rad}}{2} \\ &\approx 253\,\mathrm{mJ} \end{aligned}$$

The spring contains about 253 mJ, i.e. 0.25 J. Only the part delivered by the barrel while running down is usable (formula (3.16)).

Maximum Storable Energy

(3.15) ADVANCED

$$W_{\max} = \frac{\sigma_{\mathrm{zul}}^2 \cdot b \cdot s \cdot l}{6 \cdot E}$$

The product $b \cdot s \cdot l$ is the volume of the spring. The storable energy therefore increases with the spring volume and with $\sigma_{\mathrm{zul}}^2 : E$. This is why the barrel is given as much space as possible in the movement, and why materials with a high elastic limit are worthwhile.

Valid: ideal spring, bent to the permissible stress over its whole length. Upper limit; real springs do not reach it.

$W_{\max}$	maximum elastically storable energy	mJ
σ_{zul}	permissible bending stress	$\frac{\mathrm{N}}{\mathrm{mm^2}}$
b	blade height (width of the spring)	mm
s	blade thickness (thickness of the spring)	mm
l	length (of the mainspring)	mm
E	modulus of elasticity (of the spring material)	$\frac{\mathrm{N}}{\mathrm{mm^2}}$

EXAMPLE Spring 1.3 mm × 0.12 mm × 420 mm, $\sigma_{\mathrm{zul}} = 2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}$, $E = 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}$.

Given: $\sigma_{\mathrm{zul}} = 2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}; \quad b = 1.3\,\mathrm{mm}; \quad s = 0.12\,\mathrm{mm}; \quad l = 420\,\mathrm{mm}; \quad E = 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}$

$$\begin{aligned} W_{\max} &= \frac{\sigma_{\mathrm{zul}}^2 \cdot b \cdot s \cdot l}{6 \cdot E} \\ &= \frac{\left(2500\,\frac{\mathrm{N}}{\mathrm{mm^2}}\right)^2 \cdot 1.3\,\mathrm{mm} \cdot 0.12\,\mathrm{mm} \cdot 420\,\mathrm{mm}}{6 \cdot 200\,000\,\frac{\mathrm{N}}{\mathrm{mm^2}}} \\ &\approx 341\,\mathrm{mJ} \end{aligned}$$

At most about 0.34 J. This is the energy needed to lift 35 g by one metre.

Work Delivered while Running Down

(3.16) JOURNEYMAN

$$W = 2 \cdot \pi \cdot N \cdot M_{\mathrm{m}}$$

Work is torque times angle of rotation; in each turn the barrel rotates through 2π. The mean torque is read from the measured let-down curve.

Valid: torque averaged over the run-down.

W	work (delivered by the barrel while running down)	mJ
N	turns (of the barrel)	1
M_{m}	mean torque over these turns	$\mathrm{N{\cdot}mm}$

EXAMPLE A barrel delivers 6.0 $\mathrm{N \cdot mm}$ on average over 6.5 turns.

Given: $N = 6.5$; $M_{\mathrm{m}} = 6.0\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} W &= 2 \cdot \pi \cdot N \cdot M_{\mathrm{m}} \\ &= 2 \cdot \pi \cdot 6.5 \cdot 6.0\,\mathrm{N{\cdot}mm} \\ &\approx 245\,\mathrm{mJ} \end{aligned}$$

The movement receives about 245 mJ, just under a quarter of a joule for almost two days of running.

Efficiency of the Mainspring

(3.17) MASTER

$$\eta_{\mathrm{F}} = \frac{W_{\mathrm{ab}}}{W_{\mathrm{auf}}}$$

Friction between the coils and at the wall consumes part of the winding work. Clean blades and the right grease in the barrel keep the loss small.

Valid: both quantities of work measured over the same turns.

η_{F}	efficiency of the spring in the barrel	1
W_{ab}	work delivered while running down (area under the let-down curve)	mJ
W_{auf}	work expended in winding (area under the winding curve)	mJ

EXAMPLE The barrel from the previous example delivers 245 mJ while running down. 288 mJ were expended in winding over the same turns.

Given: $W_{\mathrm{ab}} = 245\,\mathrm{mJ}$; $W_{\mathrm{auf}} = 288\,\mathrm{mJ}$

$$\begin{aligned} \eta_{\mathrm{F}} &= \frac{W_{\mathrm{ab}}}{W_{\mathrm{auf}}} \\ &= \frac{245\,\mathrm{mJ}}{288\,\mathrm{mJ}} \\ &= 0.85 \end{aligned}$$

The spring returns 85 % of the winding work; 15 % is lost as friction.

Key point: Double the blade height, double the torque. Double the blade thickness, eight times the torque.

3.3 Running Time

The running time is the time for which the movement runs on one full wind. It follows from the turns of the barrel and the time taken by one turn. The power reserve is the part of the running time that still lies ahead of the movement. As in Chapter 4, the number of teeth of a pinion carries a prime and the index of its arbor.

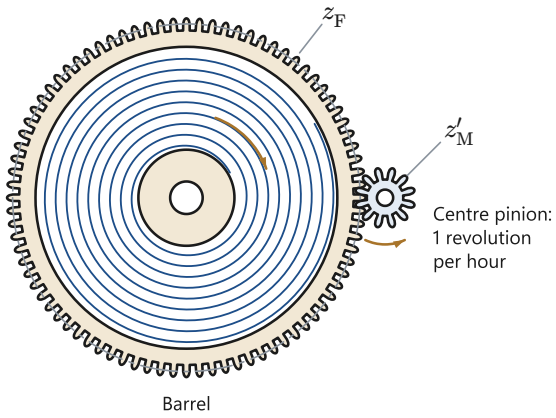


Fig. 3.9 The barrel with z_F teeth drives the centre pinion with z'_M teeth

Rotation Period of the Barrel

(3.18) JOURNEYMAN

$$t_F = \frac{z_F}{z'_M} \cdot t_M$$

The barrel meshes with the pinion of the centre wheel, which turns once per hour. If the barrel has seven times as many teeth as the pinion, one turn takes seven hours.

Valid: barrel meshes directly with the pinion of a wheel that turns once per hour. If the driven wheel turns differently, insert its rotation period.

t_F	rotation period of the barrel (time for one turn)	h
z_F	number of teeth of the barrel	1
z'_M	number of teeth of the centre pinion	1
t_M	rotation period of the centre wheel (1 h)	h

EXAMPLE A barrel with 84 teeth drives a centre pinion with 12 teeth.

Given: $z_F = 84$; $z'_M = 12$; $t_M = 1\,\mathrm{h}$

$$\begin{aligned} t_F &= \frac{z_F}{z'_M} \cdot t_M \\ &= \frac{84}{12} \cdot 1\,\mathrm{h} \\ &= 7\,\mathrm{h} \end{aligned}$$

The barrel takes 7 hours for one turn.

Running Time from the Turns of the Barrel

(3.19) JOURNEYMAN

$$t_G = N \cdot t_F$$

REARRANGED $N = \frac{t_G}{t_F}$

t_G	running time	h
N	turns (of the barrel)	1
t_F	rotation period of the barrel (time for one turn)	h

The turns that the barrel actually delivers give the running time. Rearranged, the formula gives the turns required for a specified running time.

Valid: evenly running movement; turns from full wind to standstill. For the usable running time count only the turns in which the torque is sufficient for an adequate amplitude.

Good to know: The power reserve is the running time remaining after a certain period of running: remaining turns times rotation period of the barrel.

EXAMPLE The barrel from the previous example (7 h per turn) delivers 6.5 usable turns.

Given: $N = 6.5$; $t_F = 7\,\text{h}$

$$\begin{aligned} t_G &= N \cdot t_F \\ &= 6.5 \cdot 7\,\text{h} \\ &= 45.5\,\text{h} \end{aligned}$$

The watch runs for 45.5 hours. After 24 hours, 21.5 hours of power reserve remain.

Rotation Period of the Barrel with Intermediate Wheel

(3.20) JOURNEYMAN

$$t_F = \frac{z_F \cdot z_B}{z'_B \cdot z'_M} \cdot t_M$$

t_F	rotation period of the barrel (time for one turn)	h
z_F	number of teeth of the barrel	1
z_B	number of teeth of the intermediate wheel	1
z'_B	number of teeth of the intermediate pinion	1
z'_M	number of teeth of the centre pinion	1
t_M	rotation period of the centre wheel (1 h)	h

For long running times an intermediate wheel sits between barrel and centre wheel. The numerator contains the driving wheels, the denominator the driven pinions.

Valid: barrel drives the intermediate pinion, the intermediate wheel drives the centre pinion; centre wheel once per hour.

EXAMPLE Eight-day movement: barrel 84 teeth, intermediate pinion 12, intermediate wheel 72, centre pinion 12.

Given: $z_F = 84$; $z_B = 72$; $z'_B = 12$; $z'_M = 12$; $t_M = 1\,\text{h}$

$$\begin{aligned} t_F &= \frac{z_F \cdot z_B}{z'_B \cdot z'_M} \cdot t_M \\ &= \frac{84 \cdot 72}{12 \cdot 12} \cdot 1\,\text{h} \\ &= 42\,\text{h} \end{aligned}$$

One turn takes 42 hours. For eight days, i.e. 192 hours, the movement needs about 4.6 turns according to formula (3.19).

In an automatic movement the outer end of the spring is not hooked in. It carries a slipping bridle that presses outwards against the barrel wall and slips as soon as the spring is fully wound. Its slipping torque determines the maximum torque the spring delivers to the movement. If the bridle slips too early, the spring is never fully wound and power reserve is lost. If it holds too firmly, the torque becomes too high and the balance knocks.

3.4 Weight Drive

A weight pulls on the barrel via a cable, gut line or chain. Its torque remains the same over the entire running time; this makes the weight drive the most uniform power source. The price is the height of fall.

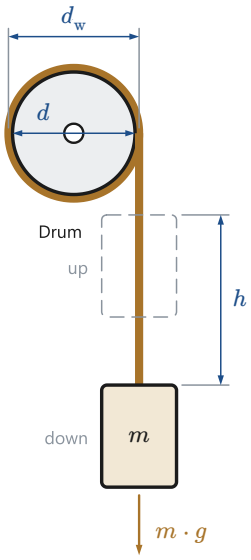


Fig. 3.10 Weight hanging directly on the line ($k = 1$): effective diameter d_w to the centre of the line, height of fall h

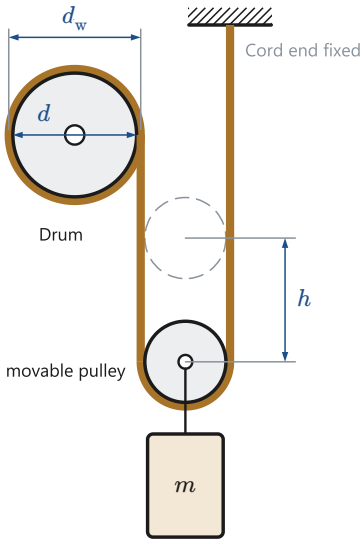


Fig. 3.11 Weight on a loose pulley ($k = 2$): the line runs off twice as far as the weight falls

Effective Barrel Diameter

(3.21) JOURNEYMAN

$$d_w = d + d_S$$

The force of the line acts at the centre of the line. The lever arm is therefore half the barrel diameter plus half the thickness of the line.

Valid: smooth barrel, line in one layer. For grooved barrels measure the diameter to the centre of the line.

d_w	effective barrel diameter (to the centre of the line)	mm
d	diameter (of the barrel)	mm
d_S	thickness of the cable or gut line	mm

EXAMPLE A line 1.5 mm thick lies on a barrel of 36 mm diameter.

Given: $d = 36\,\mathrm{mm}$; $d_S = 1.5\,\mathrm{mm}$

$$\begin{aligned} d_w &= d + d_S \\ &= 36\,\mathrm{mm} + 1.5\,\mathrm{mm} \\ &= 37.5\,\mathrm{mm} \end{aligned}$$

The calculation uses 37.5 mm.

Torque at the Weight Barrel

(3.22) JOURNEYMAN

$$M = \frac{m \cdot g \cdot d_\mathrm{w}}{2 \cdot k}$$

The weight force acts at half the effective diameter. If the weight hangs on a loose pulley, each of the two strands carries half the load, so only half the weight force pulls on the barrel.

Valid: friction of the pulley and weight of line and pulley neglected.

M	torque (at the barrel)	$\mathrm{N{\cdot}mm}$
m	mass (of the weight)	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
d_w	effective barrel diameter (to the centre of the line)	mm
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1

EXAMPLE A longcase clock has a weight of 4 kg on a loose pulley; effective barrel diameter 37.5 mm.

Given: $m = 4\,\mathrm{kg}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $d_\mathrm{w} = 37.5\,\mathrm{mm}$; $k = 2$

$$\begin{aligned} M &= \frac{m \cdot g \cdot d_\mathrm{w}}{2 \cdot k} \\ &= \frac{4\,\mathrm{kg} \cdot 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot 37.5\,\mathrm{mm}}{2 \cdot 2} \\ &\approx 368\,\mathrm{N{\cdot}mm} \end{aligned}$$

About $368\,\mathrm{N \cdot mm}$, i.e. $0.368\,\mathrm{N \cdot m}$, about fifty times that of a wristwatch. Without a pulley the torque would be twice as large.

Height of Fall of the Weight

(3.23) JOURNEYMAN

$$h = \frac{\pi \cdot d_\mathrm{w} \cdot N_\mathrm{W}}{k}$$

In each revolution the barrel releases one circumference of line. With a loose pulley the line is divided between two strands, so the weight falls only half as far.

Valid: line in one layer on the barrel.

h	height (height of fall of the weight)	mm
d_w	effective barrel diameter (to the centre of the line)	mm
N_W	revolutions of the barrel	1
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1

Good to know: The space required in the case must also include the height of the weight and, with a loose pulley, the pulley with its hook.

EXAMPLE The barrel of the longcase clock (effective diameter 37.5 mm, loose pulley) makes 16 revolutions in eight days.

Given: $d_\mathrm{w} = 37.5\,\mathrm{mm}$; $N_\mathrm{W} = 16$; $k = 2$

$$\begin{aligned} h &= \frac{\pi \cdot d_\mathrm{w} \cdot N_\mathrm{W}}{k} \\ &= \frac{\pi \cdot 37.5\,\mathrm{mm} \cdot 16}{2} \\ &\approx 942\,\mathrm{mm} \end{aligned}$$

The weight falls about 94 cm in eight days. Without a pulley it would be 1.88 m, more than a usual case provides.

Running Time from the Height of Fall

(3.24) JOURNEYMAN

$$t_{\mathrm{G}} = \frac{k \cdot h}{\pi \cdot d_{\mathrm{w}}} \cdot t_{\mathrm{W}}$$

The fraction is the number of barrel revolutions that fit into the height of fall. As with the mainspring barrel, the rotation period of the weight barrel follows from the numbers of teeth (formula (3.18)).

Valid: line in one layer. Only the distance the weight can actually fall counts as height of fall, excluding the height of weight and pulley.

t_{G}	running time	h
k	number (of supporting strands of the line: 1 without pulley, 2 with loose pulley)	1
h	height (height of fall of the weight)	mm
d_{w}	effective barrel diameter (to the centre of the line)	mm
t_{W}	rotation period of the barrel	h

EXAMPLE In a wall regulator the weight can fall 1100 mm. The barrel (effective diameter 31 mm) turns once in 12 hours. How long does the clock run with a loose pulley?

Given: $k = 2$; $h = 1100\,\mathrm{mm}$; $d_{\mathrm{w}} = 31\,\mathrm{mm}$; $t_{\mathrm{W}} = 12\,\mathrm{h}$

$$\begin{aligned} t_{\mathrm{G}} &= \frac{k \cdot h}{\pi \cdot d_{\mathrm{w}}} \cdot t_{\mathrm{W}} \\ &= \frac{2 \cdot 1100\,\mathrm{mm}}{\pi \cdot 31\,\mathrm{mm}} \cdot 12\,\mathrm{h} \\ &\approx 271\,\mathrm{h} \end{aligned}$$

About 271 hours, i.e. a little over 11 days. Without a pulley it would be only 5.6 days; for an eight-day movement this case therefore needs the loose pulley.

Energy of the Weight

(3.25) JOURNEYMAN

$$W = m \cdot g \cdot h$$

The energy depends only on the mass and height of fall of the weight. A loose pulley changes nothing: half the force at the barrel, but twice the length of line.

Valid: friction of the pulley neglected.

W	work (delivered by the weight, potential energy)	J
m	mass (of the weight)	kg
g	gravitational acceleration ($9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	height (height of fall of the weight)	m

EXAMPLE The weight of the longcase clock (4 kg) falls 0.942 m in eight days.

Given: $m = 4\,\mathrm{kg}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 0.942\,\mathrm{m}$

$$\begin{aligned} W &= m \cdot g \cdot h \\ &= 4\,\mathrm{kg} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 0.942\,\mathrm{m} \\ &\approx 37.0\,\mathrm{J} \end{aligned}$$

The longcase clock converts 37 J in eight days. The wristwatch from Section 3.2 manages on 0.245 J in just under two days.

Key point: The loose pulley halves height of fall and torque; the running time remains the same. For the same torque it needs twice the weight.

3.5 Fusee and Chain

The fusee compensates for the decreasing torque of the mainspring. With the spring fully wound, the chain pulls at the small radius of the fusee, with the spring almost run down at the large radius. The torque at the fusee wheel thus remains almost constant.

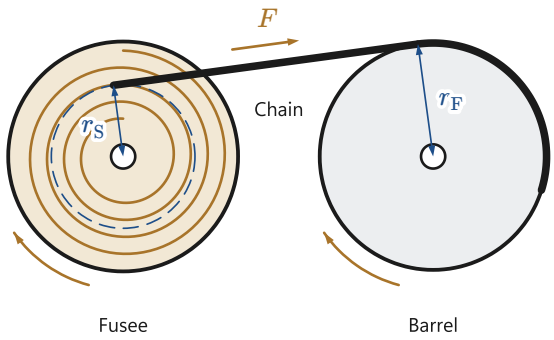


Fig. 3.12 Fusee and barrel seen from above: the chain pulls with the force F at the effective radius r_S

Torque at the Fusee

(3.26) ADVANCED

$$M_S = \frac{M_F \cdot r_S}{r_F}$$

REARRANGED $r_S = \dfrac{M_S \cdot r_F}{M_F}$

The chain transmits the force $F = M_F : r_F$ from the barrel to the fusee; there it acts at the radius r_S. Rearranged, the formula gives for every state of winding the radius at which the fusee delivers the required torque.

Valid: both radii measured to the centre of the chain; friction of chain and pivots neglected.

M_S	torque at the fusee	$\mathrm{N{\cdot}mm}$
M_F	torque of the barrel at the same moment	$\mathrm{N{\cdot}mm}$
r_S	effective radius of the fusee at the point where the chain runs off	mm
r_F	radius of the barrel (to the centre of the chain)	mm

EXAMPLE The barrel of a fusee pocket watch has a radius of 9 mm. The fusee is to deliver a constant $32\ \mathrm{N \cdot mm}$. Fully wound, the spring delivers $48\ \mathrm{N \cdot mm}$. What radius does the fusee need at this point?

Given: $M_S = 32\,\mathrm{N{\cdot}mm}$; $r_F = 9\,\mathrm{mm}$; $M_F = 48\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} r_S &= \frac{M_S \cdot r_F}{M_F} \\ &= \frac{32\,\mathrm{N{\cdot}mm} \cdot 9\,\mathrm{mm}}{48\,\mathrm{N{\cdot}mm}} \\ &= 6.0\,\mathrm{mm} \end{aligned}$$

6 mm at the thin end of the fusee. When the spring has dropped to $32\ \mathrm{N \cdot mm}$, the radius must be 9 mm.

3.6 Power of the Drive

Power is energy per unit time. A mechanical watch needs very little of it: at the barrel of a wristwatch it is one to a few microwatts, at the weight barrel of a longcase clock some tens of microwatts.

Mean Power

(3.27) JOURNEYMAN

$$P = \frac{W}{t_\mathrm{G}}$$

For substitution, convert the running time to seconds: $1\,\mathrm{h} = 3600\,\mathrm{s}$. Then $1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{W}$.

Valid: mean value over the entire running time.

P	power (of the drive)	$\mu\mathrm{W}$
W	work (delivered during the running time)	mJ
t_G	running time	h

EXAMPLE The barrel from Section 3.2 delivers 245 mJ in 45.5 hours.

Given: $W = 245\,\mathrm{mJ}$; $t_\mathrm{G} = 45.5\,\mathrm{h}$

$$\begin{aligned} P &= \frac{W}{t_\mathrm{G}} \\ &= \frac{245\,\mathrm{mJ}}{45.5\,\mathrm{h}} \\ &= \frac{0{,}245\,\mathrm{J}}{163\,800\,\mathrm{s}} \\ &\approx 1.5\,\mu\mathrm{W} \end{aligned}$$

About 1.5 microwatts. A 1 W LED needs about 700 000 times as much.

Power from Torque and Rotation Period

(3.28) MASTER

$$P = \frac{2 \cdot \pi \cdot M}{t_\mathrm{U}}$$

Power is torque times angular velocity. The driving arbor turns once through 2π in the rotation period.

Valid: torque constant over the period considered; insert the rotation period in seconds.

P	power (of the drive)	$\mu\mathrm{W}$
M	torque (at the driving arbor: mainspring barrel or weight barrel)	$\mathrm{N{\cdot}m}$
t_U	rotation period of the driving arbor	h

EXAMPLE $0.368\,\mathrm{N \cdot m}$ act at the weight barrel of the longcase clock; it turns once in 12 hours.

Given: $M = 0.368\,\mathrm{N{\cdot}m}$; $t_\mathrm{U} = 12\,\mathrm{h}$

$$\begin{aligned} P &= \frac{2 \cdot \pi \cdot M}{t_\mathrm{U}} \\ &= \frac{2 \cdot \pi \cdot 0.368\,\mathrm{N{\cdot}m}}{12\,\mathrm{h}} \\ &= \frac{2 \cdot \pi \cdot 0{,}368\,\mathrm{N{\cdot}m}}{43\,200\,\mathrm{s}} \\ &\approx 53.5\,\mu\mathrm{W} \end{aligned}$$

About 54 microwatts, roughly 36 times as much as for the wristwatch. The same calculation with $6\,\mathrm{N \cdot mm}$ and 7 hours again gives 1.5 microwatts for the wristwatch.

Power at the Oscillator

(3.29) MASTER

$$P_{\mathrm{U}} = \eta \cdot P$$

On the way through gear train and escapement, the greater part of the drive power is lost. The overall efficiency is the product of the individual efficiencies (Chapters 2 and 4).

Valid: steady-state running with constant amplitude. The efficiency of 0.3 in the example is a rough guide value for watches with lever escapement.

P_{U}	power reaching the balance or pendulum	$\mu\mathrm{W}$
η	efficiency (of gear train and escapement together)	1
P	power (of the drive)	$\mu\mathrm{W}$

EXAMPLE 1.5 microwatts come from the barrel. Gear train and escapement together have an efficiency of 0.3.

Given: $\eta = 0.3$; $P = 1.5\,\mu\mathrm{W}$

$$\begin{aligned} P_{\mathrm{U}} &= \eta \cdot P \\ &= 0.3 \cdot 1.5\,\mu\mathrm{W} \\ &= 0.45\,\mu\mathrm{W} \end{aligned}$$

About half a microwatt arrives at the balance. Depending on calibre and beat rate, the power at the balance lies between a few tenths of a microwatt and about one microwatt.

3.7 Winding and Hand Setting

In manual winding, the crown turns the barrel arbor via winding pinion, crown wheel and ratchet wheel. In an automatic movement this is done by the oscillating weight, the rotor, via a strongly reducing gear train. In the pulled-out position the crown drives the motion work via the sliding pinion. In winding, small wheels drive large ones; the gearing is to slow speed. The calculation therefore uses, as with the motion work (Section 4.3), the reduction ratio: numbers of teeth of the driven wheels divided by those of the driving ones. This is the reciprocal of the gear ratio from Section 4.1.

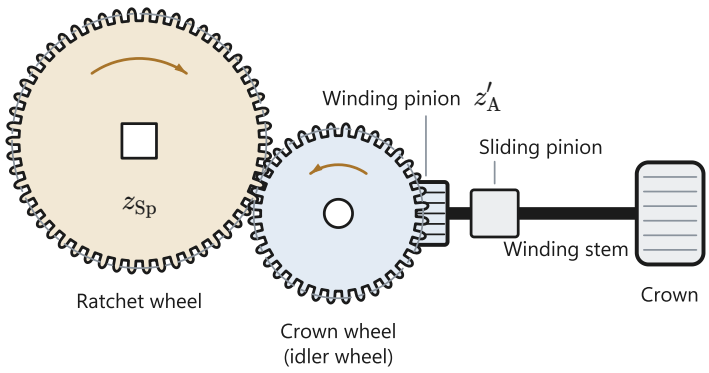


Fig. 3.13 Crown winding seen from above (schematic): the winding pinion with z'_{A} teeth drives, via the crown wheel, the ratchet wheel with z_{Sp} teeth on the barrel arbor

Crown Revolutions for Winding

(3.30) JOURNEYMAN

$$N_{\mathrm{K}} = \frac{N \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{A}}}$$

REARRANGED $$N = \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{A}}}{z_{\mathrm{Sp}}}$$

N_{K}	revolutions of the crown	1
N	turns (by which the barrel arbor tensions the spring)	1
z_{Sp}	number of teeth of the ratchet wheel	1
z'_{A}	number of teeth of the winding pinion (on the winding stem)	1

The crown wheel sits between winding pinion and ratchet wheel and is only an idler: its number of teeth drops out of the calculation. The reduction ratio of the winding is the number of teeth of the ratchet wheel divided by that of the winding pinion.

Valid: winding pinion, crown wheel and ratchet wheel in one chain; the crown wheel meshes with both using the same toothing. If the arbor of the crown wheel carries two different wheels, this stage counts as in formula (4.3).

Good to know: Each revolution of the crown adds as much running time as the rotation period of the barrel divided by the reduction ratio. In the example, at 7 hours per turn, this is just under 2 hours per revolution of the crown.

EXAMPLE The winding pinion has 15 teeth, the crown wheel 36, the ratchet wheel 54. The spring can be tensioned by 6.5 turns. How often must the crown be turned for full winding?

Given: $N = 6.5$; $z_{\mathrm{Sp}} = 54$; $z'_{\mathrm{A}} = 15$

$$\begin{aligned} N_{\mathrm{K}} &= \frac{N \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{A}}} \\ &= \frac{6.5 \cdot 54}{15} \\ &= 23.4 \end{aligned}$$

23.4 full revolutions. By hand one usually turns half revolutions, i.e. about 47 strokes. The 36 teeth of the crown wheel play no part in the calculation.

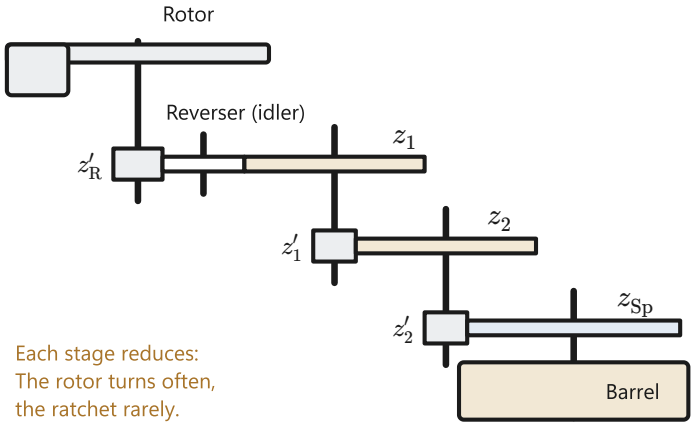


Fig. 3.14 Automatic winding, developed in side view (schematic): rotor pinion, reverser, two reduction stages and ratchet wheel

Reduction Ratio of Automatic Winding

(3.31) MASTER

$$i_{\mathrm{R}} = \frac{z_1 \cdot z_2 \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{R}} \cdot z'_1 \cdot z'_2}$$

The rotor has little torque but plenty of travel. The reduction gear train therefore reduces strongly: the numerator contains the driven wheels, the denominator the driving pinions. Reverser wheels that only rectify the direction of rotation and mesh with both neighbours using the same toothing are idlers and drop out.

Valid: two reduction stages between rotor pinion and ratchet wheel; with a different number of stages, correspondingly more or fewer factors. Guide value: the reduction ratio usually lies between 100 and 180.

i_{R}	reduction ratio of automatic winding (rotor revolutions per turn of the ratchet wheel)	1
z_1	number of teeth of the first reduction wheel	1
z_2	number of teeth of the second reduction wheel	1
z_{Sp}	number of teeth of the ratchet wheel	1
z'_{R}	number of teeth of the rotor pinion	1
z'_1	number of teeth of the pinion on the first reduction wheel	1
z'_2	number of teeth of the pinion on the second reduction wheel	1

EXAMPLE The rotor pinion has 24 teeth. The first reduction wheel has 66 teeth and a pinion with 8, the second 64 teeth and a pinion with 10, the ratchet wheel 70 teeth.

Given: $z_1 = 66;\quad z_2 = 64;\quad z_{\mathrm{Sp}} = 70;\quad z'_{\mathrm{R}} = 24;\quad z'_1 = 8;\quad z'_2 = 10$

$$\begin{aligned} i_{\mathrm{R}} &= \frac{z_1 \cdot z_2 \cdot z_{\mathrm{Sp}}}{z'_{\mathrm{R}} \cdot z'_1 \cdot z'_2} \\ &= \frac{66 \cdot 64 \cdot 70}{24 \cdot 8 \cdot 10} \\ &= 154 \end{aligned}$$

The rotor turns 154 times until the ratchet wheel has made one revolution and tensioned the spring by one turn.

Rotor Revolutions for a Power Reserve

(3.32) MASTER

$$N_{\mathrm{R}} = \frac{i_{\mathrm{R}} \cdot t_{\mathrm{R}}}{t_{\mathrm{F}}}$$

N_{R}	revolutions of the rotor (in the winding direction)	1
i_{R}	reduction ratio of automatic winding (rotor revolutions per turn of the ratchet wheel)	1
t_{R}	power reserve (to be built up)	h
t_{F}	rotation period of the barrel (time for one turn)	h

One turn of the ratchet wheel costs i_{R} rotor revolutions and gives as many hours of running as the barrel needs for one turn. With 24 hours the formula gives the daily requirement: this is how often the rotor must turn per day for the movement to maintain its state of wind.

Valid: minimum value without losses: the idle travel of the reverser on change of direction is not included, the slipping bridle does not slip. With unidirectional winding only the revolutions in the winding direction count.

Good to know: When worn, more revolutions usually accumulate than necessary; the slipping bridle lets the excess slip (Section 3.3). If an automatic watch stops despite being worn, either the movement is insufficient for the daily requirement, or the winding loses travel, for example through gummed reverser wheels.

EXAMPLE The movement from the previous example has a barrel that turns once in 7 hours. How many rotor revolutions does it need for 24 hours of running?

Given: $i_{\mathrm{R}} = 154$; $t_{\mathrm{R}} = 24\,\mathrm{h}$; $t_{\mathrm{F}} = 7\,\mathrm{h}$

$$\begin{aligned} N_{\mathrm{R}} &= \frac{i_{\mathrm{R}} \cdot t_{\mathrm{R}}}{t_{\mathrm{F}}} \\ &= \frac{154 \cdot 24\,\mathrm{h}}{7\,\mathrm{h}} \\ &= 528 \end{aligned}$$

528 revolutions per day, i.e. 22 per hour of running. For 42 hours of power reserve from standstill it is 924 revolutions.

Hand Setting: Travel of the Minute Hand

(3.33) JOURNEYMAN

$$N_{\mathrm{M}} = \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{K}}}{z_{\mathrm{V}}}$$

N_{M}	revolutions of the minute hand	1
N_{K}	revolutions of the crown	1
z'_{K}	number of teeth of the sliding pinion (toothing for hand setting)	1
z_{V}	number of teeth of the cannon pinion	1

In the hand-setting position the sliding pinion meshes with the setting wheel. This drives the cannon pinion with the minute hand via the minute wheel. Setting wheel and minute wheel are idlers here. One revolution of the minute hand is 60 minutes of hand travel.

Valid: setting wheel, further idlers and minute wheel each mesh with their two neighbours using the same toothing. The hour hand follows via the motion work in the ratio 12 to 1 (Section 4.3).

EXAMPLE The sliding pinion has 9 teeth for hand setting, the cannon pinion 12. How far does the minute hand advance for one revolution of the crown?

Given: $N_{\mathrm{K}} = 1$; $z'_{\mathrm{K}} = 9$; $z_{\mathrm{V}} = 12$

$$\begin{aligned} N_{\mathrm{M}} &= \frac{N_{\mathrm{K}} \cdot z'_{\mathrm{K}}}{z_{\mathrm{V}}} \\ &= \frac{1 \cdot 9}{12} \\ &= 0.75 \end{aligned}$$

By three quarters of a revolution, i.e. 45 minutes of hand travel per revolution of the crown.

Key point: Crown wheel, setting wheel and minute wheel are only idlers here. What counts are the first driving and the last driven wheel.

4 Gear Train and Toothing

The gear train transmits the force of the drive to the escapement and in doing so counts the oscillations of the oscillator: its ratios determine how fast each arbor turns and which beat rate belongs to a movement. This is why calculating an unknown movement starts with counting teeth.

Sections 4.1 to 4.3 calculate with numbers of teeth, Section 4.4 with the dimensions of the toothing. Striking work, calendar and moon phase follow the same rules; friction in the gear train concludes the chapter. Depending on the formula, the symbol k stands for the addendum coefficient or for a number; the table of quantities of the formula is authoritative.

4.1 Gear Ratio

A wheel drives a pinion: this is the basic element of every gear train. Pinions are small gear wheels with usually 6 to 16 leaves; they are fixed on the arbor of the next wheel.

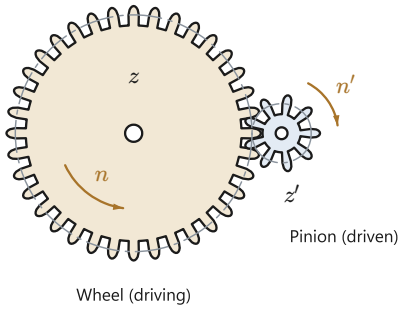


Fig. 4.1 Wheel drives pinion: the pinion turns faster by the gear ratio and in the opposite direction

Gear Ratio of One Mesh

(4.1) JOURNEYMAN

$$i = \frac{z}{z'}$$

The gear ratio states how often the pinion turns while the wheel makes one revolution. In this chapter z is the number of teeth of a wheel and z' that of a pinion; an index names the arbor.

Valid: wheel drives pinion, as in the going train and the striking work: the speed is stepped up. Where a pinion drives a wheel (motion work, winding), the speed is stepped down; the calculation then uses the reduction ratio, the number of teeth of the driven wheel divided by that of the driving pinion, formula (4.11).

i	gear ratio (greater than 1 means: stepping up in speed)	1
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The centre wheel has 80 teeth and drives the third pinion with 10 leaves.

Given: $z = 80$; $z' = 10$

$$\begin{aligned} i &= \frac{z}{z'} \\ &= \frac{80}{10} \\ &= 8 \end{aligned}$$

The third pinion turns eight times while the centre wheel makes one revolution.

Rotational Speed of the Pinion

(4.2) JOURNEYMAN

$$n' = \frac{n \cdot z}{z'}$$

In the mesh, the same number of teeth pass on both sides: $z \cdot n = z' \cdot n'$. The pinion therefore turns faster than the wheel by the gear ratio.

n'	rotational speed of the driven pinion	$\frac{1}{\mathrm{h}}$
n	rotational speed (of the driving wheel)	$\frac{1}{\mathrm{h}}$
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The centre wheel (80 teeth) turns once per hour. How fast does the third pinion with 10 leaves turn?

Given: $n = 1\,\frac{1}{\mathrm{h}}$; $z = 80$; $z' = 10$

$$n' = \frac{n \cdot z}{z'} = \frac{1\,\frac{1}{\mathrm{h}} \cdot 80}{10} = 8\,\frac{1}{\mathrm{h}}$$

Pinion and third wheel turn eight times per hour; one revolution takes 7.5 min.

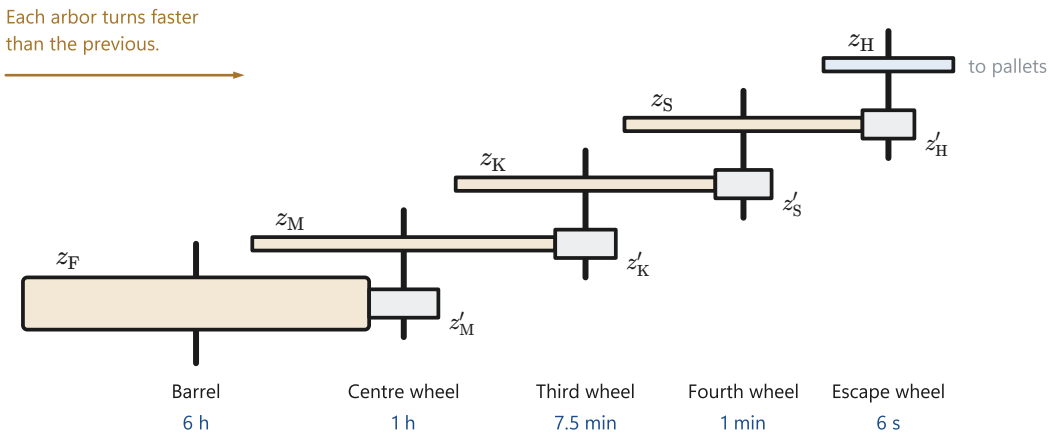


Fig. 4.2 Gear train of a watch, developed in side view. Below, the rotation periods for the tooth counts 72 : 12, 80 : 10, 75 : 10 and 80 : 8

Overall Gear Ratio

(4.3) JOURNEYMAN

$$i_{\text{ges}} = \frac{z_1 \cdot z_2 \cdot z_3}{z'_1 \cdot z'_2 \cdot z'_3}$$

If several meshes follow each other, the individual gear ratios are multiplied: product of the numbers of teeth of all driving wheels divided by the product of all driven pinions. With more or fewer meshes the fraction becomes correspondingly longer or shorter.

i_{ges}	overall gear ratio (from the first to the last arbor)	1
z_1	number of teeth of the first driving wheel	1
z_2	number of teeth of the second driving wheel	1
z_3	number of teeth of the third driving wheel	1
z'_1	number of leaves of the first driven pinion	1
z'_2	number of leaves of the second driven pinion	1
z'_3	number of leaves of the third driven pinion	1

Good to know: An idler that is driven and drives on with the same teeth appears once in the numerator and once in the denominator. It does not change the gear ratio, only the direction of rotation.

EXAMPLE A pocket watch movement has centre wheel 80, third wheel 75 and fourth wheel 80 teeth; third and fourth pinions have 10 leaves each, the escape pinion 8 leaves.

Given: $z_1 = 80;\quad z_2 = 75;\quad z_3 = 80;\quad z'_1 = 10;\quad z'_2 = 10;\quad z'_3 = 8$

$$\begin{aligned} i_{\text{ges}} &= \frac{z_1 \cdot z_2 \cdot z_3}{z'_1 \cdot z'_2 \cdot z'_3} \\ &= \frac{80 \cdot 75 \cdot 80}{10 \cdot 10 \cdot 8} \\ &= 600 \end{aligned}$$

The escape wheel turns 600 times while the centre wheel makes one revolution. The individual gear ratios are 8, 7.5 and 10.

Rotation Period

(4.4) JOURNEYMAN

$$t'_{\text{U}} = \frac{t_{\text{U}} \cdot z'}{z}$$

The rotation period is the reciprocal of the rotational speed. From arbor to arbor it becomes shorter by the gear ratio. In Fig. 4.2 the rotation periods are shown below the arbors.

t'_{U}	rotation period of the pinion (time for one revolution)	s
t_{U}	rotation period of the driving wheel	s
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE The fourth wheel (80 teeth) turns once in 60 s and drives the escape pinion with 8 leaves.

Given: $t_{\text{U}} = 60\,\text{s};\quad z' = 8;\quad z = 80$

$$\begin{aligned} t'_{\text{U}} &= \frac{t_{\text{U}} \cdot z'}{z} \\ &= \frac{60\,\text{s} \cdot 8}{80} \\ &= 6\,\text{s} \end{aligned}$$

The escape wheel needs 6 s for one revolution.

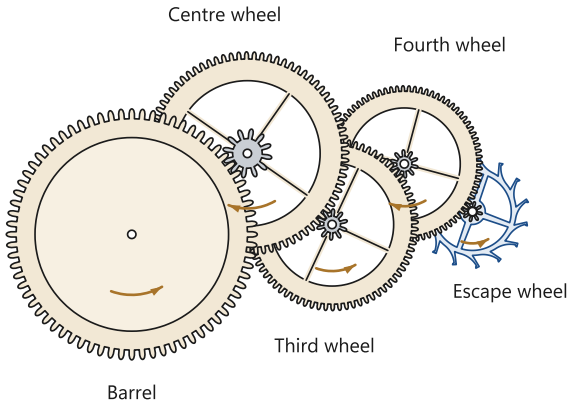


Fig. 4.3 The same gear train in plan view: each arbor turns opposite to the previous one

Key point: Each mesh reverses the direction of rotation. After an even number of meshes the last arbor turns like the first, after an odd number in the opposite direction.

Torque at the Pinion

(4.5) JOURNEYMAN

$$M' = \frac{M \cdot \eta \cdot z'}{z}$$

What the gear train gains in rotational speed it loses in torque. Friction at the tooth flanks and in the bearings takes away an additional part.

Valid: uniform rotation. The torque fluctuates somewhat during the engagement of one tooth; the formula gives the mean value.

M'	torque at the arbor of the driven pinion	$\mathrm{N{\cdot}mm}$
M	torque (at the arbor of the driving wheel)	$\mathrm{N{\cdot}mm}$
η	efficiency (per mesh; guide value 0.90 to 0.95; without friction 1)	1
z	number of teeth (of the driving wheel)	1
z'	number of leaves of the pinion (driven)	1

EXAMPLE A barrel with 72 teeth delivers 8 N·mm and drives the centre pinion with 12 leaves. The mesh has an efficiency of 0.95.

Given: $M = 8\,\mathrm{N{\cdot}mm}$; $\eta = 0.95$; $z' = 12$; $z = 72$

$$\begin{aligned} M' &= \frac{M \cdot \eta \cdot z'}{z} \\ &= \frac{8\,\mathrm{N{\cdot}mm} \cdot 0.95 \cdot 12}{72} \\ &\approx 1.27\,\mathrm{N{\cdot}mm} \end{aligned}$$

1.27 N·mm arrive at the centre arbor. Without friction it would be 1.33 N·mm.

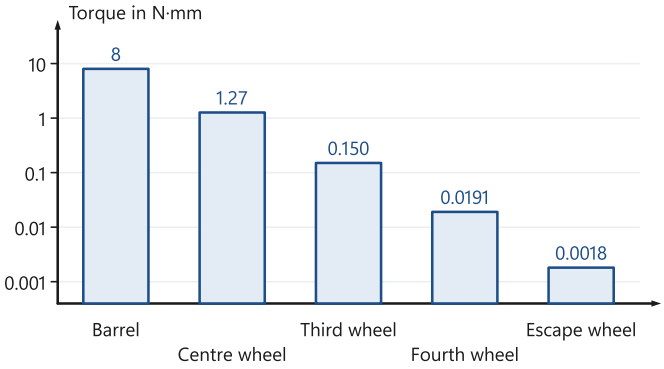


Fig. 4.4 Torque at the arbors of the gear train from Fig. 4.2 with 8 N·mm at the barrel and $\eta = 0.95$ per mesh

4.2 Calculating the Going Train

The going train connects the centre wheel, which turns once per hour, with the escape wheel. Its gear ratio and the number of teeth of the escape wheel determine how many vibrations per hour the oscillator must make for the hands to run correctly.

These formulas are used to determine the beat rate of an unknown movement, the length of a missing pendulum (Chapter 6) and the number of teeth of a missing wheel or pinion.

Beat Rate from the Gear Train

(4.6) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot z_\mathrm{H} \cdot i \cdot n_\mathrm{M}$$

Each tooth of the escape wheel makes the oscillator perform two vibrations: one at the entry, one at the exit. Per revolution of the escape wheel these are $2 \cdot z_\mathrm{H}$ beats.

Valid: all common escapements; i according to formula (4.3). The centre wheel turns once per hour; if the calculation starts from another arbor, its rotational speed is inserted.

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_H	number of teeth of the escape wheel	1
i	gear ratio (from the centre wheel to the escape wheel)	1
n_M	rotational speed of the centre wheel (one revolution per hour)	$\frac{1}{\mathrm{h}}$

EXAMPLE A regulator has a centre wheel with 80 and an intermediate wheel with 75 teeth, two pinions with 10 leaves each and an escape wheel with 30 teeth. The gear ratio from the centre wheel to the escape wheel is $8 \cdot 7.5 = 60$.

Given: $z_\mathrm{H} = 30; \quad i = 60; \quad n_\mathrm{M} = 1\ \frac{1}{\mathrm{h}}$

$$\begin{aligned} n_\mathrm{h} &= 2 \cdot z_\mathrm{H} \cdot i \cdot n_\mathrm{M} \\ &= 2 \cdot 30 \cdot 60 \cdot 1\ \tfrac{1}{\mathrm{h}} \\ &= 3600\ \tfrac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

3600 beats per hour are 60 per minute. This movement requires a seconds pendulum 994 mm long (Chapter 6).

Beat Rate of a Watch from the Numbers of Teeth

(4.7) JOURNEYMAN

$$n_{\mathrm{h}} = \frac{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{M}} \cdot z_{\mathrm{M}} \cdot z_{\mathrm{K}} \cdot z_{\mathrm{S}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}} \cdot z'_{\mathrm{H}}}$$

The numerator contains the driving wheels, the denominator the driven pinions. Each pinion carries the index of the arbor on which it sits (Fig. 4.2).

Valid: gear train with three meshes between centre wheel and escape wheel. For a different design, adapt the fraction according to formula (4.3).

n_{h}	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_{H}	number of teeth of the escape wheel	1
n_{M}	rotational speed of the centre wheel (one revolution per hour)	$\frac{1}{\mathrm{h}}$
z_{M}	number of teeth of the centre wheel	1
z_{K}	number of teeth of the third wheel	1
z_{S}	number of teeth of the fourth wheel	1
z'_{K}	number of leaves of the third pinion	1
z'_{S}	number of leaves of the fourth pinion	1
z'_{H}	number of leaves of the escape pinion	1

Good to know: If a wheel or pinion is missing, the formula is rearranged for its number of teeth. The result must be a whole number; otherwise a number of teeth has been miscounted or the beat rate wrongly assumed.

EXAMPLE Counted were: centre wheel 80, third wheel 75, fourth wheel 84, escape wheel 20 teeth; third pinion 10, fourth pinion 10, escape pinion 7 leaves.

Given: $z_{\mathrm{H}} = 20$; $n_{\mathrm{M}} = 1\,\frac{1}{\mathrm{h}}$; $z_{\mathrm{M}} = 80$; $z_{\mathrm{K}} = 75$; $z_{\mathrm{S}} = 84$; $z'_{\mathrm{K}} = 10$; $z'_{\mathrm{S}} = 10$; $z'_{\mathrm{H}} = 7$

$$\begin{aligned} n_{\mathrm{h}} &= \frac{2 \cdot z_{\mathrm{H}} \cdot n_{\mathrm{M}} \cdot z_{\mathrm{M}} \cdot z_{\mathrm{K}} \cdot z_{\mathrm{S}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}} \cdot z'_{\mathrm{H}}} \\ &= \frac{2 \cdot 20 \cdot 1\,\frac{1}{\mathrm{h}} \cdot 80 \cdot 75 \cdot 84}{10 \cdot 10 \cdot 7} \\ &= 28\,800\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

The movement beats 28 800 vibrations per hour; the balance oscillates at 4 Hz.

Rotational Speed of the Escape Wheel

(4.8) JOURNEYMAN

$$n_{\mathrm{H}} = \frac{n_{\mathrm{h}}}{2 \cdot z_{\mathrm{H}}}$$

This gives the rotational speed that the gear train must deliver at the escape wheel. More teeth on the escape wheel means the gear train can run more slowly.

n_{H}	rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$
n_{h}	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_{H}	number of teeth of the escape wheel	1

EXAMPLE How fast does an escape wheel with 20 teeth turn at 28 800 A/h?

Given: $n_{\mathrm{h}} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$; $z_{\mathrm{H}} = 20$

$$\begin{aligned} n_{\mathrm{H}} &= \frac{n_{\mathrm{h}}}{2 \cdot z_{\mathrm{H}}} \\ &= \frac{28\,800\ \frac{\mathrm{A}}{\mathrm{h}}}{2 \cdot 20} \\ &= 12\ \frac{1}{\mathrm{min}} \end{aligned}$$

Twelve revolutions per minute; one revolution takes 5 s. An escape wheel with 15 teeth would need 16 revolutions per minute at the same beat rate.

Watches up to 21 600 A/h almost always have an escape wheel with 15 teeth, movements with 28 800 A/h and more often one with 20 teeth. For clocks the range extends from about 15 to 40 teeth. If the seconds hand sits on the arbor of the escape wheel and the pendulum beats seconds, the escape wheel has 30 teeth.

Condition for the Fourth Wheel

(4.9) JOURNEYMAN

$$\frac{z_{\mathrm{M}} \cdot z_{\mathrm{K}}}{z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}} = 60$$

REARRANGED $z_{\mathrm{K}} = \frac{60 \cdot z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}}{z_{\mathrm{M}}}$

If the arbor of the fourth wheel carries the seconds hand, it must turn 60 times as fast as the centre wheel. The usual division is $8 \cdot 7.5$.

Valid: movements whose fourth wheel turns once per minute (small seconds or central seconds wheel).

z_{M}	number of teeth of the centre wheel	1
z_{K}	number of teeth of the third wheel	1
z'_{K}	number of leaves of the third pinion	1
z'_{S}	number of leaves of the fourth pinion	1

EXAMPLE In a movement the third wheel is missing. The centre wheel has 80 teeth, the third and fourth pinions have 10 leaves each.

Given: $z'_{\mathrm{K}} = 10$; $z'_{\mathrm{S}} = 10$; $z_{\mathrm{M}} = 80$

$$\begin{aligned} z_{\mathrm{K}} &= \frac{60 \cdot z'_{\mathrm{K}} \cdot z'_{\mathrm{S}}}{z_{\mathrm{M}}} \\ &= \frac{60 \cdot 10 \cdot 10}{80} \\ &= 75 \end{aligned}$$

The missing third wheel has 75 teeth. The module is given by the centre distance, formula (4.16).

Beat Rate from the Last Mesh

(4.10) JOURNEYMAN

$$n_\mathrm{h} = \frac{2 \cdot z_\mathrm{H} \cdot n_\mathrm{S} \cdot z_\mathrm{S}}{z'_\mathrm{H}}$$

REARRANGED $$z_\mathrm{S} = \frac{n_\mathrm{h} \cdot z'_\mathrm{H}}{2 \cdot z_\mathrm{H} \cdot n_\mathrm{S}}$$

If the fourth wheel turns once per minute, the last mesh is sufficient to check the beat rate or to calculate a missing fourth wheel.

Valid: fourth wheel turns once per minute.

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z_H	number of teeth of the escape wheel	1
n_S	rotational speed of the fourth wheel (one revolution per minute)	$\frac{1}{\mathrm{min}}$
z_S	number of teeth of the fourth wheel	1
z'_H	number of leaves of the escape pinion	1

EXAMPLE A movement with 28 800 A/h has an escape wheel with 20 teeth and an escape pinion with 7 leaves. The fourth wheel is missing.

Given: $n_\mathrm{h} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$; $z'_\mathrm{H} = 7$; $z_\mathrm{H} = 20$; $n_\mathrm{S} = 1\ \frac{1}{\mathrm{min}}$

$$z_\mathrm{S} = \frac{n_\mathrm{h} \cdot z'_\mathrm{H}}{2 \cdot z_\mathrm{H} \cdot n_\mathrm{S}} = \frac{28\,800\ \frac{\mathrm{A}}{\mathrm{h}} \cdot 7}{2 \cdot 20 \cdot 1\ \frac{1}{\mathrm{min}}} = 84$$

The fourth wheel must have 84 teeth.

Tooth-Count Sets for Watches (Calculated Examples, Fourth Wheel Turns Once per Minute)

Beat rate in A/h	Centre wheel : third pinion	Third wheel : fourth pinion	Fourth wheel : escape pinion	Escape wheel	Gear ratio
18 000	80 : 10	75 : 10	80 : 8	15	600
18 000	64 : 8	60 : 8	70 : 7	15	600
21 600	80 : 10	75 : 10	72 : 6	15	720
21 600	64 : 8	60 : 8	84 : 7	15	720
28 800	80 : 10	75 : 10	84 : 7	20	720
36 000	80 : 10	75 : 10	90 : 6	20	900

The sets are calculated with formula (4.7) and show how the beat rate arises from the numbers of teeth. Actual calibres differ from them; the counted teeth are always authoritative.

Key point: Beat rate equals twice the escape-wheel teeth times the gear ratio from centre wheel to escape wheel. Driving wheels in the numerator, driven pinions in the denominator.

4.3 Motion Work

The motion work sits under the dial and derives the rotation of the hour hand from that of the minute hand. The cannon pinion sits with friction on the arbor of the centre wheel so that the hands can be set.

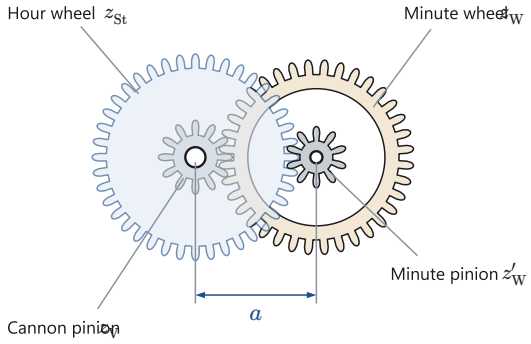


Fig. 4.5 Motion work in plan view: both meshes have the same centre distance a

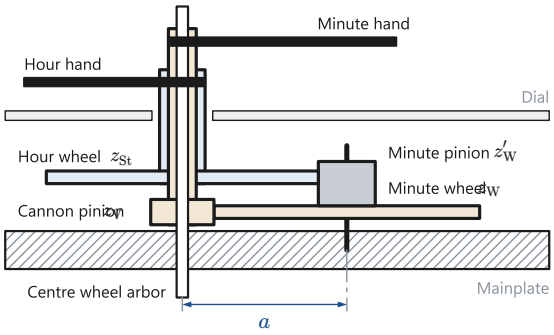


Fig. 4.6 Motion work in section (schematic): the hour wheel turns loosely on the cannon pinion

Reduction Ratio of the Motion Work

(4.11) JOURNEYMAN

$$\frac{z_W \cdot z_{St}}{z_V \cdot z'_W} = 12$$

REARRANGED $$z_{St} = \frac{12 \cdot z_V \cdot z'_W}{z_W}$$

The cannon pinion carries the minute hand and turns once per hour, the hour wheel once in 12 hours. Because pinions drive here, the driven wheels are in the numerator: the motion work reduces 12 to 1.

Valid: display with 12 hours per revolution of the hour hand. After two meshes the hour wheel turns in the same direction as the cannon pinion.

z_W	number of teeth of the minute wheel	1
z_{St}	number of teeth of the hour wheel	1
z_V	number of leaves of the cannon pinion (drives the minute wheel)	1
z'_W	number of leaves of the minute pinion (drives the hour wheel)	1

EXAMPLE The hour wheel of a motion work is missing. The cannon pinion has 14 leaves, the minute wheel 42 teeth, the minute pinion 12 leaves.

Given: $z_V = 14$; $z'_W = 12$; $z_W = 42$

$$\begin{aligned} z_{St} &= \frac{12 \cdot z_V \cdot z'_W}{z_W} \\ &= \frac{12 \cdot 14 \cdot 12}{42} \\ &= 48 \end{aligned}$$

The hour wheel has 48 teeth. The reduction is divided into 3 at the first and 4 at the second mesh.

24-Hour Display

(4.12) MASTER

$$z_{24} = 2 \cdot z_{A}$$

The 24-hour hand turns half as fast as the hour hand. An idler does not change the reduction ratio but makes the hand run clockwise. The same 2 to 1 reduction drives the date driving wheel, which advances once a day (Section 4.6).

Valid: drive from the hour wheel. Calculated from the cannon pinion, the reduction ratio is 24 to 1.

z_{24}	number of teeth of the 24-hour wheel	1
z_{A}	number of teeth of the driving wheel on the hour-wheel pipe	1

EXAMPLE A wheel with 24 teeth sits on the hour-wheel pipe and drives the 24-hour wheel via an idler.

Given: $z_{A} = 24$

$$\begin{aligned} z_{24} &= 2 \cdot z_{A} \\ &= 2 \cdot 24 \\ &= 48 \end{aligned}$$

The 24-hour wheel needs 48 teeth. The number of teeth of the idler can be chosen freely.

Equal Centre Distance in the Motion Work

(4.13) MASTER

$$m_1 \cdot (z_{V} + z_{W}) = m_2 \cdot (z'_{W} + z_{St})$$

REARRANGED $$m_2 = \frac{m_1 \cdot (z_{V} + z_{W})}{z'_{W} + z_{St}}$$

Cannon pinion and hour wheel turn about the same axis, and so do minute wheel and minute pinion. Both meshes therefore have the same centre distance according to formula (4.16). If the sums of the numbers of teeth differ, the modules must differ too.

m_1	module of the cannon pinion and minute wheel mesh	mm
z_{V}	number of leaves of the cannon pinion (drives the minute wheel)	1
z_{W}	number of teeth of the minute wheel	1
m_2	module of the minute pinion and hour wheel mesh	mm
z'_{W}	number of leaves of the minute pinion (drives the hour wheel)	1
z_{St}	number of teeth of the hour wheel	1

EXAMPLE A motion work has cannon pinion 12, minute wheel 36, minute pinion 10 and hour wheel 40 teeth. The first mesh has a module of 0.25 mm.

Given: $m_1 = 0.25\,\mathrm{mm}$; $z_{V} = 12$; $z_{W} = 36$; $z'_{W} = 10$; $z_{St} = 40$

$$\begin{aligned} m_2 &= \frac{m_1 \cdot (z_{V} + z_{W})}{z'_{W} + z_{St}} \\ &= \frac{0.25\,\mathrm{mm} \cdot (12 + 36)}{10 + 40} \\ &= 0.24\,\mathrm{mm} \end{aligned}$$

The second mesh needs a module of 0.24 mm. With the tooth counts 10 and 30 as well as 8 and 32, both meshes could use the same module.

4.4 Tooth Geometry

All dimensions of a toothing depend on the module. Module and number of teeth fix pitch circle, centre distance and tooth size; the tooth form then determines tip and root circles.

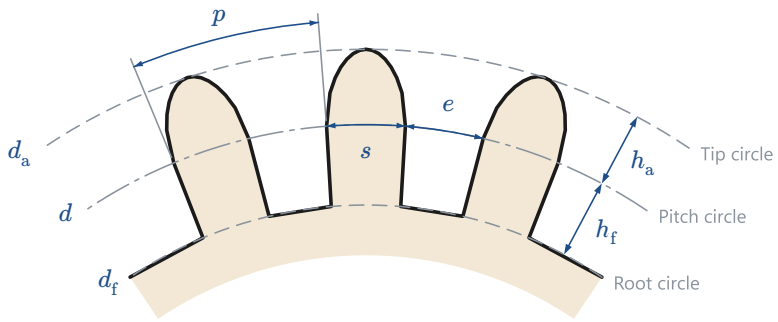


Fig. 4.7 Dimensions on the gear wheel: tip circle d_a, pitch circle d, root circle d_f, pitch p, tooth thickness s, space e, addendum h_a and dedendum h_f

Pitch and Module

(4.14) JOURNEYMAN

$$p = \pi \cdot m$$

REARRANGED $m = \frac{p}{\pi}$

The module is the pitch divided by π. It is the measure of tooth size: only wheel and pinion with the same module fit together.

p	circular pitch (distance from tooth to tooth, as an arc on the pitch circle)	mm
m	module	mm

EXAMPLE What is the pitch for module 0.2 mm?

Given: $m = 0.2\,\mathrm{mm}$

$$\begin{aligned} p &= \pi \cdot m \\ &= \pi \cdot 0.2\,\mathrm{mm} \\ &\approx 0.628\,\mathrm{mm} \end{aligned}$$

From tooth to tooth it is 0.628 mm on the pitch circle; tooth and space are therefore each about 0.3 mm wide.

Pitch Circle Diameter

(4.15) JOURNEYMAN

$$d = m \cdot z$$

REARRANGED $m = \frac{d}{z}$

The pitch circle is the imaginary circle on which wheel and pinion roll on each other like two smooth friction wheels.

d	pitch circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE A wheel has 80 teeth and a module of 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}$; $z = 80$

$$\begin{aligned} d &= m \cdot z \\ &= 0.2\,\mathrm{mm} \cdot 80 \\ &= 16\,\mathrm{mm} \end{aligned}$$

The pitch circle measures 16 mm. It cannot be seen on the finished wheel; only the tip circle can be measured.

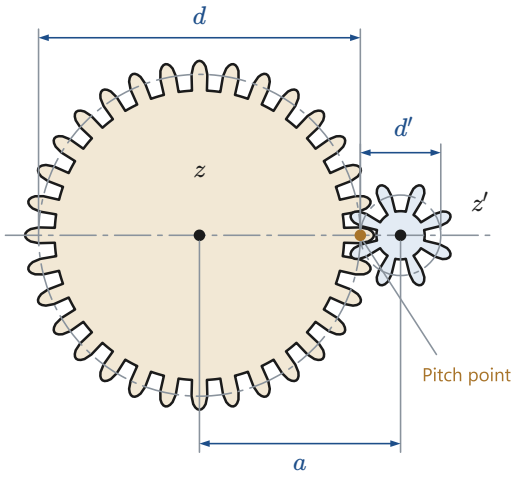


Fig. 4.8 Mesh: the pitch circles d and d' touch at the pitch point

Centre Distance

(4.16) JOURNEYMAN

$$a = \frac{m \cdot (z + z')}{2}$$

REARRANGED $m = \dfrac{2 \cdot a}{z + z'}$

a	centre distance	mm
m	module	mm
z	number of teeth (of the wheel)	1
z'	number of leaves of the pinion	1

The pitch circles touch at the pitch point. The centre distance is the sum of the two pitch circle radii. Rearranged, the formula gives the module from the measured centre distance.

Valid: external toothing with correctly set depth of engagement.

EXAMPLE A centre distance of 8.5 mm is measured between two bearings. The wheel has 75 teeth, the pinion 10.

Given: $a = 8.5\,\mathrm{mm}; \quad z = 75; \quad z' = 10$

$$\begin{aligned} m &= \frac{2 \cdot a}{z + z'} \\ &= \frac{2 \cdot 8.5\,\mathrm{mm}}{75 + 10} \\ &= 0.2\,\mathrm{mm} \end{aligned}$$

The mesh has a module of 0.2 mm.

Pitch Circle from the Centre Distance

(4.17) MASTER

$$d = \frac{2 \cdot a \cdot z}{z + z'}$$

The centre distance is divided between wheel and pinion in the ratio of the numbers of teeth. For the pinion, z' is in the numerator.

d	pitch circle diameter (of the wheel)	mm
a	centre distance	mm
z	number of teeth (of the wheel)	1
z'	number of leaves of the pinion	1

EXAMPLE Centre distance 8.5 mm, wheel 75 teeth, pinion 10 leaves.

Given: $a = 8.5\,\mathrm{mm}; \quad z = 75; \quad z' = 10$

$$\begin{aligned} d &= \frac{2 \cdot a \cdot z}{z + z'} \\ &= \frac{2 \cdot 8.5\,\mathrm{mm} \cdot 75}{75 + 10} \\ &= 15\,\mathrm{mm} \end{aligned}$$

The pitch circle of the wheel measures 15 mm, leaving 2 mm for the pinion.

Horological toothing. In the going train the wheel drives the pinion, with the most uniform force possible and little friction. For this the cycloidal toothing has become established, today usually executed as circular-arc toothing according to the standards of the watch industry: above the pitch circle the wheel tooth carries an ogival addendum, the pinion leaf has straight radial flanks and a round or ogival tip. The addenda depend on the numbers of teeth. The following formulas combine them in an allowance added to the number of teeth.

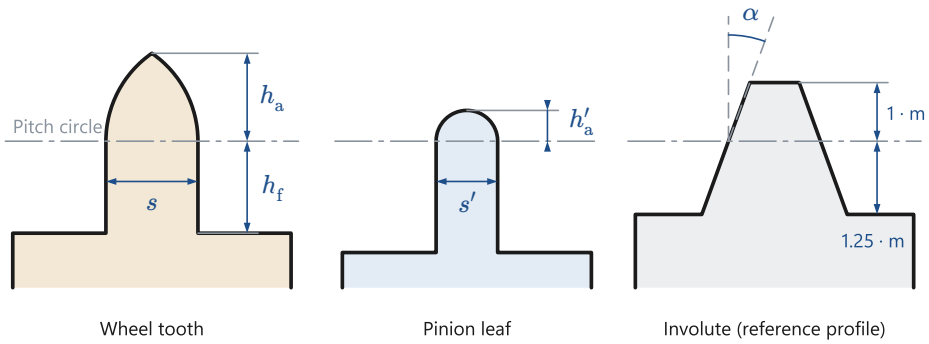


Fig. 4.9 Tooth forms drawn on a straight pitch line: wheel tooth with ogival addendum, pinion leaf with semicircular tip, basic rack profile of involute toothing with $\alpha = 20°$

Tip Circle of the Wheel

(4.18) APPROXIMATION JOURNEYMAN

$$d_\mathrm{a} \approx m \cdot (z + k)$$

The tip circle is the outside diameter of the wheel and the only dimension that is easy to measure. The addendum is $h_\mathrm{a} = \frac{k}{2} \cdot m$.

Valid: horological toothing, wheel drives pinion. k from the table; without a table use $k \approx 3$, in course exercises also $k \approx \pi$.

d_a	tip circle diameter (of the wheel)	mm
m	module	mm
z	number of teeth (of the wheel)	1
k	addendum coefficient (of the wheel: twice the addendum, measured in modules)	1

EXAMPLE A wheel with 80 teeth and module 0.2 mm meshes with a pinion with 10 leaves; according to the table $k = 3.19$.

Given: $m = 0.2\,\mathrm{mm}$; $z = 80$; $k = 3.19$

$$\begin{aligned} d_\mathrm{a} &\approx m \cdot (z + k) \\ &= 0.2\,\mathrm{mm} \cdot (80 + 3.19) \\ &\approx 16.64\,\mathrm{mm} \end{aligned}$$

The wheel is turned to a tip circle of 16.64 mm; the pitch circle lies 0.32 mm below the tooth tips.

Addendum Coefficient k of the Wheel at a Glance (Rounded, for Gear Ratios from 6 to 10)

Number of leaves of the pinion	6	7	8	10	12
Addendum coefficient k	2.6	2.8	2.9	3.2	3.4

The more leaves the pinion has, the higher the ogival addendum of the wheel tooth. The exact values also depend on the number of teeth of the wheel; they are given in the following table.

Addendum Coefficient k of the Wheel by Numbers of Teeth of Wheel and Pinion (Table Values for Horological Cycloidal Toothing)

Number of teeth z of the wheel	Pinion $z' = 6$	$z' = 7$	$z' = 8$	$z' = 9$	$z' = 10$	$z' = 12$	$z' = 15$
20	2.53	2.65	2.76	2.86	2.95	3.10	3.24
24	2.56	2.68	2.79	2.90	2.99	3.14	3.31
25	2.57	2.69	2.80	2.91	3.00	3.15	3.33
28	2.57	2.71	2.82	2.93	3.03	3.18	3.38
30	2.58	2.71	2.83	2.94	3.04	3.20	3.40
32	2.59	2.72	2.84	2.95	3.06	3.22	3.43
35	2.60	2.73	2.85	2.96	3.07	3.25	3.46
36	2.60	2.73	2.85	2.96	3.08	3.26	3.47
40	2.60	2.74	2.86	2.98	3.10	3.29	3.51
42	2.61	2.74	2.87	2.98	3.10	3.30	3.53
45	2.61	2.74	2.88	3.00	3.12	3.32	3.55
48	2.62	2.75	2.88	3.00	3.12	3.33	3.56
50	2.62	2.75	2.89	3.01	3.12	3.33	3.57
54	2.62	2.75	2.89	3.01	3.13	3.35	3.59
56	2.62	2.76	2.90	3.02	3.13	3.35	3.60
60	2.63	2.76	2.90	3.02	3.14	3.36	3.61
64	2.63	2.77	2.91	3.03	3.15	3.37	3.62
70	2.63	2.77	2.92	3.05	3.17	3.39	3.64
72	2.64	2.78	2.92	3.05	3.17	3.39	3.64
75	2.64	2.78	2.93	3.06	3.18	3.40	3.65
80	2.64	2.79	2.93	3.07	3.19	3.41	3.66
84	2.64	2.79	2.93	3.07	3.20	3.42	3.67
90	2.65	2.80	2.94	3.08	3.21	3.44	3.69
96	2.65	2.80	2.94	3.09	3.22	3.45	3.70
100	2.66	2.81	2.95	3.10	3.23	3.46	3.71
110	2.66	2.81	2.96	3.11	3.24	3.47	3.73
120	2.67	2.82	2.97	3.12	3.25	3.49	3.75
140	2.68	2.84	2.99	3.13	3.27	3.51	3.78
150	2.69	2.84	3.00	3.14	3.28	3.52	3.80
160	2.69	2.85	3.01	3.15	3.29	3.52	3.81
180	2.70	2.86	3.01	3.15	3.30	3.53	3.83
200	2.71	2.87	3.02	3.16	3.31	3.55	3.85

Course and examination exercises calculate with these values. For numbers of teeth between the rows, take the nearest row; from row to row k changes by only a few hundredths. The addendum of the wheel tooth is $h_{\mathrm{a}} = \frac{k}{2} \cdot m$. The values belong to the ogival tooth of cycloidal toothing: its flank is generated when a circle of half the diameter of the pinion pitch circle rolls on the outside of the wheel's pitch circle (epicycloid). Recalculated, the table values lie at about 95 % of the height at which the two flanks meet at the tip when the tooth is half a pitch thick on the pitch circle. For manufacture, the toothing standards of the watch industry (NHS, NIHS) apply. Depending on standard and profile, they give somewhat different values from this table, for wheel toothing by a few hundredths up to a few tenths; anyone making a wheel therefore calculates with the standard sheet.

Module from the Measured Tip Circle

(4.19) APPROXIMATION JOURNEYMAN

$$m \approx \frac{d_a}{z + k}$$

The usual route to a missing wheel or pinion: count the teeth on the existing mating part, measure the tip circle and calculate the module. Check the result against the centre distance according to formula (4.16).

Valid: as for formula (4.18). For an existing pinion, formula (4.20) applies accordingly.

m	module	mm
d_a	tip circle diameter (of the wheel)	mm
z	number of teeth (of the wheel)	1
k	addendum coefficient (of the wheel: twice the addendum, measured in modules)	1

EXAMPLE The pinion of a wheel is missing. The wheel has 64 teeth and a tip circle of 10.06 mm; it is to drive a pinion with 8 leaves, i.e. $k = 2.9$.

Given: $d_a = 10.06\,\text{mm}; \quad z = 64; \quad k = 2.9$

$$\begin{aligned} m &\approx \frac{d_a}{z + k} \\ &= \frac{10.06\,\text{mm}}{64 + 2.9} \\ &\approx 0.150\,\text{mm} \end{aligned}$$

The module is 0.15 mm. The missing pinion therefore has a pitch circle of 1.2 mm, the centre distance is 5.4 mm.

Tip Circle of the Pinion

(4.20) APPROXIMATION MASTER

$$d'_a \approx m \cdot (z' + k')$$

In the going train, the pinion tip works only briefly before the line of centres. It is therefore low; with a semicircular tip the addendum equals half the leaf thickness. In general the addendum is $h'_a = \frac{k'}{2} \cdot m$.

Valid: horological toothing, pinion is driven. k' from the two tables.

d'_a	tip circle diameter of the pinion	mm
m	module	mm
z'	number of leaves of the pinion	1
k'	addendum coefficient of the pinion (table)	1

EXAMPLE A pinion with 8 leaves and module 0.15 mm is given semicircular tips; from the leaf thickness $k' = 1.05$.

Given: $m = 0.15\,\text{mm}; \quad z' = 8; \quad k' = 1.05$

$$\begin{aligned} d'_a &\approx m \cdot (z' + k') \\ &= 0.15\,\text{mm} \cdot (8 + 1.05) \\ &\approx 1.36\,\text{mm} \end{aligned}$$

The pinion has an outside diameter of 1.36 mm with a pitch circle of 1.2 mm. With the rounded table value $k' = 1.0$ the result is 1.35 mm.

Pinion Leaf of Horological Toothing: Leaf Thickness and Resulting Addendum Coefficient (Guide Values)

Number of leaves of the pinion	Leaf thickness s'	Addendum coefficient k' for semicircular tip
6 to 10	about one third of the pitch, $1.05 \cdot m$	1.05
from 11	about two fifths of the pitch, $1.25 \cdot m$	1.25

With a semicircular tip the addendum is half the leaf thickness; these values follow from this. The boundary between 10 and 11 leaves follows the toothing standard of the watch industry.

Addendum Coefficient k' of the Pinion by Tip Form (Rounded Table Values of Course and Examination Exercises)

Tip form of the pinion leaf	Pinion with 6 to 10 leaves	Pinion from 11 leaves
semicircular (rounded)	1.0	1.2
ogival (pointed)	1.5	1.8

The values for the semicircular tip are the rounded values 1.05 and 1.25 from the table above. The difference changes the tip circle by only $0.05 \cdot m$, i.e. by 0.01 mm for module 0.2 mm. As in the toothing standard of the watch industry, the boundary lies between 10 and 11 leaves: up to 10 leaves the pinion leaf must stay thin so that the wheel tooth has room in the space. Some course materials already count the pinion with 10 leaves in the second group; in the examination the value given by the exercise or its table applies. Ogival tips are higher; depending on the profile they reach up to about $k' = 2$.

Root Circle of the Wheel

(4.21) APPROXIMATION MASTER

$$d_\mathrm{f} \approx m \cdot (z - \pi)$$

d_f	root circle diameter (of the wheel)	mm
m	module	mm
z	number of teeth (of the wheel)	1

The dedendum of the wheel tooth is about half the pitch, $h_\mathrm{f} \approx \frac{\pi}{2} \cdot m$. This lets the pinion tip pass freely through the space.

Valid: guide value for horological toothing. The root depth of the wheel is not critical as long as the pinion tip does not bottom.

EXAMPLE Wheel with 80 teeth, module 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}; \quad z = 80$

$$\begin{aligned} d_\mathrm{f} &\approx m \cdot (z - \pi) \\ &= 0.2\,\mathrm{mm} \cdot (80 - \pi) \\ &\approx 15.37\,\mathrm{mm} \end{aligned}$$

The tooth root lies on a diameter of 15.37 mm. The whole tooth depth is therefore about 0.63 mm.

Root Circle of the Pinion (Guide Value)

(4.22) APPROXIMATION JOURNEYMAN

$$d'_\mathrm{f} \approx m \cdot (z' - 4)$$

d'_f	root circle diameter of the pinion (core diameter)	mm
m	module	mm
z'	number of leaves of the pinion	1

The dedendum of the pinion leaf is about $2 \cdot m$. This is sufficient for every wheel addendum in the table: the highest measures $1.93 \cdot m$, the usual ones about $1.3 \cdot m$ to $1.7 \cdot m$. The remainder is tip clearance.

Valid: guide value of horological toothing for course and examination exercises. The following formula calculates more precisely with the addendum coefficient of the mating wheel.

EXAMPLE A pinion with 12 leaves has a module of 0.31 mm. What is its root circle?

Given: $m = 0.31\,\mathrm{mm}; \quad z' = 12$

$$\begin{aligned} d'_\mathrm{f} &\approx m \cdot (z' - 4) \\ &= 0.31\,\mathrm{mm} \cdot (12 - 4) \\ &= 2.48\,\mathrm{mm} \end{aligned}$$

2.48 mm with a pitch circle of 3.72 mm.

Root Circle of the Pinion

(4.23) ADVANCED

$$d'_{\mathrm{f}} = m \cdot (z' - k) - 2 \cdot c$$

The space of the pinion must take the tip of the wheel tooth and still leave clearance below it: dedendum of the pinion equals addendum of the wheel plus tip clearance.

d'_{f}	root circle diameter of the pinion (core diameter)	mm
m	module	mm
z'	number of leaves of the pinion	1
k	addendum coefficient (of the mating wheel)	1
c	tip clearance between wheel tip and pinion root (guide value $0.4 \cdot m$)	mm

EXAMPLE Pinion with 10 leaves, module 0.2 mm, mating wheel with $k = 3.2$, tip clearance 0.08 mm.

Given: $m = 0.2\,\mathrm{mm}$; $z' = 10$; $k = 3.2$; $c = 0.08\,\mathrm{mm}$

$$\begin{aligned} d'_{\mathrm{f}} &= m \cdot (z' - k) - 2 \cdot c \\ &= 0.2\,\mathrm{mm} \cdot (10 - 3.2) - 2 \cdot 0.08\,\mathrm{mm} \\ &= 1.20\,\mathrm{mm} \end{aligned}$$

The core of the pinion measures 1.2 mm with a pitch circle of 2 mm. Without tip clearance it would be 1.36 mm; at this dimension the wheel tooth would already bottom.

Tooth Thickness of the Wheel

(4.24) JOURNEYMAN

$$s = \frac{\pi \cdot m}{2}$$

Tooth and space share the pitch: $p = s + e$. On the wheel both are equal, so the tooth thickness is half the pitch.

Valid: wheel of horological toothing (guide value). Pinion leaves are thinner, see table at formula (4.20).

s	tooth thickness (of the wheel on the pitch circle)	mm
m	module	mm

EXAMPLE Module 0.2 mm.

Given: $m = 0.2\,\mathrm{mm}$

$$\begin{aligned} s &= \frac{\pi \cdot m}{2} \\ &= \frac{\pi \cdot 0.2\,\mathrm{mm}}{2} \\ &\approx 0.314\,\mathrm{mm} \end{aligned}$$

Tooth and space of the wheel are each 0.314 mm wide on the pitch circle.

Backlash

(4.25) MASTER

$$j = p - s - s'$$

The pinion leaf is narrower than the space of the wheel. The difference is the backlash. It leaves room for oil, dirt, pitch errors and runout.

Valid: measurement on the pitch circle at the correct centre distance.

j	backlash on the pitch circle	mm
p	circular pitch (distance from tooth to tooth, as an arc on the pitch circle)	mm
s	tooth thickness (of the wheel on the pitch circle)	mm
s'	leaf thickness of the pinion on the pitch circle	mm

EXAMPLE Module 0.2 mm: pitch 0.628 mm, tooth thickness of the wheel 0.314 mm, of the pinion with 8 leaves 0.209 mm.

Given: $p = 0.628\,\mathrm{mm}$; $s = 0.314\,\mathrm{mm}$; $s' = 0.209\,\mathrm{mm}$

$$\begin{aligned} j &= p - s - s' \\ &= 0.628\,\mathrm{mm} - 0.314\,\mathrm{mm} - 0.209\,\mathrm{mm} \\ &= 0.105\,\mathrm{mm} \end{aligned}$$

About 0.1 mm of clearance remains between the flanks, one sixth of the pitch.

Depthing. If the arbors are too close, the pitch circles intersect: the depthing is too deep, the teeth jam against each other, the movement runs heavily and in the worst case locks. If they are too far apart, the depthing is too shallow: the tooth tips lead, the force is transmitted unevenly and the mesh drops audibly. Checking is done with the depthing tool or by feeling the play between the teeth; correction is made via the centre distance or with a suitable pinion.

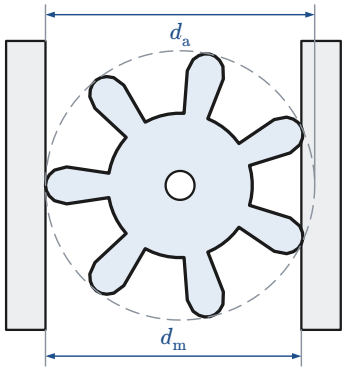


Fig. 4.10 Odd number of teeth: the measured dimension d_m is smaller than the tip circle d_a

Measuring the Tip Circle with an Odd Number of Teeth

(4.26) MASTER

$$d_a = \frac{d_m}{\cos^2\left(\frac{90^\circ}{z}\right)}$$

d_a	tip circle diameter	mm
d_m	measured dimension between the measuring faces	mm
z	number of teeth (odd)	1

With an odd number of teeth, each tooth is opposite a space. One measuring face rests on one tooth tip, the other on two. The measured dimension is therefore smaller than the tip circle.

Valid: odd number of teeth; measuring faces flat, parallel and wider than one pitch. With an even number of teeth the tip circle is measured directly.

EXAMPLE A pinion with 7 leaves measures 1.62 mm between the jaws of the calliper.

Given: $d_m = 1.62\,\text{mm}; \quad z = 7$

$$d_a = \frac{d_m}{\cos^2\left(\frac{90^\circ}{z}\right)} = \frac{1.62\,\text{mm}}{\cos^2\left(\frac{90^\circ}{7}\right)} \approx 1.70\,\text{mm}$$

The tip circle is 0.08 mm larger than the measured dimension. With 15 teeth the difference still amounts to about 1 %, from about 60 teeth less than 0.1 %.

Involute toothing. The toothing of mechanical engineering is found in the watch where the speed is stepped down or the direction of force changes: in winding, in hand setting, in automatic gearing and in calendar mechanisms. Its flanks are involutes, i.e. curves traced by a thread unwound from the base circle. The toothing tolerates small errors in the centre distance: the rotation is still transmitted uniformly. Pitch, pitch circle and centre distance are calculated as above.

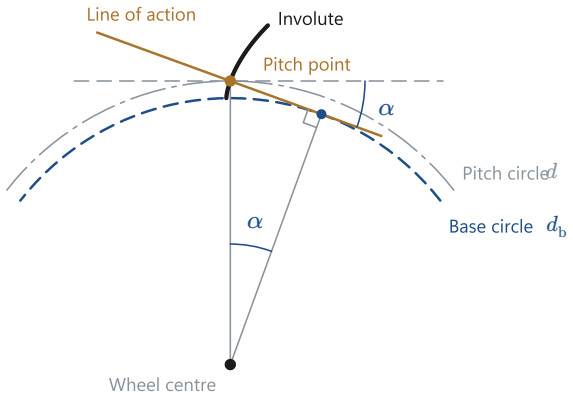


Fig. 4.11 Involute toothing: the line of action touches the base circle and is inclined by the pressure angle α

Tip Circle of Involute Toothing

(4.27) MASTER

$$d_\mathrm{a} = m \cdot (z + 2)$$

REARRANGED $m = \dfrac{d_\mathrm{a}}{z + 2}$

The addendum equals the module. Rearranged, the formula gives the module of an involute wheel from tip circle and number of teeth.

Valid: standard toothing without profile shift, pressure angle 20°, addendum $1 \cdot m$.

d_a	tip circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE A wheel with 24 teeth and module 0.5 mm.

Given: $m = 0.5\,\mathrm{mm}$; $z = 24$

$$\begin{aligned} d_\mathrm{a} &= m \cdot (z + 2) \\ &= 0.5\,\mathrm{mm} \cdot (24 + 2) \\ &= 13\,\mathrm{mm} \end{aligned}$$

Tip circle 13 mm with a pitch circle of 12 mm.

Root Circle of Involute Toothing

(4.28) MASTER

$$d_\mathrm{f} = m \cdot (z - 2.5)$$

The dedendum is 1.25 times the module: addendum of the mating wheel plus $0.25 \cdot m$ tip clearance.

Valid: standard toothing without profile shift. In precision engineering larger dedenda are also common; then twice the dedendum factor replaces 2.5.

d_f	root circle diameter	mm
m	module	mm
z	number of teeth	1

EXAMPLE The same wheel: 24 teeth, module 0.5 mm.

Given: $m = 0.5\,\mathrm{mm}$; $z = 24$

$$\begin{aligned} d_\mathrm{f} &= m \cdot (z - 2.5) \\ &= 0.5\,\mathrm{mm} \cdot (24 - 2.5) \\ &= 10.75\,\mathrm{mm} \end{aligned}$$

Root circle 10.75 mm; the tooth depth is $2.25 \cdot m$, i.e. 1.125 mm.

Base Circle

(4.29) ADVANCED

$$d_\mathrm{b} = d \cdot \cos\alpha$$

The involute is unwound from the base circle. The teeth always touch on the line of action, which is tangent to both base circles and inclined by the pressure angle to the common tangent of the pitch circles.

d_b	base circle diameter	mm
d	pitch circle diameter	mm
α	pressure angle (standardized 20°)	°

EXAMPLE Pitch circle 12 mm, pressure angle 20°.

Given: $d = 12\,\mathrm{mm}$; $\alpha = 20°$

$$\begin{aligned} d_\mathrm{b} &= d \cdot \cos\alpha \\ &= 12\,\mathrm{mm} \cdot \cos(20°) \\ &\approx 11.28\,\mathrm{mm} \end{aligned}$$

The base circle lies 0.36 mm below the pitch circle. There is no involute below the base circle.

4.5 Striking Work

The striking work is a separate train with its own drive, gear train and fly. The going train releases it; a counting mechanism determines the number of blows: the count wheel or the rack.

Sum of the Hour Blows

(4.30) MASTER

$$N = \frac{k \cdot (k+1)}{2}$$

N	number (of blows for the hours 1 to k)	1
k	last hour struck	1

The hour blows 1 + 2 + 3 and so on add up according to the sum formula.

EXAMPLE How many blows are struck from 1 o'clock to 12 o'clock?

Given: $k = 12$

$$\begin{aligned} N &= \frac{k \cdot (k+1)}{2} \\ &= \frac{12 \cdot (12+1)}{2} \\ &= 78 \end{aligned}$$

78 blows in 12 hours, 156 per day.

Blows per Day

(4.31) MASTER

$$N_{\mathrm{d}} = 2 \cdot (N_{12} + 12 \cdot b)$$

N_{d}	blows in 24 hours	1
N_{12}	hour blows in 12 hours (78)	1
b	blows at each half hour (0 or 1)	1

The half-hour blow adds 12 blows in 12 hours.

Valid: hour striking with at most one blow at the half hour. Quarter-striking work has its own train and is calculated separately.

EXAMPLE A striking work strikes the hours and one blow at each half hour.

Given: $N_{12} = 78; \quad b = 1$

$$\begin{aligned} N_{\mathrm{d}} &= 2 \cdot (N_{12} + 12 \cdot b) \\ &= 2 \cdot (78 + 12 \cdot 1) \\ &= 180 \end{aligned}$$

180 blows per day: 90 in 12 hours instead of 78.

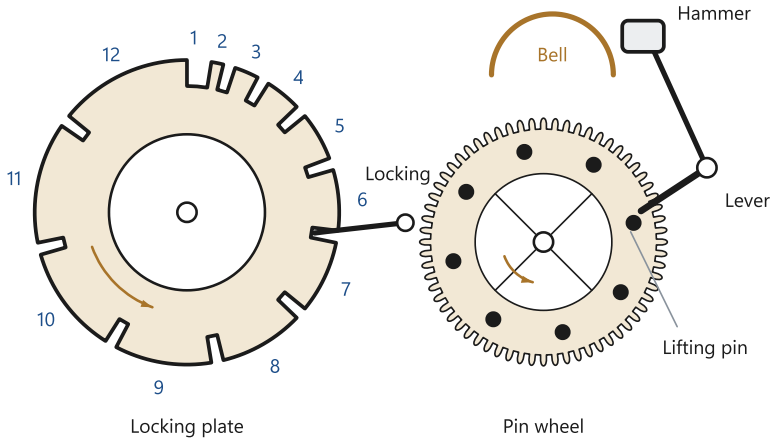


Fig. 4.12 Count wheel with 78 divisions (each hour occupies as many divisions as it has blows) and pin wheel with hammer

Count Wheel

(4.32) MASTER

$$N_{\mathrm{Sch}} = \frac{z_{\mathrm{N}} \cdot z_{\mathrm{Sch}}}{z'_{\mathrm{Sch}}}$$

The count wheel counts the blows. It turns once in 12 hours; its circumference is divided into 78 or 90 parts, and after each sequence of blows the locking detent drops into a notch and stops the train.

Valid: count wheel driven from the pin-wheel arbor and turning once in 12 hours.

N_{Sch}	blows per revolution of the count wheel (78, with half-hour striking 90)	1
z_{N}	number of lifting pins	1
z_{Sch}	number of teeth of the count-wheel drive wheel	1
z'_{Sch}	number of leaves of the pinion on the pin-wheel arbor that drives the count wheel	1

EXAMPLE A pin wheel carries 10 lifting pins. On its arbor sits a pinion with 8 leaves that drives a count-wheel drive wheel with 72 teeth.

Given: $z_{\mathrm{N}} = 10$; $z_{\mathrm{Sch}} = 72$; $z'_{\mathrm{Sch}} = 8$

$$\begin{aligned} N_{\mathrm{Sch}} &= \frac{z_{\mathrm{N}} \cdot z_{\mathrm{Sch}}}{z'_{\mathrm{Sch}}} \\ &= \frac{10 \cdot 72}{8} \\ &= 90 \end{aligned}$$

90 blows per revolution: a striking work with half-hour striking. For 78 blows, 13 lifting pins with a reduction ratio of 6 are suitable.

In rack striking, the hour snail, a stepped cam with 12 steps on the hour wheel, determines how far the rack falls. With each blow the gathering pallet gathers the rack back by one tooth until the train is locked again. Unlike the count wheel, rack striking cannot get out of step with the hands.

Blows per Revolution of the Driving Wheel

(4.33) MASTER

$$N_{\mathrm{A}} = \frac{z_{\mathrm{N}} \cdot z_{\mathrm{A}}}{z'_{\mathrm{N}}}$$

Each lifting pin raises the hammer once. The pin wheel turns faster than the driving wheel by the gear ratio.

Valid: driving wheel meshes directly with the pinion of the pin wheel. If an intermediate wheel lies between them, its gear ratio is added as a factor.

N_{A}	blows per revolution of the driving wheel (great wheel or barrel)	1
z_{N}	number of lifting pins	1
z_{A}	number of teeth of the driving wheel	1
z'_{N}	number of leaves of the pinion on the pin-wheel arbor	1

EXAMPLE A great wheel with 96 teeth drives the pinion (8 leaves) of a pin wheel with 8 lifting pins.

Given: $z_{\mathrm{N}} = 8$; $z_{\mathrm{A}} = 96$; $z'_{\mathrm{N}} = 8$

$$\begin{aligned} N_{\mathrm{A}} &= \frac{z_{\mathrm{N}} \cdot z_{\mathrm{A}}}{z'_{\mathrm{N}}} \\ &= \frac{8 \cdot 96}{8} \\ &= 96 \end{aligned}$$

96 blows are struck per revolution of the barrel.

Revolutions per Winding

(4.34) MASTER

$$U = \frac{t_\mathrm{G} \cdot n_\mathrm{d}}{N_\mathrm{A}}$$

The number of blows per day is given by formula (4.31). Going train and striking work are designed so that both run equally long on one winding.

U	required revolutions of the driving wheel	1
t_G	running time	d
n_d	blows per day	$\frac{1}{\mathrm{d}}$
N_A	blows per revolution of the driving wheel	1

EXAMPLE The striking work from the previous example is to run for 8 days and strikes 180 times a day.

Given: $t_\mathrm{G} = 8\,\mathrm{d}$; $\quad n_\mathrm{d} = 180\,\frac{1}{\mathrm{d}}$; $\quad N_\mathrm{A} = 96$

$$\begin{aligned} U &= \frac{t_\mathrm{G} \cdot n_\mathrm{d}}{N_\mathrm{A}} \\ &= \frac{8\,\mathrm{d} \cdot 180\,\frac{1}{\mathrm{d}}}{96} \\ &= 15 \end{aligned}$$

1440 blows are struck in 8 days; for this the barrel turns 15 times. Line thickness and barrel diameter then give the height of fall of the weight (Chapter 3).

Revolutions of the Fly per Blow

(4.35) MASTER

$$U_\mathrm{W} = \frac{i_\mathrm{W}}{z_\mathrm{N}}$$

The fly brakes the train by the air resistance of its vanes. Because the resistance increases sharply with rotational speed, a uniform speed is established and the blows follow at equal intervals. Larger or more steeply set vanes make the striking slower.

U_W	revolutions of the fly per blow	1
i_W	gear ratio from pin wheel to fly	1
z_N	number of lifting pins	1

EXAMPLE From the pin wheel (64 teeth, 8 lifting pins) to the fly there are three meshes: 64 to 8, 56 to 7 and 49 to 7. The gear ratio is $8 \cdot 8 \cdot 7 = 448$.

Given: $i_\mathrm{W} = 448$; $\quad z_\mathrm{N} = 8$

$$\begin{aligned} U_\mathrm{W} &= \frac{i_\mathrm{W}}{z_\mathrm{N}} \\ &= \frac{448}{8} \\ &= 56 \end{aligned}$$

The fly turns 56 times per blow.

4.6 Calendar and Moon Phase

Calendar mechanisms count days. Their wheels are not driven continuously but advanced by one tooth once a day; the number of teeth is then the number of days per revolution.

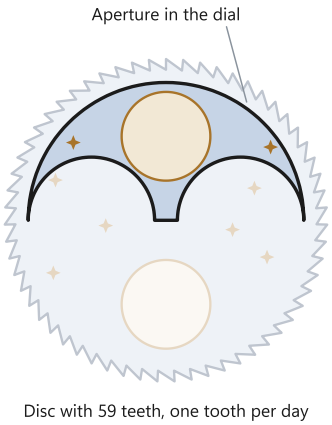


Fig. 4.13 Moon-phase disc behind the aperture in the dial

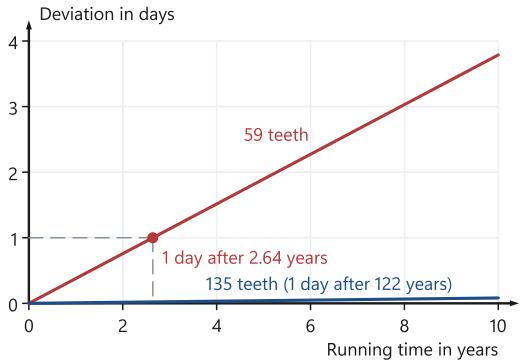


Fig. 4.14 Deviation of the moon-phase display from the mean moon

Moon-Phase Disc

(4.36) MASTER

$$t_{\mathrm{L}} = \frac{z \cdot t_{\mathrm{s}}}{k}$$

From new moon to new moon, an average of 29.530 59 days passes, the synodic month. Because a wheel cannot have half teeth, the disc carries two moons and 59 teeth.

t_{L}	displayed duration of one lunation	d
z	number of teeth (of the moon disc)	1
t_{s}	time between two advances (one tooth per day)	d
k	number (of moons on the disc)	1

EXAMPLE A moon disc with 59 teeth and two moons is advanced by one tooth daily.

Given: $z = 59; \quad t_{\mathrm{s}} = 1\,\mathrm{d}; \quad k = 2$

$$\begin{aligned} t_{\mathrm{L}} &= \frac{z \cdot t_{\mathrm{s}}}{k} \\ &= \frac{59 \cdot 1\,\mathrm{d}}{2} \\ &= 29.5\,\mathrm{d} \end{aligned}$$

The display passes through all phases in 29.5 days. The moon needs 29.530 59 days on average, so the display runs slightly fast.

Running Time until the Moon Phase Deviates

(4.37) MASTER

$$t_{\mathrm{F}} = \frac{\Delta t \cdot t_{\mathrm{L}}}{|T_{\mathrm{syn}} - t_{\mathrm{L}}|}$$

With each lunation the display deviates by the difference between the displayed and the true lunation. The deviations add up.

Valid: mean lunar motion. The true moon deviates from it by up to about half a day in the course of a month.

t_{F}	running time until the display deviates by Δt	d
Δt	deviation of the display from the true age of the moon	d
t_{L}	displayed duration of one lunation	d
T_{syn}	synodic month (mean value 29.530 59 d, Section 11.1)	d

EXAMPLE After what time does the display with 59 teeth deviate by one day?

Given: $\Delta t = 1\,\mathrm{d}$; $t_{\mathrm{L}} = 29.5\,\mathrm{d}$; $T_{\mathrm{syn}} = 29.530\,59\,\mathrm{d}$

$$\begin{aligned} t_{\mathrm{F}} &= \frac{\Delta t \cdot t_{\mathrm{L}}}{|T_{\mathrm{syn}} - t_{\mathrm{L}}|} \\ &= \frac{1\,\mathrm{d} \cdot 29.5\,\mathrm{d}}{|29.530\,59\,\mathrm{d} - 29.5\,\mathrm{d}|} \\ &\approx 964\,\mathrm{d} \end{aligned}$$

After 964 days, i.e. about 2.6 years, the display is one day ahead and must be corrected.

Precise Moon Phase with Gearing

(4.38) ADVANCED

$$t_{\mathrm{L}} = \frac{t_{\mathrm{A}} \cdot z}{z' \cdot k}$$

A gear ratio between the advancing wheel and the moon disc matches the synodic month much more precisely than the simple disc with 59 teeth.

Valid: moon disc driven via wheels. The disc with 135 teeth is widespread; the drive in the example is one possible tooth-count set, others are in use.

t_{L}	displayed duration of one lunation	d
t_{A}	rotation period of the driving arbor (star wheel with 7 teeth, advanced daily: 7 d)	d
z	number of teeth (of the moon disc)	1
z'	number of leaves of the pinion (drives the moon disc)	1
k	number (of moons on the disc)	1

EXAMPLE A star wheel with 7 teeth is advanced daily. On its arbor sits a pinion with 16 leaves that drives, via an idler, a moon disc with 135 teeth and two moons.

Given: $t_{\mathrm{A}} = 7\,\mathrm{d}$; $z = 135$; $z' = 16$; $k = 2$

$$\begin{aligned} t_{\mathrm{L}} &= \frac{t_{\mathrm{A}} \cdot z}{z' \cdot k} \\ &= \frac{7\,\mathrm{d} \cdot 135}{16 \cdot 2} \\ &= 29.531\,25\,\mathrm{d} \end{aligned}$$

The display is now only 57 s too slow per lunation. According to formula (4.37) with $T_{\mathrm{syn}} = 29.530\,59\,\mathrm{d}$, it takes 44 744 days, a good 122 years, until it lags by one day.

Counting Wheels of Calendar Displays

Display	Wheel	Advances	One revolution in
date	date ring or date wheel with 31 teeth	daily	31 days
day of the week	star wheel with 7 teeth	daily	7 days
month	star wheel with 12 teeth	monthly	1 year
moon phase	disc with 59 teeth and two moons	daily	59 days
perpetual calendar	programme wheel with 48 month steps	monthly	4 years (1461 days)

The simple calendar is advanced by hand five times a year, namely after every month with fewer than 31 days. The annual calendar needs one correction at the end of February. The perpetual calendar also knows the leap years; it need not be corrected until the year 2100, which is not a leap year.

The date driving wheel turns once in 24 hours and is driven by the hour wheel with a reduction ratio of 2 to 1, formula (4.12). Leap years, the length of the year and the equation of time are given in Chapter 11.

4.7 Efficiency of the Gear Train

Every mesh and every bearing consumes part of the driving force through friction. The efficiency states what proportion of the torque remains.

Overall Efficiency of the Gear Train

(4.39) MASTER

$$\eta_{\text{ges}} = \eta^{k}$$

The efficiencies of meshes in series are multiplied. If they differ, the product of the individual values replaces the power. The general form is given in formula (2.30).

Valid: guide values for clean, oiled movements. Pinions with few leaves, dry bearings and depthings that are too deep or too shallow reduce the efficiency considerably.

η_{ges}	overall efficiency of the gear train	1
η	efficiency (per mesh; including bearing friction, guide value 0.90 to 0.95)	1
k	number (of meshes)	1

EXAMPLE From the barrel to the escape wheel there are four meshes, each with an efficiency of 0.95.

Given: $\eta = 0.95; \quad k = 4$

$$\begin{aligned}\eta_{\text{ges}} &= \eta^{k} \\ &= 0.95^{4} \\ &\approx 0.81\end{aligned}$$

About one fifth of the torque is lost in the gear train. At 0.90 per mesh it would already be one third.

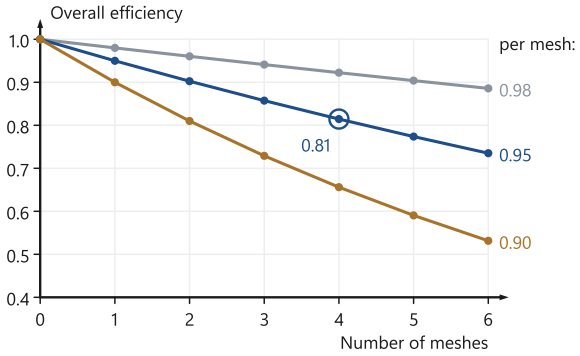


Fig. 4.15 Overall efficiency by number of meshes

Torque at the Escape Wheel

(4.40) MASTER

$$M_{\mathrm{H}} = \frac{M_{\mathrm{F}} \cdot \eta_{\mathrm{ges}}}{i_{\mathrm{ges}}}$$

From arbor to arbor the torque falls by the gear ratio and by the friction losses (Fig. 4.4).

M_{H}	torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
M_{F}	torque at the barrel	$\mathrm{N{\cdot}mm}$
η_{ges}	overall efficiency of the gear train	1
i_{ges}	overall gear ratio (from barrel to escape wheel)	1

EXAMPLE A barrel delivers 8 N·mm. The gear ratio to the escape wheel is $6 \cdot 600 = 3600$, the overall efficiency 0.81.

Given: $M_{\mathrm{F}} = 8\,\mathrm{N{\cdot}mm}$; $\eta_{\mathrm{ges}} = 0.81$; $i_{\mathrm{ges}} = 3600$

$$\begin{aligned} M_{\mathrm{H}} &= \frac{M_{\mathrm{F}} \cdot \eta_{\mathrm{ges}}}{i_{\mathrm{ges}}} \\ &= \frac{8\,\mathrm{N{\cdot}mm} \cdot 0.81}{3600} \\ &= 1.8\,\mu\mathrm{N{\cdot}m} \end{aligned}$$

1.8 µN·m remain at the escape wheel, a little over one five-thousandth of the spring torque. This is why even a little dirt on the fast arbors brings a movement to a stop.

Efficiency from Measured Torques

(4.41) ADVANCED

$$\eta_{\mathrm{ges}} = \frac{M_{\mathrm{H}} \cdot i_{\mathrm{ges}}}{M_{\mathrm{F}}}$$

Without friction, the spring torque divided by the gear ratio would arrive at the escape wheel. The efficiency is the ratio of the measured value to this ideal value.

η_{ges}	overall efficiency of the gear train	1
M_{H}	torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
i_{ges}	overall gear ratio (from barrel to escape wheel)	1
M_{F}	torque at the barrel	$\mathrm{N{\cdot}mm}$

EXAMPLE On a movement, 8 N·mm are measured at the barrel and 1.7 µN·m at the escape wheel; the gear ratio is 3600.

Given: $M_{\mathrm{H}} = 1.7\,\mu\mathrm{N{\cdot}m}$; $i_{\mathrm{ges}} = 3600$; $M_{\mathrm{F}} = 8\,\mathrm{N{\cdot}mm}$

$$\begin{aligned} \eta_{\mathrm{ges}} &= \frac{M_{\mathrm{H}} \cdot i_{\mathrm{ges}}}{M_{\mathrm{F}}} \\ &= \frac{1.7\,\mu\mathrm{N{\cdot}m} \cdot 3600}{8\,\mathrm{N{\cdot}mm}} \\ &= 0.765 \end{aligned}$$

The gear train transmits a little over three quarters of the torque. This corresponds to 0.935 per mesh with four meshes.

5 Oscillator: Balance and Balance Spring

Balance and balance spring form the oscillator of the watch. The balance is the inert mass, the balance spring the spring that returns it to its rest position. How fast the two oscillate together is determined by only two quantities: the moment of inertia of the balance and the restoring torque coefficient of the balance spring.

Almost all regulating formulas in this chapter follow from one rule: the period increases with the square root of the moment of inertia and decreases with the square root of the restoring torque coefficient. One percent more moment of inertia or one percent less restoring torque coefficient makes the watch half a percent slower, i.e. 432 s per day. In this chapter N stands for a number (coils of the balance spring, identical screws on the rim), W for energy and P for power.

5.1 Oscillation of the Balance

The balance oscillates back and forth about its rest position. One oscillation is there and back; the watch ticks twice in the process, because each beat is a vibration (half-oscillation).

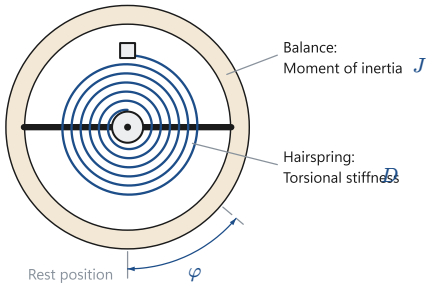


Fig. 5.1 Balance with balance spring: the balance provides the moment of inertia J, the balance spring the restoring torque coefficient D

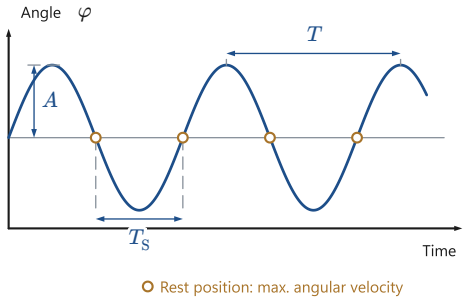


Fig. 5.2 Angle of the balance over time: amplitude A, period T, beat duration $T_S = \frac{1}{2} T$

Period of the Balance

(5.1) JOURNEYMAN

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J}{D}}$$

REARRANGED $D = \dfrac{4 \cdot \pi^2 \cdot J}{T^2}$

A more inert balance oscillates more slowly, a stronger balance spring faster. Both quantities are under the square root: four times the moment of inertia doubles the period.

Valid: restoring torque of the balance spring proportional to the angle, without friction and without influence of the escapement. J includes rim, arms, staff, double roller and collet.

T	period (full oscillation, there and back)	s
J	moment of inertia (of the balance; with everything that oscillates with it)	$\mathrm{mg{\cdot}cm^2}$
D	restoring torque coefficient (of the balance spring; torque per angle)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

Good to know: The amplitude does not appear in the formula. An ideal oscillator oscillates equally fast at small and large amplitude; this property is called isochronism (Section 5.7).

EXAMPLE A balance has a moment of inertia of $10\,\mathrm{mg{\cdot}cm^2}$, its balance spring a restoring torque coefficient of $0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. How long does one oscillation last?

Given: $J = 10\,\mathrm{mg{\cdot}cm^2}$; $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{J}{D}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{10\,\mathrm{mg{\cdot}cm^2}}{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}} \\ &\approx 0.250\,\mathrm{s} \end{aligned}$$

One oscillation lasts a quarter of a second. The balance therefore makes 4 oscillations and 8 beats per second.

Frequency of the Balance

(5.2) JOURNEYMAN

$$f = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{D}{J}}$$

The frequency is the reciprocal of the period: $f = \frac{1}{T}$.

Valid: as for formula (5.1).

f	frequency (full oscillations per second)	Hz
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE What is the frequency of the oscillator from the previous example?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned} f &= \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{D}{J}} \\ &= \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}{10\,\mathrm{mg{\cdot}cm^2}}} \\ &\approx 4.00\,\mathrm{Hz} \end{aligned}$$

4 Hz, the usual frequency of today's wristwatches.

Beat Rate from the Frequency

(5.3) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot f$$

REARRANGED $f = \frac{n_\mathrm{h}}{2}$

n_h	beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
f	frequency (full oscillations per second)	Hz

The watch beats twice per oscillation. For watches the beat rate is stated in vibrations per hour (A/h, A for alternation). The formula with the beat duration is already given in Section 1.1.

EXAMPLE A balance oscillates at 3 Hz. What beat rate is this?

Given: $f = 3\,\mathrm{Hz}$

$$\begin{aligned} n_\mathrm{h} &= 2 \cdot f \\ &= 2 \cdot 3\,\mathrm{Hz} \\ &= 6\,\frac{1}{\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &\approx 21\,600\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

21 600 vibrations per hour. The beat rate is given in the data sheet of the calibre, and the timing machine calculates with it.

Common Beat Rates

Beat rate n_h in $\frac{\mathrm{A}}{\mathrm{h}}$	Beats per second	Frequency f in Hz	Period T in s	Beat duration T_S in s
18 000	5	2.5	0.4	0.2
19 800	5.5	2.75	0.3636	0.1818
21 600	6	3	0.3333	0.1667
25 200	7	3.5	0.2857	0.1429
28 800	8	4	0.25	0.125
36 000	10	5	0.2	0.1

The beat duration T_S is the duration of one vibration, i.e. half the period.

Angular Frequency

(5.4) MASTER

$$\omega = 2 \cdot \pi \cdot f$$

ω	angular frequency	$\frac{\mathrm{rad}}{\mathrm{s}}$
f	frequency (full oscillations per second)	Hz

The angular frequency is the frequency in radians: 2π per oscillation. For the balance also $\omega = \sqrt{\frac{D}{J}}$.

EXAMPLE What is the angular frequency at 4 Hz?

Given: $f = 4\,\mathrm{Hz}$

$$\begin{aligned} \omega &= 2 \cdot \pi \cdot f \\ &= 2 \cdot \pi \cdot 4\,\mathrm{Hz} \\ &\approx 25.1\,\frac{\mathrm{rad}}{\mathrm{s}} \end{aligned}$$

About 25.1 rad per second. The following formulas calculate with this number.

Angle of the Balance over Time

(5.5) MASTER

$$\varphi = A \cdot \sin(\omega \cdot t)$$

The balance oscillates sinusoidally: it is fastest at the rest position and stands still for an instant at the turning points.

Valid: free, undamped oscillation. The product $\omega \cdot t$ is an angle in radians; the calculator must be set to rad for the sine.

φ	angle (of the balance from the rest position)	$^\circ$
A	amplitude (greatest angle from the rest position)	$^\circ$
ω	angular frequency	$\frac{\text{rad}}{\text{s}}$
t	time (since passing through the rest position)	s

EXAMPLE A balance oscillates at 4 Hz ($\omega = 25.13\ \frac{\text{rad}}{\text{s}}$) with 270° amplitude. Where is it 0.02 s after passing through the rest position?

Given: $A = 270^\circ; \quad \omega = 25.13\ \frac{\text{rad}}{\text{s}}; \quad t = 0.02\,\text{s}$

$$\begin{aligned}\varphi &= A \cdot \sin(\omega \cdot t)\\ &= 270^\circ \cdot \sin\left(25.13\ \tfrac{\text{rad}}{\text{s}} \cdot 0.02\,\text{s}\right)\\ &\approx 130^\circ\end{aligned}$$

After 0.02 s the balance has already covered about 130°, almost half the amplitude. It needs 0.0625 s in total to reach the turning point, a quarter oscillation.

Maximum Angular Velocity

(5.6) MASTER

$$\omega_{\max} = A \cdot \omega$$

At the rest position the entire energy of the oscillation is kinetic. This is where the escapement acts, because a disturbance at this point changes the rate least.

Valid: insert the amplitude in radians: degrees times $\frac{\pi}{180}$.

$\omega_{\max}$	angular velocity of the balance when passing through the rest position	$\frac{\text{rad}}{\text{s}}$
A	amplitude (in radians)	rad
ω	angular frequency	$\frac{\text{rad}}{\text{s}}$

EXAMPLE How fast does the balance at 4 Hz and 270° amplitude (4.71 rad) turn at the rest position?

Given: $A = 4.71\,\text{rad}; \quad \omega = 25.13\ \frac{\text{rad}}{\text{s}}$

$$\begin{aligned}\omega_{\max} &= A \cdot \omega\\ &= 4.71\,\text{rad} \cdot 25.13\ \tfrac{\text{rad}}{\text{s}}\\ &\approx 118\ \tfrac{\text{rad}}{\text{s}}\end{aligned}$$

About 118 rad per second; turning on uniformly, this would be almost 19 revolutions per second. A point on the rim at 5 mm radius moves at about 0.6 m per second.

Key point: 28 800 vibrations per hour are 8 beats per second and 4 Hz.

5.2 Balance Spring

The balance spring is a bending spring. Its restoring torque coefficient states how much torque it produces per angle. It depends on the material, the cross-section of the blade and the effective length.

Restoring Torque of the Balance Spring

(5.7) JOURNEYMAN

$$M = D \cdot \varphi$$

The restoring torque increases uniformly with the angle and always points towards the rest position. Only because of this does the balance oscillate equally fast at every amplitude.

Valid: balance spring in the elastic range; coils do not touch.

M	torque (restoring torque of the balance spring)	$\mu\mathrm{N{\cdot}m}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
φ	angle (of the balance from the rest position; radians)	rad

EXAMPLE With what torque does a balance spring with $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ pull the balance back at the turning point at 270° (4.71 rad)?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $\varphi = 4.71\,\mathrm{rad}$

$$\begin{aligned} M &= D \cdot \varphi \\ &= 0.632\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \cdot 4.71\,\mathrm{rad} \\ &\approx 2.98\,\mu\mathrm{N{\cdot}m} \end{aligned}$$

Just under 3 µN·m. This is the order of magnitude of the torque that arrives at the escape wheel.

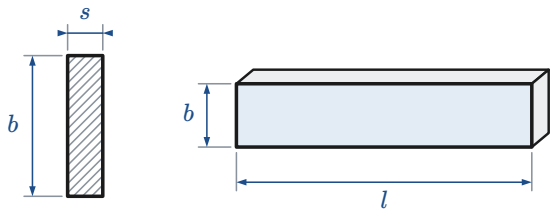


Fig. 5.3 Blade of the balance spring: blade height b, blade thickness s, effective length l

Restoring Torque Coefficient from the Dimensions of the Balance Spring

(5.8) MASTER

$$D = \frac{E \cdot b \cdot s^3}{12 \cdot l}$$

Height and modulus of elasticity enter linearly, the thickness to the third power. A longer balance spring is softer.

Valid: rectangular cross-section, uniform blade, pure bending. With N and mm, D is obtained in N·mm per rad; $1\,\mathrm{N{\cdot}mm} = 1000\,\mu\mathrm{N{\cdot}m}$.

D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
E	modulus of elasticity (of the balance-spring material)	$\frac{\mathrm{N}}{\mathrm{mm}^2}$
b	blade height (in the direction of the balance staff)	mm
s	blade thickness (in the radial direction)	mm
l	length (of the balance spring, effective)	mm

EXAMPLE A balance spring has $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2}$, a blade height of 0.14 mm, a blade thickness of 0.032 mm and an effective length of 121 mm.

Given: $E = 200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2}$; $b = 0.14\,\mathrm{mm}$; $s = 0.032\,\mathrm{mm}$; $l = 121\,\mathrm{mm}$

$$\begin{aligned} D &= \frac{E \cdot b \cdot s^3}{12 \cdot l} \\ &= \frac{200\,000\ \frac{\mathrm{N}}{\mathrm{mm}^2} \cdot 0.14\,\mathrm{mm} \cdot (0.032\,\mathrm{mm})^3}{12 \cdot 121\,\mathrm{mm}} \\ &\approx 0.632\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \end{aligned}$$

This balance spring matches the balance with $10\,\mathrm{mg{\cdot}cm}^2$ from Section 5.1 and gives 4 Hz with it.

Rate Change due to Blade Thickness

(5.9) APPROXIMATION MASTER

$$\Delta G \approx \frac{3 \cdot \Delta s}{2 \cdot s} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δs	change in blade thickness (+ means: thicker)	$\mu\mathrm{m}$
s	blade thickness	$\mu\mathrm{m}$

Because the restoring torque coefficient increases with s^3, 1 % more thickness changes the restoring torque coefficient by 3 % and the rate by 1.5 %.

Valid: small changes; height, length and balance unchanged. For the blade height, correspondingly $\Delta G \approx \frac{\Delta b}{2 \cdot b} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$.

EXAMPLE A balance spring is 0.1 µm thicker than 32 µm.

Given: $\Delta s = 0.1\,\mu\mathrm{m}; \quad s = 32\,\mu\mathrm{m}$

$$\begin{aligned}\Delta G &\approx \frac{3 \cdot \Delta s}{2 \cdot s} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{3 \cdot 0.1\,\mu\mathrm{m}}{2 \cdot 32\,\mu\mathrm{m}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 405\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

One tenth of a micrometre makes almost 7 minutes per day. No blade can be measured this precisely; this is why balance springs are not sorted by dimension but by their measured restoring torque coefficient and matched to the balance.

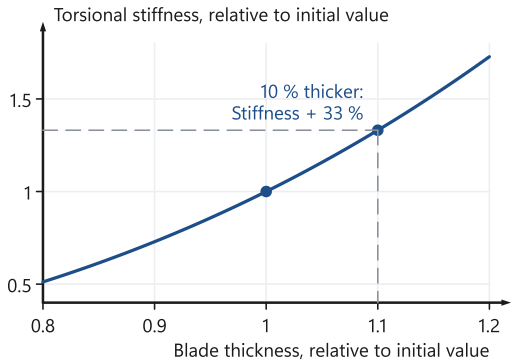


Fig. 5.4 The restoring torque coefficient increases with the third power of the blade thickness

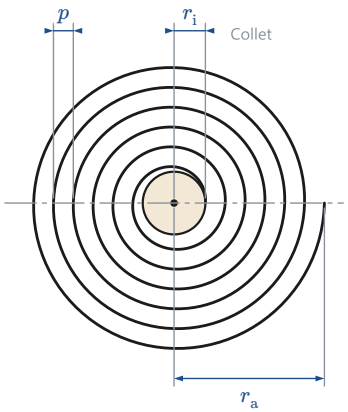


Fig. 5.5 Flat balance spring: radii r_i and r_a, coil pitch p

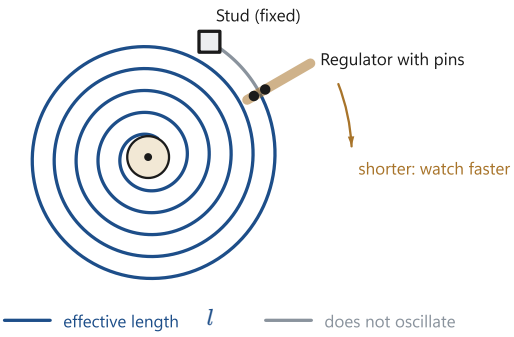


Fig. 5.6 Regulator: the effective length l extends from the collet to the regulator pins

Length of the Flat Balance Spring

(5.10) APPROXIMATION MASTER

$$l \approx \pi \cdot N \cdot (r_\mathrm{a} + r_\mathrm{i})$$

Every coil is almost a circle. The length is the number of coils times the circumference of the mean coil.

Valid: uniformly wound flat balance spring (Archimedean spiral) with at least about 5 coils; terminal curve and attachment ends not included.

l	length (of the balance spring)	mm
N	number (of coils)	1
r_a	radius of the outermost coil	mm
r_i	radius of the innermost coil (at the collet)	mm

EXAMPLE A balance spring has 12 coils between 0.6 mm and 2.6 mm radius.

Given: $N = 12; \quad r_\mathrm{a} = 2.6\,\mathrm{mm}; \quad r_\mathrm{i} = 0.6\,\mathrm{mm}$

$$\begin{aligned} l &\approx \pi \cdot N \cdot (r_\mathrm{a} + r_\mathrm{i}) \\ &= \pi \cdot 12 \cdot (2.6\,\mathrm{mm} + 0.6\,\mathrm{mm}) \\ &\approx 121\,\mathrm{mm} \end{aligned}$$

The blade is about 121 mm long, as long as the balance spring in the examples on the restoring torque coefficient and the regulator. A wristwatch thus contains a good 12 cm of spring strip on the area of a shirt button.

Coil Pitch

(5.11) MASTER

$$p = \frac{r_\mathrm{a} - r_\mathrm{i}}{N}$$

The pitch determines how much clearance the coils have when the balance spring breathes. At large amplitude they must not touch.

Valid: uniformly wound flat balance spring.

p	coil pitch (centre of blade to centre of the next blade)	mm
r_a	radius of the outermost coil	mm
r_i	radius of the innermost coil	mm
N	number (of coils)	1

EXAMPLE How far apart are the coils of the balance spring from the previous example?

Given: $r_\mathrm{a} = 2.6\,\mathrm{mm}; \quad r_\mathrm{i} = 0.6\,\mathrm{mm}; \quad N = 12$

$$\begin{aligned} p &= \frac{r_\mathrm{a} - r_\mathrm{i}}{N} \\ &= \frac{2.6\,\mathrm{mm} - 0.6\,\mathrm{mm}}{12} \\ &\approx 0.167\,\mathrm{mm} \end{aligned}$$

About 0.17 mm from blade to blade. With a blade thickness of 0.032 mm, about 0.13 mm of clearance remains.

CGS Number of the Balance Spring

(5.12) MASTER

$$K = D \cdot \left(d_\mathrm{a}^2 - d_\mathrm{i}^2\right)$$

The CGS value is the restoring torque coefficient times the difference of the squared diameters. It depends only on material, blade and coil pitch, not on the point at which the balance spring is pinned: pinned further out, the restoring torque coefficient decreases to the same extent as the difference of the squares increases. This is why unpinned balance springs are graded and ordered by this value.

Valid: uniformly wound flat balance spring; d_a at the point where the balance spring is pinned, d_i at the collet. The CGS number is the same value in the old unit $\mathrm{dyn \cdot cm^3}$ per rad: numerical value in $\mathrm{N \cdot mm^3}$ times 100.

K	CGS value of the balance spring (times 100 gives the CGS number)	$\frac{\mathrm{N \cdot mm^3}}{\mathrm{rad}}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$
d_a	diameter of the outermost coil ($2 \cdot r_\mathrm{a}$)	mm
d_i	diameter of the innermost coil ($2 \cdot r_\mathrm{i}$)	mm

Good to know: With the length and the coil pitch from the two previous formulas this becomes $K \approx \frac{E \cdot b \cdot s^3 \cdot p}{3 \cdot \pi}$. Neither the length nor the diameters appear in it.

EXAMPLE For 4 Hz, the balance with $10\,\mathrm{mg \cdot cm^2}$ from Section 5.1 requires a restoring torque coefficient of $0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$. The balance spring is to be pinned at a diameter of 5.0 mm, its innermost coil measures 1.2 mm. Which balance spring is ordered?

Given: $D = 0.632\ \frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$; $\quad d_\mathrm{a} = 5.0\,\mathrm{mm}$; $\quad d_\mathrm{i} = 1.2\,\mathrm{mm}$

$$\begin{aligned} K &= D \cdot \left(d_\mathrm{a}^2 - d_\mathrm{i}^2\right) \\ &= 0.632\ \tfrac{\mu\mathrm{N \cdot m}}{\mathrm{rad}} \cdot \left((5.0\,\mathrm{mm})^2 - (1.2\,\mathrm{mm})^2\right) \\ &\approx 0.0149\ \tfrac{\mathrm{N \cdot mm^3}}{\mathrm{rad}} \end{aligned}$$

The CGS value is 0.0149, so the CGS number is 1.49. The next standard number is ordered, here 1.50.

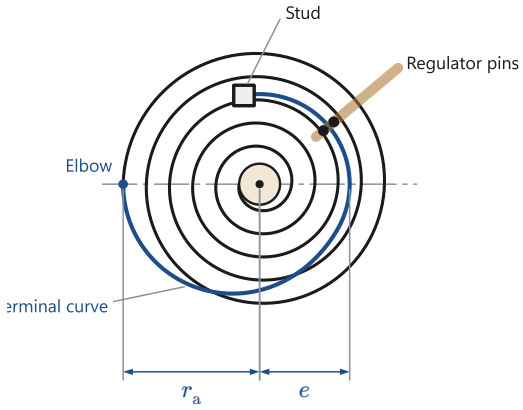


Fig. 5.7 Breguet balance spring from above: the raised terminal curve leads the blade from the outer radius r_a to the terminal radius e, on which the regulator pins move

Terminal Curve Number

(5.13) JOURNEYMAN

$$N_{\mathrm{E}} = \frac{100 \cdot e}{r_{\mathrm{a}}}$$

REARRANGED $e = \dfrac{N_{\mathrm{E}} \cdot r_{\mathrm{a}}}{100}$

N_{E}	terminal curve number	1
e	terminal radius (balance axis to midway between the regulator pins)	mm
r_{a}	radius of the outermost coil	mm

With the Breguet balance spring, the outer end is raised and led inwards in a terminal curve. Its last part is a circular arc about the balance axis, on which the regulator pins move. The terminal curve number gives the radius of this arc as a percentage of the outer radius. The shape of the curve is chosen from the curve template according to this number.

Valid: balance spring with raised overcoil. The curve templates are graded; the nearest number is chosen. Numbers between about 40 and 100 are usual.

EXAMPLE A Breguet balance spring has an outside diameter of 6.4 mm, i.e. an outer radius of 3.2 mm. The regulator pins are 2.4 mm from the balance axis. Which terminal curve is required?

Given: $e = 2.4\,\mathrm{mm}$; $r_{\mathrm{a}} = 3.2\,\mathrm{mm}$

$$\begin{aligned} N_{\mathrm{E}} &= \frac{100 \cdot e}{r_{\mathrm{a}}} \\ &= \frac{100 \cdot 2.4\,\mathrm{mm}}{3.2\,\mathrm{mm}} \\ &= 75 \end{aligned}$$

Terminal curve number 75: its arc lies at three quarters of the outer radius.

Vibrating: Oscillations in the Counting Time

(5.14) JOURNEYMAN

$$N_{\mathrm{S}} = \frac{n_{\mathrm{h}} \cdot t}{2}$$

N_{S}	number of full oscillations (in the counting time)	1
n_{h}	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$
t	time (counting time)	s

In vibrating, one looks for the point of the balance spring at which it gives the nominal beat rate with its balance. The balance spring is held there on trial, the balance is allowed to oscillate freely and the full oscillations are counted.

Valid: n_{h} and t in matching units: convert A/h to beats per second (divide by 3600) if t is in seconds.

EXAMPLE A balance for 18 000 A/h is vibrated with its new balance spring. How many full oscillations must it make in 60 s?

Given: $n_{\mathrm{h}} = 18\,000\ \frac{\mathrm{A}}{\mathrm{h}}$; $t = 60\,\mathrm{s}$

$$\begin{aligned} N_{\mathrm{S}} &= \frac{n_{\mathrm{h}} \cdot t}{2} \\ &= \frac{18\,000\ \frac{\mathrm{A}}{\mathrm{h}} \cdot 60\,\mathrm{s}}{2} \\ &= 150 \end{aligned}$$

150 oscillations per minute. If more are counted, the balance spring is held too short at this point; if fewer, too long.

Effective Length for the Nominal Beat Rate

(5.15) JOURNEYMAN

$$l_2 = l_1 \cdot \left(\frac{n_1}{n_2}\right)^2$$

l_2	required effective length	mm
l_1	effective length in the trial	mm
n_1	beat rate in the trial	$\frac{\mathrm{A}}{\mathrm{h}}$
n_2	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$

The beat rate changes with the square root of the reciprocal of the length. Shorter means faster.

Valid: the same balance, uniform blade.

EXAMPLE With an effective length of 130 mm a balance makes 28 000 A/h. It is to make 28 800 A/h.

Given: $l_1 = 130\,\mathrm{mm}$; $n_1 = 28\,000\,\frac{\mathrm{A}}{\mathrm{h}}$; $n_2 = 28\,800\,\frac{\mathrm{A}}{\mathrm{h}}$

$$l_2 = l_1 \cdot \left(\frac{n_1}{n_2}\right)^2$$

$$= 130\,\mathrm{mm} \cdot \left(\frac{28\,000\,\frac{\mathrm{A}}{\mathrm{h}}}{28\,800\,\frac{\mathrm{A}}{\mathrm{h}}}\right)^2$$

$$\approx 122.9\,\mathrm{mm}$$

The balance spring must be pinned about 7 mm shorter.

Rate Change with the Regulator

(5.16) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{\Delta l}{2 \cdot l} \cdot 86\,400\,\frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta l \approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δl	change in length (of the effective length; + means: longer)	mm
l	length (of the balance spring, effective: collet to regulator pins)	mm

The regulator moves the regulator pins along the outermost coil. Shorter effective length means a larger restoring torque coefficient, and the watch becomes faster; hence the minus sign.

Valid: small changes; the balance spring lies between the regulator pins without play. With play the regulator acts more weakly and the rate depends on the amplitude (Section 5.7).

EXAMPLE A watch with an effective balance-spring length of 121 mm loses 20 s per day. It is therefore to become faster by 20 s per day.

Given: $\Delta G = 20\,\frac{\mathrm{s}}{\mathrm{d}}$; $l = 121\,\mathrm{mm}$

$$\Delta l \approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$$

$$= -\frac{2 \cdot 121\,\mathrm{mm} \cdot 20\,\frac{\mathrm{s}}{\mathrm{d}}}{86\,400\,\frac{\mathrm{s}}{\mathrm{d}}}$$

$$\approx -0.056\,\mathrm{mm}$$

The regulator must shorten the effective length by about 0.06 mm. This is hardly visible; this is why good movements have a fine adjustment on the regulator.

Key point: Shorter balance spring, faster watch. 1 ‰ less effective length gives about 43 s per day.

5.3 Balance

The moment of inertia states how strongly the balance resists being turned. Not only the mass counts, but above all its distance from the axis: the radius enters squared. This is why almost the entire effect lies in the rim.

Moments of inertia of balances are given in mg·cm²: $1\,\mathrm{mg{\cdot}cm^2} = 10^{-10}\,\mathrm{kg{\cdot}m^2}$. Calculating with milligrams and millimetres gives mg·mm²; 100 mg·mm² are 1 mg·cm².

The general formulas for the moment of inertia of simple bodies and the parallel axis theorem are given in Section 2.6.

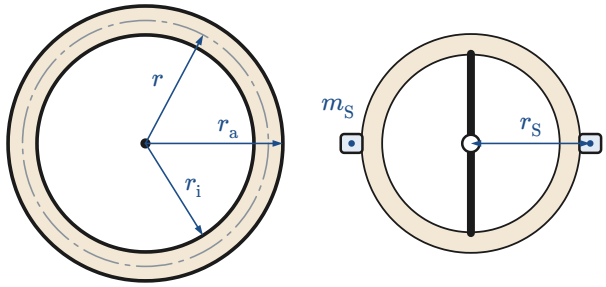


Fig. 5.8 Left, the rim with mean radius r, inner radius r_i and outer radius r_a; right, screws of mass m_S at distance r_S

Moment of Inertia of the Rim (Thin Ring)

(5.17) APPROXIMATION MASTER

$$J \approx m \cdot r^2$$

For an estimate, the entire mass of the rim is assumed to lie on the mean radius. Arms and staff contribute little because their mass lies close to the axis.

Valid: rim narrow compared with the radius; rim only, without arms, staff and screws.

J	moment of inertia (of the rim)	$\mathrm{mg{\cdot}cm^2}$
m	mass (of the rim)	mg
r	radius (of the rim, mean value)	mm

Good to know: Calculated exactly, the rim is a hollow cylinder, formula (2.37). With an outer radius of 4.9 mm and an inner radius of 4.1 mm this gives $8.16\,\mathrm{mg{\cdot}cm^2}$ for the rim from the example; the estimate lies only 0.8 % below.

EXAMPLE A balance rim weighs 40 mg and has a mean radius of 4.5 mm.

Given: $m = 40\,\mathrm{mg}$; $r = 4.5\,\mathrm{mm}$

$$\begin{aligned} J &\approx m \cdot r^2 \\ &= 40\,\mathrm{mg} \cdot (4.5\,\mathrm{mm})^2 \\ &\approx 8.1\,\mathrm{mg{\cdot}cm^2} \end{aligned}$$

About $8\,\mathrm{mg{\cdot}cm^2}$ for the rim alone. With arms, staff and screws such a balance reaches about $10\,\mathrm{mg{\cdot}cm^2}$, the value of the other examples. Balances of men's wristwatches usually lie between 5 and 20 mg·cm².

Moment of Inertia of Screws and Masselottes

(5.18) MASTER

$$\Delta J = N \cdot m_\mathrm{S} \cdot r_\mathrm{S}^2$$

Small masses on the rim count as point masses. Screws are always fitted or altered in opposite pairs so that the balance remains in poise.

Valid: screw small compared with its distance from the axis.

ΔJ	change in moment of inertia (increase due to the screws)	$\mathrm{mg{\cdot}cm^2}$
N	number (of identical screws or masselottes)	1
m_S	mass of one screw	mg
r_S	distance of its centre of gravity from the balance axis	mm

EXAMPLE Two opposite screws of 0.5 mg each are fitted in a balance. Their centre of gravity lies 5 mm from the axis.

Given: $N = 2$; $m_\mathrm{S} = 0.5\,\mathrm{mg}$; $r_\mathrm{S} = 5\,\mathrm{mm}$

$$\begin{aligned} \Delta J &= N \cdot m_\mathrm{S} \cdot r_\mathrm{S}^2 \\ &= 2 \cdot 0.5\,\mathrm{mg} \cdot (5\,\mathrm{mm})^2 \\ &= 0.25\,\mathrm{mg{\cdot}cm^2} \end{aligned}$$

The two screws add $0.25\,\mathrm{mg{\cdot}cm^2}$.

Rate Change due to the Moment of Inertia

(5.19) APPROXIMATION MASTER

$$\Delta G \approx -\frac{\Delta J}{2 \cdot J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $$\Delta J \approx -\frac{2 \cdot J \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$$

A more inert balance oscillates more slowly; hence the minus sign. 1 % more moment of inertia costs 432 s per day.

Valid: small changes (up to a few percent); balance spring unchanged.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
ΔJ	change in moment of inertia (+ means: more inert)	$\mathrm{mg{\cdot}cm^2}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE The two screws from the previous example are fitted to a balance with $10\,\mathrm{mg{\cdot}cm^2}$.

Given: $\Delta J = 0.25\,\mathrm{mg{\cdot}cm^2}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx -\frac{\Delta J}{2 \cdot J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{0.25\,\mathrm{mg{\cdot}cm^2}}{2 \cdot 10\,\mathrm{mg{\cdot}cm^2}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -1080\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The watch becomes 18 minutes per day slower. Conversely, a large losing rate can be made up with two lighter screws.

Fine Regulation with Timing Screws

(5.20) APPROXIMATION MASTER

$$\Delta G \approx -\frac{N \cdot m_\mathrm{S} \cdot r_\mathrm{S} \cdot \Delta r}{J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

Free-sprung balances without a regulator are regulated by moving masses on the rim inwards or outwards: timing screws are turned, eccentric masselottes rotated. Outwards means more inert, and the watch becomes slower.

Valid: small shifts; all N masses adjusted equally. Unequal adjustment creates an out-of-poise error.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	number (of identical screws or masselottes)	1
m_S	mass of one screw	mg
r_S	distance of its centre of gravity from the balance axis	mm
Δr	shift of the centre of gravity (+ means: outwards)	mm
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE Two timing screws of 1.5 mg each sit 4.5 mm from the axis on a balance with $10\,\mathrm{mg{\cdot}cm^2}$. Both are turned out by 0.01 mm.

Given: $N = 2$; $m_\mathrm{S} = 1.5\,\mathrm{mg}$; $r_\mathrm{S} = 4.5\,\mathrm{mm}$; $\Delta r = 0.01\,\mathrm{mm}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx -\frac{N \cdot m_\mathrm{S} \cdot r_\mathrm{S} \cdot \Delta r}{J} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{2 \cdot 1.5\,\mathrm{mg} \cdot 4.5\,\mathrm{mm} \cdot 0.01\,\mathrm{mm}}{10\,\mathrm{mg{\cdot}cm^2}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -11.7\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

One hundredth of a millimetre makes about 12 s per day. Turning them in has the same effect in the other direction.

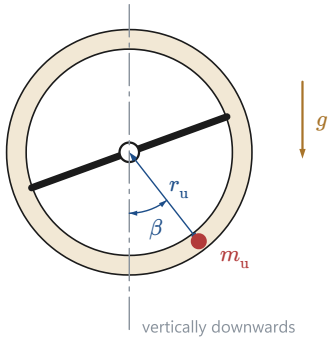


Fig. 5.9 Out-of-poise in a vertical position: excess weight m_u at distance r_u, angle β

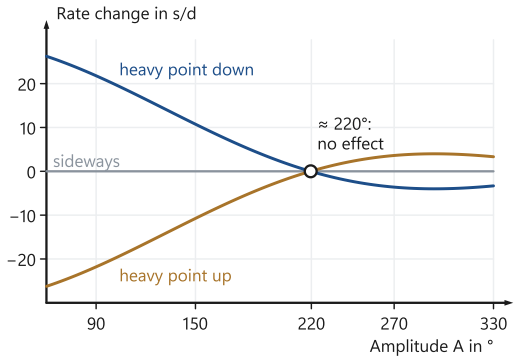


Fig. 5.10 Rate change due to 10 µg excess weight on the rim (example for formula (5.21)), as a function of amplitude

Rate Change due to Out-of-Poise (Centre-of-Gravity Error)

(5.21) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{k_A \cdot m_u \cdot g \cdot r_u \cdot \cos\beta}{2 \cdot D} \cdot 86\,400\ \frac{s}{d}$$

In the vertical positions an excess weight acts like a small pendulum on the balance. Below about 220° amplitude the watch becomes faster when the heavy point is at the bottom, above it the watch becomes slower; when it is at the top, the reverse applies. At about 220° the effect cancels out.

Valid: watch in a vertical position (balance staff horizontal), small excess weight, sinusoidal oscillation; β is measured with the balance at its rest position. In the horizontal positions the out-of-poise has no influence on the rate.

ΔG	rate change (+ means: the watch becomes faster)	$\frac{s}{d}$
k_A	amplitude factor of the out-of-poise (from the table below)	1
m_u	excess weight (mass that shifts the centre of gravity off the axis)	µg
g	gravitational acceleration	$\frac{m}{s^2}$
r_u	distance of the excess weight from the balance axis	mm
β	angle (of the excess weight from the vertical downwards; balance at rest position)	°
D	restoring torque coefficient (of the balance spring)	$\frac{\mu N \cdot m}{rad}$

Good to know: This is how an out-of-poise error is recognized: the rates of opposite vertical positions deviate from the mean in opposite directions, and the difference grows as the amplitude falls. Poising is therefore checked at low amplitude.

EXAMPLE An excess weight of 10 µg sits on a rim 4.5 mm from the axis, exactly at the bottom in the vertical position tested ($\beta = 0°$). The watch runs at 270° amplitude ($k_A = -0.12$), the balance spring has $D = 0.632\ \frac{\mu N \cdot m}{rad}$.

Given: $k_A = -0.12$; $m_u = 10\,\mu g$; $g = 9.81\ \frac{m}{s^2}$; $r_u = 4.5\,mm$; $\beta = 0°$; $D = 0.632\ \frac{\mu N \cdot m}{rad}$

$$\Delta G \approx \frac{k_A \cdot m_u \cdot g \cdot r_u \cdot \cos\beta}{2 \cdot D} \cdot 86\,400\ \frac{s}{d}$$

$$= \frac{-0.12 \cdot 10\,\mu g \cdot 9.81\ \frac{m}{s^2} \cdot 4.5\,mm \cdot \cos(0°)}{2 \cdot 0.632\ \frac{\mu N \cdot m}{rad}} \cdot 86\,400\ \frac{s}{d}$$

$$\approx -3.6\ \frac{s}{d}$$

The out-of-poise makes the watch 3.6 s per day slower in this position, 3.6 s per day faster in the opposite position. If the amplitude falls to 150° ($k_A = 0.36$), this becomes a gain of about 11 s per day.

Amplitude Factor k_A of the Out-of-Poise

Amplitude A	very small	90°	120°	150°	180°	200°	220°	240°	270°	300°	330°
k_A	1	0.72	0.54	0.36	0.18	0.08	0.00	−0.06	−0.12	−0.13	−0.11

Calculated from the sinusoidal oscillation of the balance in the gravitational field; zero at 219.5°.

Key point: Heavy point at the bottom: below 220° amplitude the watch gains, above 220° it loses.

5.4 Regulation from Nominal and Actual Values

Regulating means matching moment of inertia and restoring torque coefficient to each other so that the nominal frequency results. The rate shows how far the frequency is still off.

Rate from the Frequency Deviation

(5.22) JOURNEYMAN

$$G = \frac{f_{\text{ist}} - f_{\text{soll}}}{f_{\text{soll}}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}}$$

G	rate (+ means: the watch gains)	$\frac{\text{s}}{\text{d}}$
f_{ist}	measured frequency	Hz
f_{soll}	nominal frequency	Hz

If the balance oscillates too fast, the motion work counts too many beats: the watch gains. The factor $86\,400\ \frac{\text{s}}{\text{d}}$ converts the relative deviation into seconds per day. The same calculation with beat rates is given in Section 8.1, for quartz in Section 10.5.

Valid: frequency averaged over the measuring time. The beat rates can be inserted instead of the frequencies.

EXAMPLE A balance with a nominal frequency of 4 Hz oscillates at 4.0005 Hz.

Given: $f_{\text{ist}} = 4.0005\,\text{Hz}$; $f_{\text{soll}} = 4\,\text{Hz}$

$$\begin{aligned} G &= \frac{f_{\text{ist}} - f_{\text{soll}}}{f_{\text{soll}}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &= \frac{4.0005\,\text{Hz} - 4\,\text{Hz}}{4\,\text{Hz}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &= 10.8\ \tfrac{\text{s}}{\text{d}} \end{aligned}$$

The watch gains 10.8 s per day. Even one eight-thousandth too much frequency is clearly visible as rate.

Rate Change from Restoring Torque Coefficient and Moment of Inertia (5.23) APPROXIMATION MASTER

$$\Delta G \approx \frac{1}{2} \cdot \left(\frac{\Delta D}{D} - \frac{\Delta J}{J} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

This is the basic formula of regulation. The regulator formula (5.16) and the formulas for screws and masselottes follow from it. To eliminate a rate G, the rate change $\Delta G = -G$ is required.

Valid: small changes (up to a few percent).

ΔG	rate change (+ means: the watch becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
ΔD	change in restoring torque coefficient (+ means: stronger balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
ΔJ	change in moment of inertia (+ means: more inert balance)	$\mathrm{mg{\cdot}cm^2}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE During a repair the balance spring is replaced; the new one has a restoring torque coefficient $0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ higher than the old one with $0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. At the same time the balance with $10\,\mathrm{mg{\cdot}cm^2}$ becomes more inert by $0.010\,\mathrm{mg{\cdot}cm^2}$ due to washers.

Given: $\Delta D = 0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $\Delta J = 0.010\,\mathrm{mg{\cdot}cm^2}$; $J = 10\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}\Delta G &\approx \frac{1}{2} \cdot \left(\frac{\Delta D}{D} - \frac{\Delta J}{J} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{1}{2} \cdot \left(\frac{0.002\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}} - \frac{0.010\,\mathrm{mg{\cdot}cm^2}}{10\,\mathrm{mg{\cdot}cm^2}} \right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 93.5\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

As a result the watch becomes about one and a half minutes per day faster: the stronger balance spring gives 137 s, the more inert balance takes away 43 s.

Matching: Restoring Torque Coefficient for Balance and Beat Rate (5.24) MASTER

$$D = \pi^2 \cdot n_\mathrm{h}^2 \cdot J$$

This is how the balance spring is selected for the balance. Written with the frequency, the formula is $D = 4 \cdot \pi^2 \cdot f^2 \cdot J$.

Valid: insert n_h in beats per second and J in kg·m²; then D is obtained in N·m per rad.

D	restoring torque coefficient (of the balance spring; sought)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
n_h	beat rate (nominal value)	$\frac{\mathrm{A}}{\mathrm{h}}$
J	moment of inertia (of the balance)	$\mathrm{mg{\cdot}cm^2}$

EXAMPLE A balance with $12\,\mathrm{mg{\cdot}cm^2}$ is to beat 21 600 A/h, i.e. 6 beats per second.

Given: $n_\mathrm{h} = 21\,600\ \frac{\mathrm{A}}{\mathrm{h}}$; $J = 12\,\mathrm{mg{\cdot}cm^2}$

$$\begin{aligned}D &= \pi^2 \cdot n_\mathrm{h}^2 \cdot J \\ &= \pi^2 \cdot \left(21\,600\ \tfrac{\mathrm{A}}{\mathrm{h}}\right)^2 \cdot 12\,\mathrm{mg{\cdot}cm^2} \\ &\approx 0.426\ \tfrac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}\end{aligned}$$

The balance spring needs about $0.43\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$. For 28 800 A/h it would have to be almost twice as strong on the same balance.

Measuring the Moment of Inertia with an Additional Disc

(5.25) ADVANCED

$$J = \frac{J_Z}{\left(\frac{T_2}{T_1}\right)^2 - 1}$$

J	moment of inertia (of the balance; sought)	$\mathrm{mg{\cdot}cm^2}$
J_Z	known moment of inertia of the attached disc	$\mathrm{mg{\cdot}cm^2}$
T_1	period without disc	s
T_2	period with disc	s

An unknown moment of inertia cannot be weighed, but it can be measured via the period: a disc with a known moment of inertia makes the oscillation slower by a calculable amount.

Valid: the same balance spring for both measurements, disc mounted concentrically, free oscillation without escapement.

EXAMPLE A balance oscillates on its balance spring with 0.250 s. With an attached disc of $5\,\mathrm{mg{\cdot}cm^2}$ the oscillation lasts 0.306 s.

Given: $J_Z = 5\,\mathrm{mg{\cdot}cm^2}$; $T_1 = 0.250\,\mathrm{s}$; $T_2 = 0.306\,\mathrm{s}$

$$J = \frac{J_Z}{\left(\frac{T_2}{T_1}\right)^2 - 1} = \frac{5\,\mathrm{mg{\cdot}cm^2}}{\left(\frac{0.306\,\mathrm{s}}{0.250\,\mathrm{s}}\right)^2 - 1} \approx 10.0\,\mathrm{mg{\cdot}cm^2}$$

The balance has about $10\,\mathrm{mg{\cdot}cm^2}$. From J and T_1, formula (5.1) also gives the restoring torque coefficient of the balance spring.

5.5 Temperature

Heat changes the oscillator in two ways. The balance spring becomes softer because its modulus of elasticity decreases, and all parts expand. With a steel balance spring the first effect predominates by far: the watch loses heavily in warmth.

Two solutions have become established. The compensation balance has a cut rim of steel on the inside and brass on the outside; in warmth the free ends bend inwards, the moment of inertia decreases and compensates for the softer balance spring. Today self-compensating balance springs made of nickel-iron alloys such as Nivarox or of oxidized silicon are used, whose modulus of elasticity hardly changes with temperature, together with a monometallic balance.

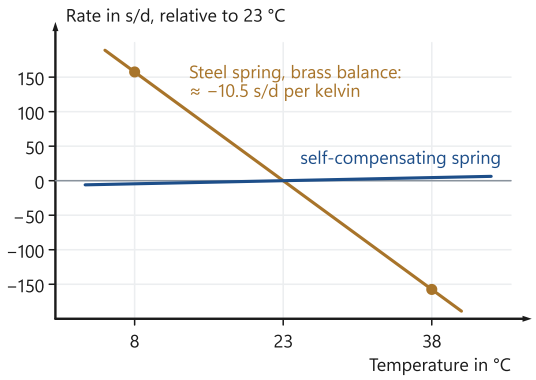


Fig. 5.11 Rate over temperature: steel balance spring with monometallic balance and self-compensating balance spring

Temperature Coefficient from Material Data

(5.26) APPROXIMATION ADVANCED

$$C \approx \frac{1}{2} \cdot (\beta_E + 3 \cdot \alpha_S - 2 \cdot \alpha_U) \cdot 86\,400\ \frac{s}{d}$$

The balance spring becomes softer in warmth (β_E is negative for steel) and at the same time larger in all dimensions; the balance becomes larger and thus more inert. The formula adds up the three contributions.

Valid: monometallic, uncut balance; uniform heating; small changes in temperature. The influence of oil, escapement and regulator pins is not included.

C	temperature coefficient of the rate (+ means: the watch gains in warmth)	$\frac{s}{d \cdot K}$
β_E	temperature coefficient of the modulus of elasticity of the balance spring	$\frac{1}{K}$
α_S	coefficient of linear expansion of the balance spring	$\frac{1}{K}$
α_U	coefficient of linear expansion of the balance	$\frac{1}{K}$

EXAMPLE A steel balance spring ($\beta_E = -240 \cdot 10^{-6}\ \frac{1}{K}$, $\alpha_S = 11.5 \cdot 10^{-6}\ \frac{1}{K}$) works with a monometallic brass balance ($\alpha_U = 18.5 \cdot 10^{-6}\ \frac{1}{K}$).

Given: $\beta_E = -240 \cdot 10^{-6}\ \frac{1}{K}$; $\alpha_S = 11.5 \cdot 10^{-6}\ \frac{1}{K}$; $\alpha_U = 18.5 \cdot 10^{-6}\ \frac{1}{K}$

$$\begin{aligned} C &\approx \frac{1}{2} \cdot (\beta_E + 3 \cdot \alpha_S - 2 \cdot \alpha_U) \cdot 86\,400\ \frac{s}{d} \\ &= \frac{1}{2} \cdot \left(-240 \cdot 10^{-6}\ \frac{1}{K} + 3 \cdot 11.5 \cdot 10^{-6}\ \frac{1}{K} - 2 \cdot 18.5 \cdot 10^{-6}\ \frac{1}{K}\right) \cdot 86\,400\ \frac{s}{d} \\ &\approx -10.5\ \frac{s}{d \cdot K} \end{aligned}$$

The watch loses about 10.5 s per day per kelvin of warming, i.e. more than 5 minutes per day between 8 °C and 38 °C. Almost all of it comes from the modulus of elasticity.

Materials of Balance Spring and Balance (Guide Values at 20 °C)

Material	Use	β_E in $10^{-6}\ \frac{1}{K}$	α in $10^{-6}\ \frac{1}{K}$
carbon steel, hardened	balance spring of older watches	−200 … −270	11 … 13
Nivarox and similar nickel-iron alloys	self-compensating balance spring	about 0, adjustable	um 8
silicon, uncoated	for comparison; not compensated without oxide layer	um −60	2.6
silicon with oxide layer	self-compensating balance spring	about 0, adjustable	um 2.6
brass	monometallic balance		18 … 21
beryllium bronze (Glucydur)	monometallic balance		17

For self-compensating balance springs, the manufacturer sets β_E so that it just compensates for the expansion of balance spring and balance.

Temperature Coefficient from Measurement

(5.27) MASTER

$$C = \frac{G_2 - G_1}{\vartheta_2 - \vartheta_1}$$

REARRANGED $G_2 = G_1 + C \cdot (\vartheta_2 - \vartheta_1)$

Testing is done at two temperatures in the same position and with the same state of wind. Usual values are 8 °C and 38 °C; the denominator is then 30 K. In chronometer testing both temperatures are fixed (Section 8.4).

Valid: mean coefficient between the two test temperatures; the watch must be thoroughly warmed or cooled before each measurement.

C	temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}}$
G_2	rate at the higher temperature	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	rate at the lower temperature	$\frac{\mathrm{s}}{\mathrm{d}}$
ϑ_2	higher test temperature	$^\circ\mathrm{C}$
ϑ_1	lower test temperature	$^\circ\mathrm{C}$

EXAMPLE A watch gains 6.0 s per day at 8 °C and 1.5 s per day at 38 °C, each in the same position.

Given: $G_2 = 1.5\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 6.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $\vartheta_2 = 38^\circ\mathrm{C}$; $\vartheta_1 = 8^\circ\mathrm{C}$

$$C = \frac{G_2 - G_1}{\vartheta_2 - \vartheta_1} = \frac{1.5\ \frac{\mathrm{s}}{\mathrm{d}} - 6.0\ \frac{\mathrm{s}}{\mathrm{d}}}{38^\circ\mathrm{C} - 8^\circ\mathrm{C}} = -0.15\ \frac{\mathrm{s}}{\mathrm{d}\cdot\mathrm{K}}$$

The watch loses 0.15 s per day per kelvin of warming. Chronometer testing according to ISO 3159 permits up to 0.6 s per day per kelvin in either direction for movements over 20 mm in diameter.

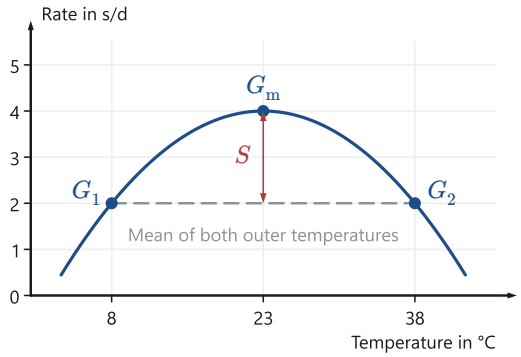


Fig. 5.12 Middle temperature error S: at the middle temperature the watch gains, although the rates G_1 and G_2 at 8 °C and 38 °C are equal

Middle Temperature Error

(5.28) ADVANCED

$$S = G_{\mathrm{m}} - \frac{G_1 + G_2}{2}$$

The compensation balance compensates almost linearly, but the steel balance spring does not soften linearly. If the rate is correct at the two outer temperatures, the watch gains in between. Self-compensating balance springs hardly have this error any more.

Valid: three measurements in the same position; middle temperature exactly between the two outer ones.

S	middle temperature error (+ means: the watch gains at the middle temperature)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{m}	rate at the middle temperature (23 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	rate at the lower temperature (8 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_2	rate at the higher temperature (38 °C)	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE A pocket watch with compensation balance and steel balance spring gains 2.0 s per day at both 8 °C and 38 °C, but 4.0 s per day at 23 °C.

Given: $G_{\mathrm{m}} = 4.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_2 = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} S &= G_{\mathrm{m}} - \frac{G_1 + G_2}{2} \\ &= 4.0\ \tfrac{\mathrm{s}}{\mathrm{d}} - \frac{2.0\ \frac{\mathrm{s}}{\mathrm{d}} + 2.0\ \frac{\mathrm{s}}{\mathrm{d}}}{2} \\ &= 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The middle temperature error is 2 s per day; a few seconds per day are typical for this design. The temperature coefficient according to formula (5.27) is zero here and does not reveal the error.

Key point: Steel balance spring with plain balance: about 11 s per day loss per kelvin of warming.

5.6 Energy and Quality Factor

The oscillating balance is an energy store. In each oscillation a small part of the energy is lost through air and bearing friction; the escapement must supply exactly this part. The smaller the loss, the less the escapement disturbs the rate.

Energy of the Oscillating Balance

(5.29) MASTER

$$W = \frac{D \cdot A^2}{2}$$

At the turning point the entire energy is in the tensioned balance spring, at the rest position in the motion of the balance: $W = \frac{1}{2} \cdot J \cdot \omega_{\max}^2$. The energy increases with the square of the amplitude. It is the energy of a torsion spring according to formula (2.26).

Valid: insert the amplitude in radians: degrees times $\frac{\pi}{180}$.

W	energy (of the oscillation)	$\mu\mathrm{J}$
D	restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$
A	amplitude (in radians)	rad

EXAMPLE What energy is contained in a balance with $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$ at 270° amplitude (4.71 rad)?

Given: $D = 0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$; $A = 4.71\,\mathrm{rad}$

$$\begin{aligned} W &= \frac{D \cdot A^2}{2} \\ &= \frac{0.632\ \frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}} \cdot (4.71\,\mathrm{rad})^2}{2} \\ &\approx 7.01\,\mu\mathrm{J} \end{aligned}$$

About 7 µJ. At 220° it would be only 4.7 µJ, two thirds of this.

Quality Factor

(5.30) ADVANCED

$$Q = \frac{2 \cdot \pi \cdot W}{\Delta W}$$

Q	quality factor	1
W	energy (of the oscillation)	μJ
ΔW	energy loss per full oscillation	μJ

The quality factor compares the stored energy with the loss per oscillation. A high quality factor means: the oscillator needs little replenishment and is little disturbed by the escapement.

Valid: weakly damped oscillator (Q much greater than 1). For the balance, Q depends somewhat on amplitude and position.

EXAMPLE The balance from the previous example loses 0.176 µJ per oscillation.

Given: $W = 7.01\,\mu J$; $\Delta W = 0.176\,\mu J$

$$\begin{aligned} Q &= \frac{2 \cdot \pi \cdot W}{\Delta W} \\ &= \frac{2 \cdot \pi \cdot 7.01\,\mu J}{0.176\,\mu J} \\ &\approx 250 \end{aligned}$$

A quality factor of around 250 is usual for a wristwatch. A good seconds pendulum reaches a few thousand to over ten thousand, a watch quartz several tens of thousands.

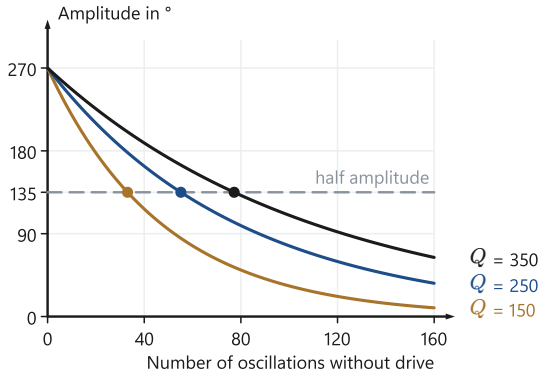


Fig. 5.13 Free decay without drive: the amplitude halves after about $0.22 \cdot Q$ oscillations

Free Decay without Drive

(5.31) APPROXIMATION ADVANCED

$$A_N \approx A_0 \cdot e^{-\frac{\pi \cdot N_S}{Q}}$$

REARRANGED $$Q \approx \frac{\pi \cdot N_S}{\ln\left(\frac{A_0}{A_N}\right)}$$

A_N	amplitude after N_S oscillations	°
A_0	amplitude at the start	°
N_S	number of full oscillations (without drive)	1
Q	quality factor	1

Without drive the oscillation decays. The amplitude halves after about $0.22 \cdot Q$ oscillations. Conversely, a decay test gives the quality factor.

Valid: friction proportional to speed (air friction), Q constant during the test. Bearing friction makes the amplitude fall faster towards the end.

EXAMPLE A balance with $Q = 250$ oscillates at 270° and is no longer driven. What is the amplitude after 55 oscillations?

Given: $A_0 = 270°$; $N_S = 55$; $Q = 250$

$$A_N \approx A_0 \cdot e^{-\frac{\pi \cdot N_S}{Q}}$$
$$= 270° \cdot e^{-\frac{\pi \cdot 55}{250}}$$
$$\approx 135°$$

After 55 oscillations, i.e. after just under 14 s at 4 Hz, the amplitude has fallen to half.

Power Requirement of the Balance

(5.32) MASTER

$$P = \Delta W \cdot f$$

P	power (that the escapement must deliver to the balance)	µW
ΔW	energy loss per full oscillation	µJ
f	frequency (full oscillations per second)	Hz

In equilibrium the escapement replaces exactly the loss in each oscillation. Written with the quality factor: $P = \frac{2 \cdot \pi \cdot f \cdot W}{Q}$. Here P is the power, not the thread pitch.

Valid: steady state with constant amplitude.

EXAMPLE The balance from the previous examples loses 0.176 µJ per oscillation and oscillates at 4 Hz.

Given: $\Delta W = 0.176\,\mu J$; $f = 4\,Hz$

$$P = \Delta W \cdot f$$
$$= 0.176\,\mu J \cdot 4\,Hz$$
$$= 0.704\,\mu W$$

The balance needs about 0.7 µW. The barrel must supply several times this, because gear train and escapement pass on only part of it (Chapters 3 and 7).

Key point: Double the amplitude, four times the energy.

5.7 Isochronism

Isochronous means of equal duration: an ideal oscillator needs the same time for large and small oscillations. Formula (5.1) satisfies this, the real watch only approximately. Because the amplitude decreases as the mainspring runs down and in the vertical positions, every isochronism error shows up as a rate difference between the fully wound and the almost run-down watch and between the positions.

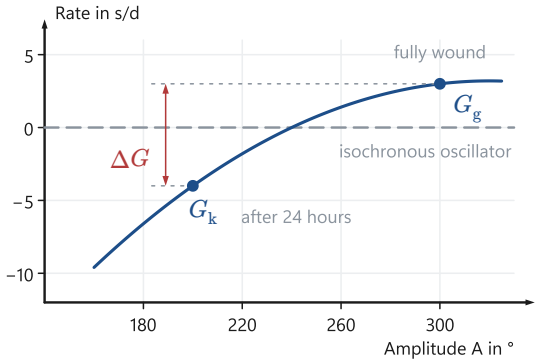


Fig. 5.14 Rate–amplitude diagram (principle): the isochronism error ΔG is the rate difference between small and large amplitude

The **isochronism error** is the rate difference between small and large amplitude, measured in the same position and at the same temperature (Fig. 5.14). The simplest way is to measure fully wound and after 24 hours of running; the formula and an example are given in Section 8.3, formula (8.14). If the watch becomes slower at small amplitude, this is often due to too much play of the balance spring between the regulator pins. Further causes are listed in the table.

What Disturbs Isochronism

Cause	Effect at small amplitude	Remedy
play of the balance spring between the regulator pins	losing: the balance spring rests against the pin for a smaller part of the oscillation and acts longer on average	keep play small, place the balance spring centrally between the pins
escapement (unlocking before, impulse around the rest position)	usually losing, because the engagement takes up a larger part of the oscillation	free-running escapement, high amplitude, high quality factor
out-of-poise of the balance	in the vertical positions gaining or losing depending on the position of the heavy point (Section 5.3)	poise
eccentric breathing of the flat balance spring	centre of gravity of the balance spring shifts, the pivots are loaded laterally; the rate depends on amplitude and position	Breguet terminal curve or balance spring with adapted shape
position of the inner pinning point at the collet	gaining or losing in the vertical positions depending on the position of the pinning point	pinning point according to manufacturer's specification; used deliberately in fine adjustment

The effects are superimposed. In fine adjustment they are matched so that they cancel each other as far as possible.

The Breguet terminal curve leads the outermost coil upwards and in a tighter arc to the stud. With the correct shape the balance spring opens and closes evenly on all sides, its centre of gravity remains on the axis and the pivots are not pushed sideways. There are design rules for the shape of the terminal curve, but no simple formula; it is bent according to a template and checked against the rate.

Key point: An oscillator is isochronous if the rate does not depend on the amplitude.

6 Pendulum

The pendulum is the oscillator of the clock. Its period depends only on its length and on gravitational acceleration, not on the mass of the bob, and at small amplitudes hardly on the amplitude. That is why a pendulum clock is regulated by its length: bob higher, clock faster.

The formulas apply to small amplitudes of a few degrees, as usual in clocks. The effects of larger amplitudes, temperature and location are covered in Sections 6.4 and 6.5.

6.1 Simple pendulum

The simple pendulum is the model we calculate with: a point mass on a weightless thread. Real pendulums deviate only slightly from it; how to calculate them exactly is shown in Section 6.3.

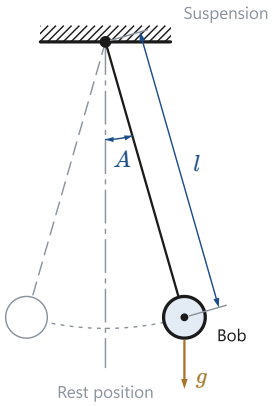


Fig. 6.1 Simple pendulum: length l to the centre of gravity of the bob, amplitude A (exaggerated for clarity)

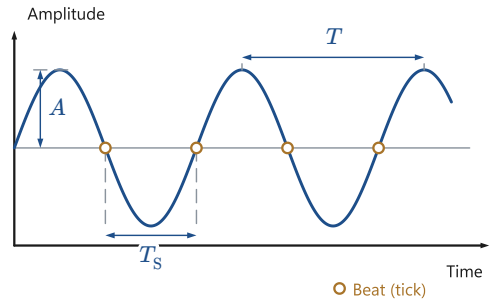


Fig. 6.2 A beat is a vibration (half-oscillation): $T_S = \frac{1}{2} T$

Period

(6.1) JOURNEYMAN

$$T = 2 \cdot \pi \cdot \sqrt{\frac{l}{g}}$$

REARRANGED $l = \frac{g \cdot T^2}{4 \cdot \pi^2}$

T	Period (full oscillation, there and back)	s
l	Pendulum length (point of suspension to centre of gravity of the bob)	m
g	Gravitational acceleration (in Central Europe $9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$)	$\frac{\mathrm{m}}{\mathrm{s}^2}$

The period grows with the square root of the length: a pendulum four times as long swings half as fast.

Valid: small amplitudes (up to about 5°), point-shaped bob, no friction. For a real pendulum, replace l by the equivalent pendulum length l_r (Section 6.3).

Good to know: The mass of the bob does not appear in the formula. A heavier bob does not change the period; it only makes the pendulum less sensitive to disturbances from the escapement and the gear train.

EXAMPLE A pendulum measures 0.5 m from the point of suspension to the centre of the bob. What is its period?

Given: $l = 0.5\,\mathrm{m}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{l}{g}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{0.5\,\mathrm{m}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}} \\ &\approx 1.42\,\mathrm{s} \end{aligned}$$

One full oscillation, there and back, takes about 1.42 s; each beat lasts 0.71 s.

Beat duration (vibration)

(6.2) JOURNEYMAN

$$T_\mathrm{S} = \pi \cdot \sqrt{\frac{l}{g}}$$

T_S	Beat duration (vibration, from one end point of the swing to the other)	s
l	Pendulum length	m
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$

Watchmakers count beats. A beat is a vibration (half-oscillation), i.e. half the period; each beat can be heard as a tick.

Valid: as for formula (6.1).

EXAMPLE How long does one beat last for a pendulum 994 mm long?

Given: $l = 0.994\,\mathrm{m}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned} T_\mathrm{S} &= \pi \cdot \sqrt{\frac{l}{g}} \\ &= \pi \cdot \sqrt{\frac{0.994\,\mathrm{m}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}} \\ &\approx 1.00\,\mathrm{s} \end{aligned}$$

Exactly one second: a pendulum of 994 mm is a seconds pendulum, as found in regulators and many longcase clocks.

Beat rate from the period

(6.3) JOURNEYMAN

$$n_{\mathrm{h}} = \frac{2}{T}$$

REARRANGED $T = \frac{2}{n_{\mathrm{h}}}$

n_{h}	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
T	Period	s

The pendulum beats twice per full oscillation. For clocks the beat rate is often given per minute, for watches in vibrations per hour (A/h).

EXAMPLE A pendulum has a period of 1.5 s. What is its beat rate per hour?

Given: $T = 1.5\,\mathrm{s}$

$$\begin{aligned} n_{\mathrm{h}} &= \frac{2}{T} \\ &= \frac{2}{1.5\,\mathrm{s}} \\ &= \frac{2}{1{,}5\,\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &= 4800\,\frac{\mathrm{A}}{\mathrm{h}} \end{aligned}$$

4800 beats per hour, i.e. 80 beats per minute.

Pendulum length from the beat rate

(6.4) JOURNEYMAN

$$l = \frac{g}{\pi^2 \cdot n_{\mathrm{h}}^2}$$

l	Pendulum length	m
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
n_{h}	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$

This is how the length of a missing pendulum is found: calculate the beat rate from the tooth counts of the gear train (Chapter 4), then the length.

Valid: as for the period, formula (6.1); insert n_{h} in beats per second when g is in $\frac{\mathrm{m}}{\mathrm{s}^2}$.

EXAMPLE A movement is designed for 90 beats per minute, i.e. 1.5 beats per second. How long must the pendulum be?

Given: $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$; $\quad n_{\mathrm{h}} = 1.5\,\frac{1}{\mathrm{s}}$

$$\begin{aligned} l &= \frac{g}{\pi^2 \cdot n_{\mathrm{h}}^2} \\ &= \frac{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{\pi^2 \cdot \left(1.5\,\frac{1}{\mathrm{s}}\right)^2} \\ &\approx 0.442\,\mathrm{m} \end{aligned}$$

The pendulum must be about 442 mm long from the point of suspension to the centre of the bob.

Pendulum length from beats per minute (workshop formula)

(6.5) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$l = \frac{3\,578\,400}{n_{\mathrm{h}}^2}$$

l	Pendulum length in mm	mm
n_{h}	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$

The number 3 578 400 is the length of the seconds pendulum (994 mm) times 60^2. Insert values only in the units stated.

Valid: $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; l in mm, n_{h} in beats per minute.

EXAMPLE How long is the pendulum for 90 beats per minute?

Given: $n_{\mathrm{h}} = 90\ \frac{1}{\mathrm{min}}$

$$\begin{aligned} l &= \frac{3\,578\,400}{n_{\mathrm{h}}^2} \\ &= \frac{3\,578\,400}{90^2} \\ &\approx 442\,\mathrm{mm} \end{aligned}$$

About 442 mm, as with the previous formula.

Key point: A pendulum of 994 mm beats seconds. A pendulum four times as long beats half as often.

6.2 Regulating by the pendulum length

If a pendulum clock gains, the pendulum is lengthened; if it loses, it is shortened. Because the period grows with the square root of the length, 1 ‰ more length makes the oscillation 0.5 ‰ slower, which is 43 s per day.

Rate change from a change in length

(6.6) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{\Delta l}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta l \approx -\dfrac{2 \cdot l \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δl	Change in length (+ means: pendulum longer, bob lower)	mm
l	Pendulum length	mm

A longer pendulum swings more slowly and the clock loses; hence the minus sign. The factor $86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$ converts the relative change into seconds per day.

Valid: small changes (Δl much smaller than l); pendulum without a heavy rod.

Good to know: On pendulums with a heavy rod, the centre of oscillation moves less than the bob. The bob must then be moved further than calculated. Calculating with the pendulum length from the beat rate, a rod with 10 %, 20 % and 30 % of the bob mass needs about 3 %, 7 % and 10 % more travel. For the real pendulum, the factor 2.2 is therefore often used instead of 2, see formula (6.7).

EXAMPLE A longcase clock with a pendulum length of 994 mm gains 30 s per day. It must therefore become 30 s per day slower. By how much must the bob be lowered?

Given: $\Delta G = -30\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad l = 994\,\mathrm{mm}$

$$\begin{aligned} \Delta l &\approx -\frac{2 \cdot l \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &= -\frac{2 \cdot 994\,\mathrm{mm} \cdot \left(-30\ \frac{\mathrm{s}}{\mathrm{d}}\right)}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &\approx 0.69\,\mathrm{mm} \end{aligned}$$

The bob must go about 0.7 mm lower, i.e. the pendulum must become longer.

Bob adjustment from rate and beat rate (workshop formula)

(6.7) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$\Delta l \approx \frac{90 \cdot G}{n_{\mathrm{h}}^2}$$

Δl	Bob travel in mm (+ means: bob lower)	mm
G	Rate of the clock in seconds per day (+ means: clock gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
n_{h}	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$

If the clock gains, the bob goes lower; if it loses, it goes higher. The number 90 contains the pendulum length from the beat rate and an allowance for the real pendulum. For the simple pendulum, formulas (6.5) and (6.6) together give $\frac{2 \cdot 3\,578\,400}{86\,400} \approx 83$. With the factor 2.2 instead of 2 this becomes 91, rounded to 90.

Valid: Δl in mm, G in seconds per day, n_{h} in beats per minute. Guide value for pendulums whose rod swings with the bob; for a very light rod use 83 instead of 90. After the adjustment the rate is measured again.

EXAMPLE The longcase clock from the example for formula (6.6) beats 60 times a minute and gains 30 s per day. By how much is the bob lowered?

Given: $G = 30\ \frac{\mathrm{s}}{\mathrm{d}}$; $n_{\mathrm{h}} = 60\ \frac{1}{\mathrm{min}}$

$$\begin{aligned}\Delta l &\approx \frac{90 \cdot G}{n_{\mathrm{h}}^2} \\ &= \frac{90 \cdot 30}{60^2} \\ &= 0.75\,\mathrm{mm}\end{aligned}$$

The bob goes 0.75 mm lower. For the simple pendulum it was 0.69 mm; the difference is the allowance for the rod swinging with the bob.

Rate change with the rating nut

(6.8) APPROXIMATION JOURNEYMAN

$$\Delta G \approx -\frac{N \cdot P}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $$N \approx -\frac{2 \cdot l \cdot \Delta G}{P \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	Turns (of the nut; + means: bob lower)	1
P	Thread pitch (of the rating nut)	mm
l	Pendulum length	mm

One full turn of the nut moves the bob by one thread pitch.

Valid: Bob rests on the nut; pendulum without a heavy rod.

EXAMPLE How much does one full turn of the rating nut (pitch 0.5 mm) change the rate of a seconds pendulum when the bob is lowered?

Given: $N = 1$; $P = 0.5\,\mathrm{mm}$; $l = 994\,\mathrm{mm}$

$$\begin{aligned}\Delta G &\approx -\frac{N \cdot P}{2 \cdot l} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{1 \cdot 0.5\,\mathrm{mm}}{2 \cdot 994\,\mathrm{mm}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx -21.7\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The clock becomes about 22 s per day slower. Finer corrections are made with partial turns; many nuts carry a graduation for this.

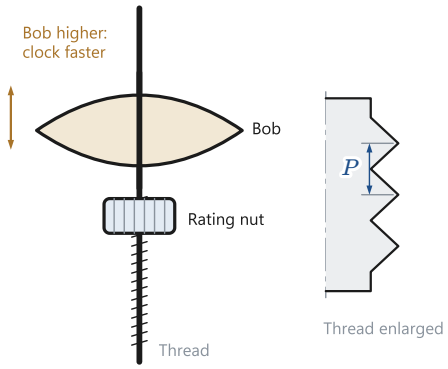


Fig. 6.3 The rating nut raises or lowers the bob by the pitch P per turn

Key point: Bob higher, clock faster. With the seconds pendulum, 1 mm gives about 43 s per day.

Timing weights. Precision pendulum clocks are finally regulated not with the nut but with small weights placed on a little tray on the pendulum rod. The pendulum keeps swinging meanwhile. Each weight added makes the clock faster, each weight removed makes it slower.

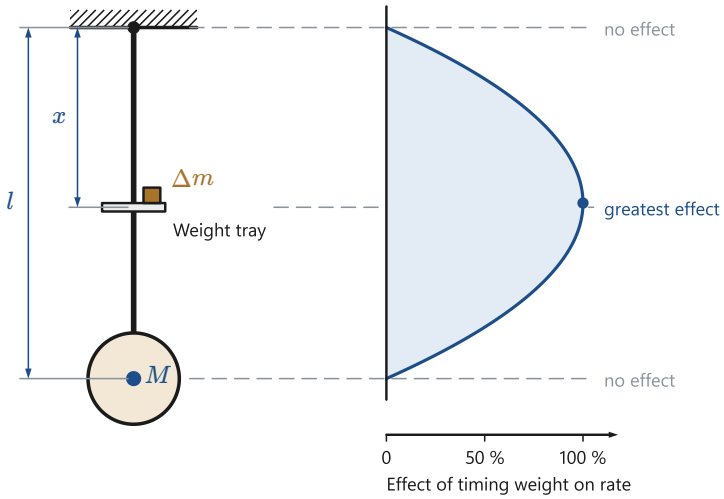


Fig. 6.4 Timing weight Δm at distance x below the suspension: the effect on the rate is greatest at half the pendulum length and vanishes at the suspension and at the centre of oscillation M

Timing weight at half the pendulum length

(6.9) APPROXIMATION JOURNEYMAN

$$\Delta G \approx \frac{\Delta m}{8 \cdot m} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

REARRANGED $\Delta m \approx \dfrac{8 \cdot m \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}}$

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δm	Mass of the timing weight (+ means: added)	g
m	Mass (of the pendulum)	kg

The weight sits above the centre of oscillation and slightly shortens the effective pendulum length. Because 86 400 divided by 8 equals 10 800, the workshop uses the shorter form $\Delta m = \frac{m \cdot \Delta G}{10\,800}$ with ΔG in seconds per day.

Valid: Tray at half the pendulum length, i.e. midway between the suspension and the centre of oscillation; weight much lighter than the pendulum; mass of the pendulum essentially in the bob. With a heavy rod the weight acts somewhat more strongly.

EXAMPLE A precision pendulum clock with a pendulum mass of 6 kg loses 0.5 s per day. Which timing weight compensates for this?

Given: $m = 6\,\mathrm{kg}$; $\Delta G = 0.5\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta m &\approx \frac{8 \cdot m \cdot \Delta G}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &= \frac{8 \cdot 6\,\mathrm{kg} \cdot 0.5\ \frac{\mathrm{s}}{\mathrm{d}}}{86\,400\ \frac{\mathrm{s}}{\mathrm{d}}} \\ &\approx 0.28\,\mathrm{g}\end{aligned}$$

About 0.28 g. Rule of thumb: the pendulum mass divided by 10 800, here 0.56 g, gives 1 s per day.

Timing weight at any height

(6.10) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{\Delta m \cdot x}{2 \cdot m \cdot l} \cdot \left(1 - \frac{x}{l}\right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

At the suspension and at the centre of oscillation an added weight has no effect. In between it makes the clock faster, most strongly in the middle. Below the centre of oscillation it makes the clock slower.

Valid: Weight much lighter than the pendulum; pendulum treated as a point mass at length l.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δm	Mass of the timing weight (+ means: added)	g
x	Distance (suspension to timing weight)	mm
m	Mass (of the pendulum)	kg
l	Pendulum length (suspension to centre of oscillation)	mm

Good to know: Around half the pendulum length the effect hardly changes with height. That is why the tray belongs there: a few millimetres do not matter. A sliding regulating weight, on the other hand, does not belong in the middle, because moving it there has almost no effect.

EXAMPLE The pendulum from the previous example (6 kg) is a seconds pendulum 994 mm long. A 1 g weight is placed on it not in the middle but 331 mm below the suspension, i.e. at one third of the length. How much does the rate change?

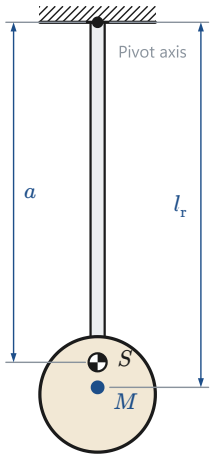
Given: $\Delta m = 1\,\mathrm{g}$; $x = 331\,\mathrm{mm}$; $m = 6\,\mathrm{kg}$; $l = 994\,\mathrm{mm}$

$$\begin{aligned}\Delta G &\approx \frac{\Delta m \cdot x}{2 \cdot m \cdot l} \cdot \left(1 - \frac{x}{l}\right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{1\,\mathrm{g} \cdot 331\,\mathrm{mm}}{2 \cdot 6\,\mathrm{kg} \cdot 994\,\mathrm{mm}} \cdot \left(1 - \frac{331\,\mathrm{mm}}{994\,\mathrm{mm}}\right) \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 1.60\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The clock becomes about 1.6 s per day faster. At half the length it would be 1.8 s.

6.3 Compound pendulum

A real pendulum has a rod with its own mass and a bob of finite size. It swings like a simple pendulum of equivalent length l_r. This length reaches to the centre of oscillation, which lies slightly below the centre of gravity.



S centre of gravity · M centre of oscillation

Fig. 6.5 Compound pendulum: centre of gravity S at distance a, centre of oscillation M at the equivalent length l_r

Period of the compound pendulum

(6.11) MASTER

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J_\mathrm{A}}{m \cdot g \cdot a}}$$

Valid: small amplitudes, no friction, rigid pendulum.

T	Period	s
J_A	Moment of inertia about the pivot axis (suspension)	$\mathrm{kg{\cdot}m^2}$
m	Mass (of the whole pendulum)	kg
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s^2}}$
a	Distance (pivot axis to centre of gravity)	m

EXAMPLE A uniform rod (length 1 m, mass 0.6 kg) swings about its upper end. Its moment of inertia about the end is $J_\mathrm{A} = \frac{1}{3} \cdot m \cdot L^2 = 0.2\,\mathrm{kg \cdot m^2}$; the centre of gravity lies in the middle.

Given: $J_\mathrm{A} = 0.2\,\mathrm{kg{\cdot}m^2}$; $m = 0.6\,\mathrm{kg}$; $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s^2}}$; $a = 0.5\,\mathrm{m}$

$$\begin{aligned} T &= 2 \cdot \pi \cdot \sqrt{\frac{J_\mathrm{A}}{m \cdot g \cdot a}} \\ &= 2 \cdot \pi \cdot \sqrt{\frac{0.2\,\mathrm{kg{\cdot}m^2}}{0.6\,\mathrm{kg} \cdot 9.81\,\frac{\mathrm{m}}{\mathrm{s^2}} \cdot 0.5\,\mathrm{m}}} \\ &\approx 1.64\,\mathrm{s} \end{aligned}$$

1.64 s; a simple pendulum of 1 m would have 2.01 s. The rod swings faster because part of its mass lies close to the pivot axis.

Equivalent pendulum length

(6.12) MASTER

$$l_\mathrm{r} = \frac{J_\mathrm{A}}{m \cdot a}$$

The equivalent pendulum length is the length of the simple pendulum with the same period. With l_r instead of l, all formulas from Sections 6.1 and 6.2 apply.

l_r	Equivalent pendulum length (pivot axis to centre of oscillation)	m
J_A	Moment of inertia about the pivot axis	$\mathrm{kg{\cdot}m^2}$
m	Mass (of the whole pendulum)	kg
a	Distance (pivot axis to centre of gravity)	m

EXAMPLE How long is the equivalent simple pendulum for the rod from the previous example?

Given: $J_\mathrm{A} = 0.2\,\mathrm{kg{\cdot}m^2}$; $m = 0.6\,\mathrm{kg}$; $a = 0.5\,\mathrm{m}$

$$\begin{aligned} l_\mathrm{r} &= \frac{J_\mathrm{A}}{m \cdot a} \\ &= \frac{0.2\,\mathrm{kg{\cdot}m^2}}{0.6\,\mathrm{kg} \cdot 0.5\,\mathrm{m}} \\ &\approx 0.667\,\mathrm{m} \end{aligned}$$

The rod swings like a simple pendulum of 667 mm, i.e. like a pendulum two thirds of its length.

Moment of inertia about the suspension (parallel axis theorem)

(6.13) MASTER

$$J_{\mathrm{A}} = J_{\mathrm{S}} + m \cdot a^2$$

This converts the moment of inertia of a component from its centre of gravity to the pivot axis of the pendulum. For several components the contributions are added.

J_{A}	Moment of inertia about the pivot axis	$\mathrm{kg{\cdot}m^2}$
J_{S}	Moment of inertia about the parallel axis through the centre of gravity	$\mathrm{kg{\cdot}m^2}$
m	Mass (of the component)	kg
a	Distance (between the two axes)	m

EXAMPLE A bob (disc, mass 1.2 kg, radius 60 mm) hangs with its centre 0.9 m below the suspension. About its own centre it has $J_{\mathrm{S}} = \frac{1}{2} \cdot m \cdot r^2 = 0.002\,16\,\mathrm{kg \cdot m^2}$.

Given: $J_{\mathrm{S}} = 0.002\,16\,\mathrm{kg{\cdot}m^2}$; $m = 1.2\,\mathrm{kg}$; $a = 0.9\,\mathrm{m}$

$$\begin{aligned} J_{\mathrm{A}} &= J_{\mathrm{S}} + m \cdot a^2 \\ &= 0.002\,16\,\mathrm{kg{\cdot}m^2} + 1.2\,\mathrm{kg} \cdot (0.9\,\mathrm{m})^2 \\ &\approx 0.974\,\mathrm{kg{\cdot}m^2} \end{aligned}$$

Almost the entire moment of inertia comes from the distance to the suspension. That is why a pendulum with a heavy bob and a light rod behaves almost like a simple pendulum.

6.4 Amplitude and circular error

The formulas from Section 6.1 are exact only for infinitely small amplitudes. The further the pendulum swings, the longer an oscillation lasts. The clock only feels this circular error when the amplitude changes, for example because the driving force drops or the oil in the escapement ages.

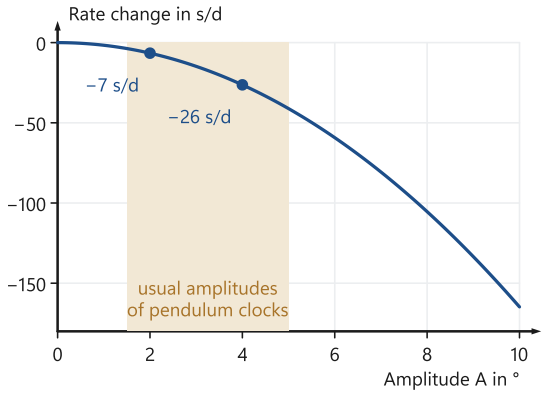


Fig. 6.6 Circular error: the larger the amplitude, the slower the pendulum swings (rate compared with very small amplitude)

Period at larger amplitude

(6.14) APPROXIMATION ADVANCED

$$T \approx T_0 \cdot \left(1 + \frac{A^2}{16}\right)$$

Valid: Amplitudes up to about 20°. Insert A in radians: degrees times $\frac{\pi}{180}$.

T	Period (at amplitude A)	s
T_0	Period at very small amplitude (formula (6.1))	s
A	Amplitude (swing of the pendulum, in radians)	rad

EXAMPLE A seconds pendulum ($T_0 = 2\,\mathrm{s}$) swings with an amplitude of 4°, i.e. 0.0698 rad.

Given: $T_0 = 2\,\mathrm{s}$; $A = 0.0698\,\mathrm{rad}$

$$\begin{aligned} T &\approx T_0 \cdot \left(1 + \frac{A^2}{16}\right) \\ &= 2\,\mathrm{s} \cdot \left(1 + \frac{(0.0698\,\mathrm{rad})^2}{16}\right) \\ &\approx 2.0006\,\mathrm{s} \end{aligned}$$

Each oscillation lasts 0.6 ms longer; compared with a very small amplitude, the clock therefore loses about 26 s per day.

Rate change when the amplitude changes

(6.15) APPROXIMATION ADVANCED

$$\Delta G \approx -\frac{A_2^2 - A_1^2}{16} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

If the amplitude decreases, the clock becomes faster.

Valid: as above.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
A_2	Amplitude after (radians)	rad
A_1	Amplitude before (radians)	rad

EXAMPLE The amplitude drops from 2.5° (0.0436 rad) to 2.0° (0.0349 rad).

Given: $A_2 = 0.0349\,\mathrm{rad}$; $A_1 = 0.0436\,\mathrm{rad}$

$$\begin{aligned} \Delta G &\approx -\frac{A_2^2 - A_1^2}{16} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{(0.0349\,\mathrm{rad})^2 - (0.0436\,\mathrm{rad})^2}{16} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx 3.7\ \frac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The clock gains about 3.7 s per day. A constant drive keeps the amplitude, and with it the rate, constant; this is one reason why precision pendulum clocks are weight-driven.

6.5 Temperature and location

Heat expands the pendulum rod and the clock loses. And because gravitational acceleration is not the same everywhere, a pendulum clock must be regulated again after it has been moved to another place.

Rate change due to temperature

(6.16) APPROXIMATION MASTER

$$\Delta G \approx -\frac{\alpha \cdot \Delta\vartheta}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

Warmer means longer, longer means slower.

Valid: Pendulum without temperature compensation; expansion of bob and suspension neglected.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
α	Coefficient of linear expansion (of the pendulum rod)	$\frac{1}{\mathrm{K}}$
$\Delta\vartheta$	Temperature change	K

EXAMPLE A pendulum clock with a steel rod ($\alpha = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) stands 10 K warmer in summer than in winter.

Given: $\alpha = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$; $\Delta\vartheta = 10\,\mathrm{K}$

$$\begin{aligned}\Delta G &\approx -\frac{\alpha \cdot \Delta\vartheta}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}} \cdot 10\,\mathrm{K}}{2} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &\approx -4.97\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

In summer the clock becomes about 5 s per day slower. With a wooden rod the error shrinks to about one third, with an Invar rod to about one tenth to one sixth.

Coefficients of linear expansion for pendulum rods (guide values at 20 °C)

Material	α in $10^{-6}\ \frac{1}{\mathrm{K}}$
Zinc	30
Aluminium	23
Brass	18 … 21
Steel, unalloyed	11 … 13
Wood (spruce, along the grain)	3 … 5
Invar (FeNi36)	1.2 … 2
Fused silica	0.5

The values vary with alloy, heat treatment and, for wood, with moisture.

Temperature compensation with two materials

(6.17) ADVANCED

$$\alpha_1 \cdot l_1 = \alpha_2 \cdot l_2$$

REARRANGED $$l_2 = \frac{\alpha_1 \cdot l_1}{\alpha_2}$$

In the gridiron pendulum and the zinc-steel pendulum, a material with high expansion raises the bob by as much as the steel lowers it.

Valid: simplified, rods only. The expansion of the bob and the position of the centre of oscillation shift the result slightly; the pendulum is therefore fine-adjusted after it has been built.

α_1	Coefficient of linear expansion of the parts acting downwards (steel)	$\frac{1}{\mathrm{K}}$
l_1	Total length of the parts acting downwards	m
α_2	Coefficient of linear expansion of the parts acting upwards (zinc, brass)	$\frac{1}{\mathrm{K}}$
l_2	Total length of the parts acting upwards	m

EXAMPLE A zinc-steel pendulum has a total of 1.2 m of steel ($\alpha_1 = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) expanding downwards. How long must the zinc tube ($\alpha_2 = 30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$) be?

Given: $\alpha_1 = 11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$; $\quad l_1 = 1.2\,\mathrm{m}$; $\quad \alpha_2 = 30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}$

$$\begin{aligned} l_2 &= \frac{\alpha_1 \cdot l_1}{\alpha_2} \\ &= \frac{11.5 \cdot 10^{-6}\ \frac{1}{\mathrm{K}} \cdot 1.2\,\mathrm{m}}{30 \cdot 10^{-6}\ \frac{1}{\mathrm{K}}} \\ &\approx 0.46\,\mathrm{m} \end{aligned}$$

The zinc tube must be about 46 cm long.

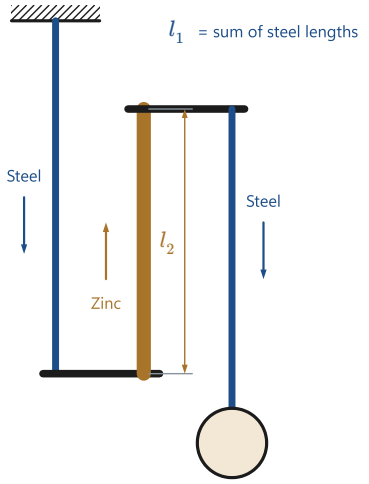


Fig. 6.7 Zinc-steel pendulum, drawn apart for clarity: steel lowers the bob, zinc raises it

Rate change due to a different location

(6.18) APPROXIMATION ADVANCED

$$\Delta G \approx \frac{\Delta g}{2 \cdot g} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

Where gravitational acceleration is greater, the pendulum swings faster.

ΔG	Rate change (+ means: clock becomes faster)	$\frac{\mathrm{s}}{\mathrm{d}}$
Δg	Change in gravitational acceleration (new location minus old location)	$\frac{\mathrm{m}}{\mathrm{s}^2}$
g	Gravitational acceleration at the old location	$\frac{\mathrm{m}}{\mathrm{s}^2}$

EXAMPLE A longcase clock regulated in Hamburg ($g = 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$) is moved to Munich ($g = 9.8074\ \frac{\mathrm{m}}{\mathrm{s}^2}$), i.e. $\Delta g = -0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

Given: $\Delta g = -0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $g = 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\begin{aligned}\Delta G &\approx \frac{\Delta g}{2 \cdot g} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{-0.0064\ \frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot 9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -28.2\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

In Munich the clock loses about 28 s per day and must be regulated again there.

Gravitational acceleration from latitude and altitude

(6.19) APPROXIMATION ADVANCED

$$g \approx 9.780\,327\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2\varphi - 5.8 \cdot 10^{-6} \cdot \sin^2\left(2 \cdot \varphi\right)\right) - 3.086 \cdot 10^{-6}\ \tfrac{1}{\mathrm{s}^2} \cdot h$$

The formula gives the normal gravity of an ideal Earth. Local ground conditions cause deviations of up to about $0.0005\ \frac{\mathrm{m}}{\mathrm{s}^2}$, more in high mountains; this is accurate enough for regulating a clock.

Valid: International gravity formula of 1980 with free-air correction for altitude.

g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
φ	Geographic latitude	°
h	Altitude (above sea level)	m

EXAMPLE Gravitational acceleration in Munich: latitude 48.14°, altitude 520 m.

Given: $\varphi = 48.14°$; $h = 520\,\mathrm{m}$

$$\begin{aligned}g &\approx 9.780\,327\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2\varphi - 5.8 \cdot 10^{-6} \cdot \sin^2\left(2 \cdot \varphi\right)\right) - 3.086 \cdot 10^{-6}\ \tfrac{1}{\mathrm{s}^2} \cdot h \\ &= 9.780\,327\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(1 + 0.005\,302\,4 \cdot \sin^2\left(48.14°\right) - 5.8 \cdot 10^{-6} \cdot \sin^2\left(2 \cdot 48.14°\right)\right) - 3.086 \cdot 10^{-6}\ \tfrac{1}{\mathrm{s}^2} \cdot 520\,\mathrm{m} \\ &\approx 9.8074\ \tfrac{\mathrm{m}}{\mathrm{s}^2}\end{aligned}$$

About $9.8074\ \frac{\mathrm{m}}{\mathrm{s}^2}$; Hamburg at sea level has $9.8138\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

7 Escapements

The escapement sits between the gear train and the oscillator. It stops the gear train, releases it tooth by tooth in step with the balance or pendulum, and in doing so supplies the oscillator with the energy it loses through friction. The escapement therefore counts the oscillations. How accurately the timepiece keeps time also depends on how little the escapement disturbs the oscillator.

The chapter begins with the quantities common to all escapements. It continues with the geometry of the Swiss lever escapement, the energy balance, the escapement noises evaluated by a timing machine, and clock escapements.

7.1 Basic quantities of all escapements

Every escapement releases the gear train in equal steps and drives the oscillator in the process. The terms are the same for all types. In this chapter the angles are uniformly named γ at the escape wheel, β at the pallet fork, λ and A at the balance. In the lever escapement the escape wheel is also called the lever escape wheel; older books sometimes call it the scape wheel.

Terms of the escapement

Term	Meaning	Angle
Lock	A tooth rests on the locking face and the wheel is stationary. Measured as the travel of the pallet fork up to the start of impulse.	β_{R} at the pallet fork
Unlocking	The oscillator pushes the pallet out of lock and releases the tooth.	Part of λ
Impulse	The tooth slides over the impulse face and drives the oscillator via the pallet fork.	γ_{H} at the wheel, β_{H} at the pallet fork
Drop	The wheel turns freely until the next tooth lands on its locking face.	γ_{F} at the wheel
Draw	Inclination of the locking face by which the wheel pulls the pallet fork against the banking.	ψ at the pallet
Run	Travel of the pallet fork after the drop up to the banking; a safety margin for the lock.	β_{V} at the pallet fork
Lift angle	Travel of the balance while the impulse pin is in the fork.	λ at the balance
Supplementary arc	Travel of the oscillator outside the impulse, up to the point of reversal.	φ_{E} at the balance or pendulum

The balance needs its lift angle for unlocking and impulse together.

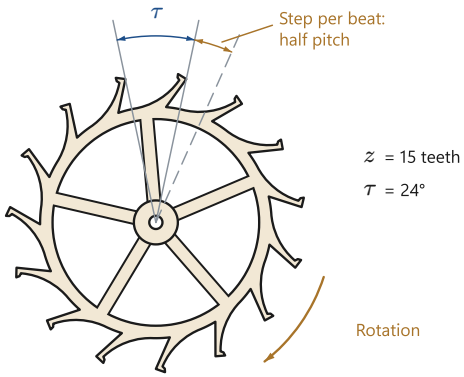


Fig. 7.1 Angular pitch τ of the escape wheel; with each beat the wheel advances by half the pitch

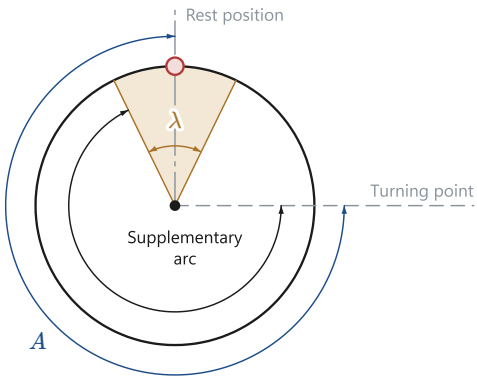


Fig. 7.2 Travel of the impulse pin from the rest position to the point of reversal: amplitude A, lift angle λ and supplementary arc

Angular pitch of the escape wheel

(7.1) JOURNEYMAN

$$\tau = \frac{360^\circ}{z}$$

τ	Angular pitch (of the escape wheel; angle from tooth to tooth)	°
z	Number of teeth (of the escape wheel)	1

The angular pitch is the angle from tooth to tooth. With each beat the wheel advances by half the pitch, because each tooth works on both pallets in turn.

Valid: Lever, cylinder, Graham and recoil escapements. In the chronometer escapement the wheel advances only on every second beat, and then by the full pitch.

EXAMPLE What is the angular pitch of an escape wheel with 15 teeth?

Given: $z = 15$

$$\begin{aligned}\tau &= \frac{360^\circ}{z} \\ &= \frac{360^\circ}{15} \\ &= 24^\circ\end{aligned}$$

The angular pitch is 24°, so the step per beat is 12°.

Beat rate and rotational speed of the escape wheel

(7.2) JOURNEYMAN

$$n_\mathrm{h} = 2 \cdot z \cdot n_\mathrm{H}$$

REARRANGED $\quad n_\mathrm{H} = \frac{n_\mathrm{h}}{2 \cdot z}$

n_h	Beat rate (vibrations per unit time)	$\frac{\mathrm{A}}{\mathrm{h}}$
z	Number of teeth (of the escape wheel)	1
n_H	Rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$

In one revolution of the escape wheel each tooth passes once and allows two beats. The formula links gear train and oscillator: from beat rate and number of teeth it follows how fast the escape wheel must turn (Chapter 4). The same calculation using the numbers of teeth of the gear train is given in Section 4.2.

Valid: all escapements in which one tooth passes per oscillation, including the cylinder and chronometer escapements.

Good to know: With a seconds pendulum and 30 teeth, the escape wheel turns exactly once a minute. Its arbor can therefore carry the seconds hand.

EXAMPLE A pendulum movement makes 4800 beats per hour, i.e. 80 per minute. The escape wheel has 32 teeth. How fast does it turn?

Given: $n_\mathrm{h} = 4800\ \frac{\mathrm{A}}{\mathrm{h}}; \quad z = 32$

$$\begin{aligned}n_\mathrm{H} &= \frac{n_\mathrm{h}}{2 \cdot z} \\ &= \frac{4800\ \frac{\mathrm{A}}{\mathrm{h}}}{2 \cdot 32} \\ &= \frac{80\ \frac{1}{\mathrm{min}}}{2 \cdot 32} \\ &\approx 1.25\ \frac{1}{\mathrm{min}}\end{aligned}$$

The escape wheel turns 1.25 times per minute, so one revolution takes 48 s. An example for the watch is given at formula (4.8).

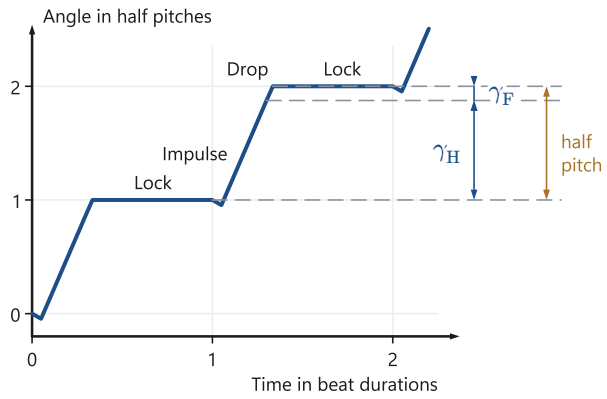


Fig. 7.3 Travel of the escape wheel in two beats: impulse γ_H and drop γ_F make up half the pitch. The small dip before each impulse is the recoil during unlocking of an escapement with draw. The time axis is not to scale: in the watch, impulse and drop last only a few hundredths of the beat duration

Impulse and drop at the wheel

(7.3) MASTER

$$\gamma_H + \gamma_F = \frac{180^\circ}{z}$$

REARRANGED $\gamma_F = \frac{180^\circ}{z} - \gamma_H$

γ_H	Impulse at the wheel (angle turned by the wheel between two drops)	°
γ_F	Drop (free angle of rotation of the wheel)	°
z	Number of teeth (of the escape wheel)	1

The step of the wheel consists of two parts. During impulse the tooth drives the pallet; during the drop the wheel turns freely until the next tooth lands on its locking face. The drop is a necessary safety margin, but it is lost energy.

Valid: γ_H counts the whole travel of the wheel between two drops. In escapements with draw, the wheel turns back by a fraction of a degree during unlocking and immediately advances again; this recoil cancels out in the total.

EXAMPLE On an escape wheel with 15 teeth, the wheel turns through 10.5° during impulse. How large is the drop?

Given: $z = 15$; $\gamma_H = 10.5^\circ$

$$\begin{aligned}\gamma_F &= \frac{180^\circ}{z} - \gamma_H \\ &= \frac{180^\circ}{15} - 10.5^\circ \\ &= 1.5^\circ\end{aligned}$$

The drop is 1.5°, a usual value for the Swiss lever escapement.

Drop as travel of the tooth tip

(7.4) MASTER

$$s_{\mathrm{F}} = R \cdot \gamma_{\mathrm{F}}$$

Under the loupe the drop appears as a small travel of the tooth tip. The formula converts the drop angle into this travel and shows how little clearance the escapement works with.

Valid: Angle in radians: $1^\circ = 0.01745\,\mathrm{rad}$.

s_{F}	Drop as travel (at the circumference of the wheel)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ_{F}	Drop (insert as angle in radians)	rad

EXAMPLE An escape wheel has a radius of 2.40 mm and the drop is 1.5°. How far does the tooth tip drop?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma_{\mathrm{F}} = 1.5^\circ$

$$\begin{aligned} s_{\mathrm{F}} &= R \cdot \gamma_{\mathrm{F}} \\ &= 2.40\,\mathrm{mm} \cdot 1.5^\circ \\ &= 2{,}40\,\mathrm{mm} \cdot 0{,}02618 \\ &\approx 0.063\,\mathrm{mm} \end{aligned}$$

The tooth tip drops freely by 0.063 mm. That is about the thickness of a hair.

Supplementary arc of the balance

(7.5) MASTER

$$\varphi_{\mathrm{E}} = A - \frac{\lambda}{2}$$

Outside the lift angle the balance swings freely, without contact with the pallet fork. It passes through this supplementary arc twice in each vibration: descending towards the impulse and, after the impulse, ascending to the point of reversal.

Valid: detached escapements (lever and chronometer escapements); lift angle symmetrical to the rest position, i.e. the watch is correctly in beat.

φ_{E}	Supplementary arc (from the end of impulse to the point of reversal)	°
A	Amplitude (from the rest position to the point of reversal)	°
λ	Lift angle (of the balance; unlocking and impulse together)	°

EXAMPLE A balance swings with an amplitude of 270°; the lift angle is 52°. How large is the supplementary arc?

Given: $A = 270^\circ$; $\lambda = 52^\circ$

$$\begin{aligned} \varphi_{\mathrm{E}} &= A - \frac{\lambda}{2} \\ &= 270^\circ - \frac{52^\circ}{2} \\ &= 244^\circ \end{aligned}$$

On each side the balance swings freely through 244°. Of the 540° of one vibration only 52° fall to the impulse; for about 90 % of its travel the balance is undisturbed.

Key point: With each beat the escape wheel advances by half the angular pitch: first the impulse, then the drop.

7.2 Swiss lever escapement: geometry

The Swiss lever escapement works in almost all mechanical wristwatches and pocket watches. Its pallet fork spans 2.5 pitches of the wheel. The pivot of the pallet fork lies at the intersection of the two tangents that touch the wheel circle at the contact points. As a result, the pressure of the tooth during lock acts along the pallet arm and does not turn the pallet fork. Only the draw gives the locking face a slight inclination.

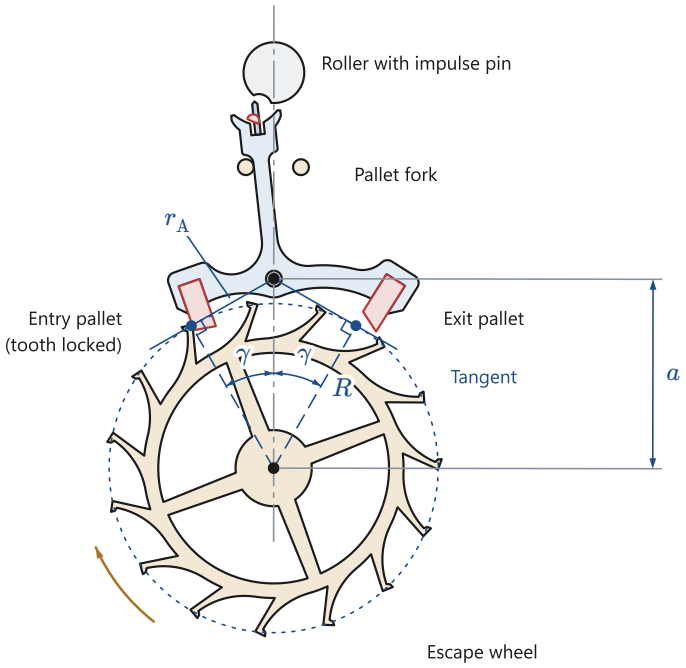


Fig. 7.4 Swiss lever escapement according to the model, tooth locked on the entry pallet: the pallet fork pivot lies at the intersection of the tangents; with 15 teeth $\gamma = 30^\circ$

Span of the pallet fork

(7.6) MASTER

$$\gamma = \frac{k \cdot 180^\circ}{z}$$

The pallet fork spans k pitches of the wheel. Because one pallet is always engaged with the wheel while the other is free, the span ends on a half pitch: 2.5 pitches in the watch, 6.5 to 10.5 in pendulum clocks. Calculations use the half span angle γ.

Valid: Contact points symmetrical to the line joining the wheel and pallet arbors.

γ	Half span angle (measured at the wheel arbor)	°
k	Span in pitches (2.5 in the Swiss lever escapement)	1
z	Number of teeth (of the escape wheel)	1

EXAMPLE What is the half span angle with 15 teeth and a span of 2.5 pitches?

Given: $k = 2.5$; $z = 15$

$$\gamma = \frac{k \cdot 180^\circ}{z} = \frac{2.5 \cdot 180^\circ}{15} = 30^\circ$$

The pallet fork spans 60°, 30° on each side of the centre line.

Centre distance of wheel and pallet fork

(7.7) MASTER

$$a = \frac{R}{\cos \gamma}$$

a	Centre distance (of escape wheel and pallet fork)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

Wheel arbor, contact point and pallet arbor form a right-angled triangle with the right angle at the contact point. The larger the span, the further the pallet fork moves away from the wheel.

Valid: tangential engagement: the line from the pallet fork pivot to the contact point is tangent to the wheel circle. R extends to the points used for the construction, depending on the design the locking corners or the centres of the impulse faces.

EXAMPLE An escape wheel with 15 teeth has a radius of 2.40 mm; the pallet fork spans 2.5 pitches. What is the centre distance?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30°$

$$\begin{aligned} a &= \frac{R}{\cos \gamma} \\ &= \frac{2.40\,\mathrm{mm}}{\cos\left(30°\right)} \\ &\approx 2.77\,\mathrm{mm} \end{aligned}$$

The arbors are 2.77 mm apart. With 15 teeth and a span of 2.5 pitches, $a = 1.155 \cdot R$ always applies.

Length of the pallet arm

(7.8) ADVANCED

$$r_{\mathrm{A}} = R \cdot \tan \gamma$$

r_{A}	Pallet arm (from the pallet fork pivot to the contact point)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

The pallet arm is the tangent segment from the pallet fork pivot to the contact point. It converts an angle of the pallet fork into the travel of the pallet.

Valid: tangential engagement as for the centre distance.

EXAMPLE How long is the pallet arm for a wheel of 2.40 mm radius and a half span angle of 30°?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30°$

$$\begin{aligned} r_{\mathrm{A}} &= R \cdot \tan \gamma \\ &= 2.40\,\mathrm{mm} \cdot \tan\left(30°\right) \\ &\approx 1.39\,\mathrm{mm} \end{aligned}$$

The pallet arm measures 1.39 mm. A lock of 1.5° at the pallet fork corresponds to a travel of the pallet of only 0.036 mm.

Distance between the contact points

(7.9) ADVANCED

$$l_\mathrm{P} = 2 \cdot R \cdot \sin\gamma$$

The chord between the contact points is the width with which the pallet fork embraces the wheel. It can be used to check whether a pallet fork matches the wheel.

Valid: Contact points on the wheel circle of radius R, symmetrical to the centre line.

l_P	Distance between the two contact points (chord in the wheel circle)	mm
R	Radius (of the escape wheel; to the contact points)	mm
γ	Half span angle (measured at the wheel arbor)	°

EXAMPLE How far apart are the contact points for a wheel radius of 2.40 mm and a half span angle of 30°?

Given: $R = 2.40\,\mathrm{mm}$; $\gamma = 30^\circ$

$$\begin{aligned} l_\mathrm{P} &= 2 \cdot R \cdot \sin\gamma \\ &= 2 \cdot 2.40\,\mathrm{mm} \cdot \sin(30^\circ) \\ &= 2.40\,\mathrm{mm} \end{aligned}$$

The distance equals the wheel radius: with a span of 60°, the wheel arbor and the two contact points form an equilateral triangle.

Key point: With 15 teeth and a span of 2.5 pitches, the centre distance is 1.155 times the wheel radius, and the contact points are exactly one wheel radius apart.

Angles of the Swiss lever escapement with 15 teeth (guide values)

Quantity	Symbol	Guide value
Angular pitch of the wheel	τ	24°
Span of the pallet fork	2γ	2.5 pitches = 60°
Drop	γ_F	1° … 1.5°
Impulse at the wheel	γ_H	10.5° … 11°
Lock at the pallet fork	β_R	1.5°
Run	β_V	0.25° … 0.5°
Impulse at the pallet fork	β_H	8.5°
Draw angle	ψ	12° … 15°
Lift angle of the balance	λ	usually 49° … 53°; older wristwatch and pocket watch movements often smaller, about 38° … 46°

With the club tooth, the impulse at the pallet fork is shared between the impulse face of the pallet and that of the tooth. The manufacturer's data always take precedence, for the lift angle also on the timing machine (Section 8.2).

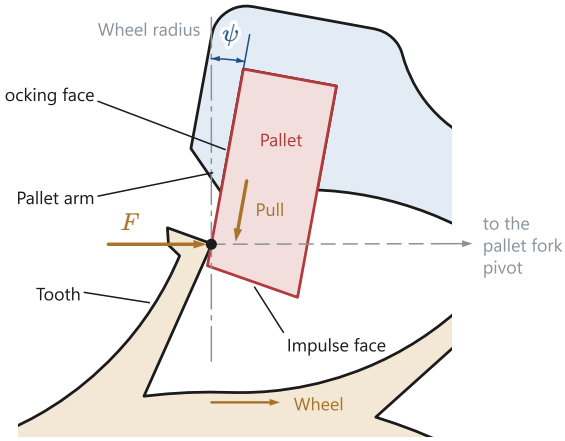


Fig. 7.5 Entry pallet locked: the locking face is inclined by the draw angle ψ to the wheel radius; the pressure F of the tooth pulls the pallet into the wheel

Draw: angle of friction as the limit

(7.10) MASTER

$$\rho = \arctan \mu$$

ρ	Angle of friction (the draw angle must be larger)	°
μ	Coefficient of friction (between tooth and locking face)	1

The locking face is inclined by the draw angle ψ. As a result, the pressure of the tooth pulls the pallet deeper into the wheel and holds the pallet fork against the banking, even under shocks. This only works if ψ is larger than the angle of friction ρ. Otherwise friction holds the pallet wherever it happens to be. The angle of friction in general is given in formula (2.33).

Valid: tangential engagement; ψ measured between the locking face and the wheel radius at the contact point.

Caution: Too much draw costs amplitude: during unlocking the balance has to turn the wheel back a little against its torque.

EXAMPLE Steel on oiled ruby has a coefficient of friction of about 0.15. What is the angle of friction?

Given: $\mu = 0.15$

$$\begin{aligned}\rho &= \arctan \mu \\ &= \arctan 0.15 \\ &\approx 8.5^\circ\end{aligned}$$

The angle of friction is 8.5°. The usual 12° to 15° of draw leaves a reserve, even as the oil ages.

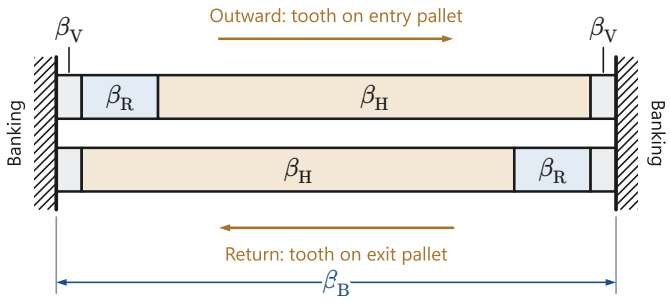


Fig. 7.6 Travel of the pallet fork between the bankings, drawn for $\beta_V = 0.5^\circ$, $\beta_R = 1.5^\circ$ and $\beta_H = 8.5^\circ$

Travel of the pallet fork between the bankings

(7.11) MASTER

$$\beta_B = \beta_R + \beta_H + 2 \cdot \beta_V$$

β_B	Travel of the pallet fork from banking to banking	°
β_R	Lock (immediately after the drop)	°
β_H	Impulse at the pallet fork	°
β_V	Run	°

Starting from the banking, the pallet fork first travels through the whole lock, then the impulse. After the drop, the draw pulls it on by the run to the other banking. The run ensures that the tooth lands safely on the locking face; it counts on both sides.

Valid: The total lock before unlocking is $\beta_R + \beta_V$. Both pallets set equally.

EXAMPLE A pallet fork has 1.5° lock, 8.5° impulse and 0.5° run. How far does it turn from banking to banking?

Given: $\beta_R = 1.5°$; $\beta_H = 8.5°$; $\beta_V = 0.5°$

$$\begin{aligned}\beta_B &= \beta_R + \beta_H + 2 \cdot \beta_V \\ &= 1.5° + 8.5° + 2 \cdot 0.5° \\ &= 11°\end{aligned}$$

The pallet fork turns through 11°. Without run it would be the frequently quoted 10°.

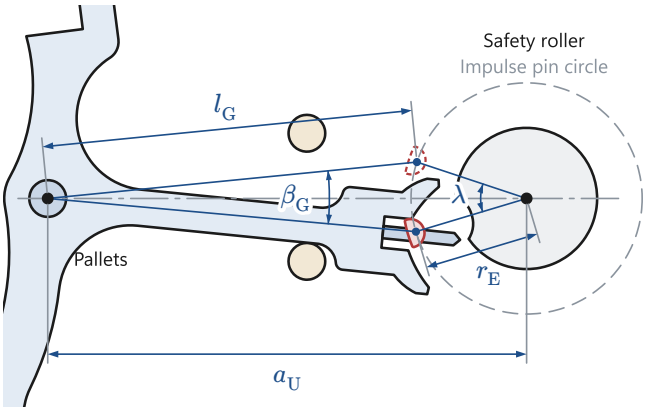


Fig. 7.7 Fork, safety roller and impulse pin according to the model ($\beta_G \approx 11^\circ$, $\lambda \approx 35^\circ$): impulse pin at entry (solid) and at exit (dashed)

Fork and impulse pin: lengths and angles

(7.12) MASTER

$$l_{\mathrm{G}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right) = r_{\mathrm{E}} \cdot \sin\left(\frac{\lambda}{2}\right)$$

REARRANGED $$r_{\mathrm{E}} = \frac{l_{\mathrm{G}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda}{2}\right)}$$

At entry and at exit the impulse pin lies both on its circle about the balance arbor and in the notch of the fork. Its distance from the centre line is the same seen from either arbor; this is what the equation states. A long fork and a small impulse pin radius give a large lift angle of the balance.

Valid: Impulse pin treated as a point on the centre line of the fork notch; width of the jewel and shake in the fork neglected. Then β_{G} equals the travel of the pallet fork between the bankings.

l_{G}	Fork length (pallet arbor to impulse pin at entry)	mm
β_{G}	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°
r_{E}	Radius of the impulse pin about the balance arbor	mm
λ	Lift angle (of the balance; unlocking and impulse together)	°

EXAMPLE A fork is 2.89 mm long. The pallet fork turns 10° while the impulse pin is in the fork; the balance is to have a lift angle of 52°. At what radius must the impulse pin be placed?

Given: $l_{\mathrm{G}} = 2.89\,\mathrm{mm}$; $\beta_{\mathrm{G}} = 10°$; $\lambda = 52°$

$$r_{\mathrm{E}} = \frac{l_{\mathrm{G}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda}{2}\right)}$$

$$= \frac{2.89\,\mathrm{mm} \cdot \sin\left(\frac{10°}{2}\right)}{\sin\left(\frac{52°}{2}\right)}$$

$$\approx 0.575\,\mathrm{mm}$$

The impulse pin sits 0.575 mm from the balance arbor. The fork is about five times as long as this radius.

Impulse pin radius from the centre distance

(7.13) ADVANCED

$$r_{\mathrm{E}} = \frac{a_{\mathrm{U}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda + \beta_{\mathrm{G}}}{2}\right)}$$

r_{E}	Radius of the impulse pin about the balance arbor	mm
a_{U}	Centre distance of pallet fork and balance	mm
β_{G}	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°
λ	Lift angle (of the balance; unlocking and impulse together)	°

In the design, the centre distance and the angles are given. The law of sines in the triangle formed by pallet arbor, balance arbor and impulse pin gives the radius at which the impulse pin must be placed.

Valid: as above: impulse pin treated as a point, without shake in the fork.

EXAMPLE Pallet fork and balance are 3.40 mm apart. The pallet fork turns 10°; the balance is to have a lift angle of 52°. What is the impulse pin radius?

Given: $a_{\mathrm{U}} = 3.40\,\mathrm{mm}$; $\beta_{\mathrm{G}} = 10°$; $\lambda = 52°$

$$r_{\mathrm{E}} = \frac{a_{\mathrm{U}} \cdot \sin\left(\frac{\beta_{\mathrm{G}}}{2}\right)}{\sin\left(\frac{\lambda + \beta_{\mathrm{G}}}{2}\right)}$$

$$= \frac{3.40\,\mathrm{mm} \cdot \sin\left(\frac{10°}{2}\right)}{\sin\left(\frac{52° + 10°}{2}\right)}$$

$$\approx 0.575\,\mathrm{mm}$$

The impulse pin sits at a radius of 0.575 mm.

Fork length from the centre distance

(7.14) ADVANCED

$$l_G = \frac{a_U \cdot \sin\left(\frac{\lambda}{2}\right)}{\sin\left(\frac{\lambda + \beta_G}{2}\right)}$$

l_G	Fork length (pallet arbor to impulse pin at entry)	mm
a_U	Centre distance of pallet fork and balance	mm
λ	Lift angle (of the balance; unlocking and impulse together)	°
β_G	Angle of the pallet fork while the impulse pin is in the fork (about 10°)	°

The same triangle gives the fork length. Fork length and impulse pin radius together add up to slightly more than the centre distance, because at entry both are inclined to the centre line.

Valid: as above: impulse pin treated as a point, without shake in the fork.

EXAMPLE How long must the fork be for a centre distance of 3.40 mm, a pallet fork angle of 10° and a lift angle of 52°?

Given: $a_U = 3.40\,\text{mm}$; $\lambda = 52°$; $\beta_G = 10°$

$$\begin{aligned} l_G &= \frac{a_U \cdot \sin\left(\frac{\lambda}{2}\right)}{\sin\left(\frac{\lambda + \beta_G}{2}\right)} \\ &= \frac{3.40\,\text{mm} \cdot \sin\left(\frac{52°}{2}\right)}{\sin\left(\frac{52° + 10°}{2}\right)} \\ &\approx 2.89\,\text{mm} \end{aligned}$$

The fork is 2.89 mm long. Check along the centre line: $2.89 \cdot \cos 5° + 0.575 \cdot \cos 26° = 3.40$.

Ratio between fork and balance

(7.15) ADVANCED

$$i_G = \frac{a_U - r_E}{r_E}$$

i_G	Ratio in the central position (angular velocity of the balance divided by that of the pallet fork)	1
a_U	Centre distance of pallet fork and balance	mm
r_E	Radius of the impulse pin about the balance arbor	mm

In the central position the fork touches the impulse pin on the line joining the arbors. There the balance turns i_G times as fast as the pallet fork, and, neglecting friction, the balance receives the i_G-th part of the pallet fork torque. Averaged over the whole impulse, the ratio is $\frac{\lambda}{\beta_G}$.

Valid: central position; at entry and at exit the ratio is larger.

EXAMPLE What is the ratio for a centre distance of 3.40 mm and an impulse pin radius of 0.575 mm?

Given: $a_U = 3.40\,\text{mm}$; $r_E = 0.575\,\text{mm}$

$$\begin{aligned} i_G &= \frac{a_U - r_E}{r_E} \\ &= \frac{3.40\,\text{mm} - 0.575\,\text{mm}}{0.575\,\text{mm}} \\ &\approx 4.91 \end{aligned}$$

In the central position the ratio is 4.91; averaged over the impulse it is 52° to 10°, i.e. 5.2.

7.3 Force and energy

With each beat the escape wheel delivers work. Only part of it reaches the balance; the rest is lost in the drop, in friction on the impulse faces and in the impacts. Unlocking also costs energy: the balance must pull the pallet

out of the wheel against friction and draw.

In the steady state the escapement supplies the balance in each oscillation with exactly as much energy as the balance loses through friction. This determines the amplitude. As in Chapter 4, M_{H} is the torque at the escape wheel.

Work of the escape wheel per beat

(7.16) MASTER

$$W_{\mathrm{S}} = \frac{M_{\mathrm{H}} \cdot \pi}{z}$$

W_{S}	Work of the escape wheel per beat	$\mu\mathrm{J}$
M_{H}	Torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
z	Number of teeth (of the escape wheel)	1

Work is torque times angle of rotation. With each beat the wheel turns by half the angular pitch, in radians $\frac{\pi}{z}$. The escapement can never pass on more than this work in any beat.

Valid: Torque constant during the step. M_{H} is the torque that reaches the escape wheel after the losses of the gear train (Chapter 4).

EXAMPLE A torque of 1.6 µN·m reaches an escape wheel with 20 teeth. What work does the wheel deliver per beat?

Given: $M_{\mathrm{H}} = 1.6\,\mu\mathrm{N{\cdot}m}; \quad z = 20$

$$\begin{aligned} W_{\mathrm{S}} &= \frac{M_{\mathrm{H}} \cdot \pi}{z} \\ &= \frac{1.6\,\mu\mathrm{N{\cdot}m} \cdot \pi}{20} \\ &\approx 0.251\,\mu\mathrm{J} \end{aligned}$$

0.25 µJ is available per beat. At 8 beats per second that is 2 µW. The power at the escape wheel of a wristwatch is of this order of magnitude.

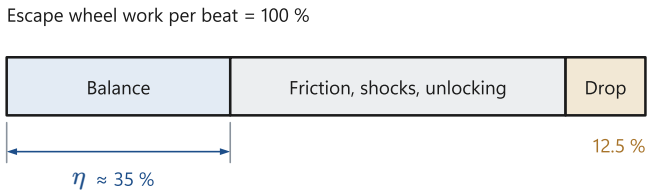


Fig. 7.8 Where the work of the escape wheel goes (guide values for the Swiss lever escapement)

Loss due to the drop

(7.17) MASTER

$$v_{\mathrm{F}} = \frac{\gamma_{\mathrm{F}} \cdot z}{180^\circ}$$

v_{F}	Proportion of the wheel's work lost in the drop	%
γ_{F}	Drop (free angle of rotation of the wheel)	°
z	Number of teeth (of the escape wheel)	1

During the drop the wheel turns without driving. The share of the drop in half the angular pitch is therefore the share of the wheel's work that is wasted when the tooth strikes the locking face.

Valid: constant torque at the wheel.

EXAMPLE What proportion of the wheel's work is lost with the wheel from Section 7.1 (15 teeth, 1.5° drop)?

Given: $\gamma_{\mathrm{F}} = 1.5^\circ; \quad z = 15$

$$\begin{aligned} v_{\mathrm{F}} &= \frac{\gamma_{\mathrm{F}} \cdot z}{180^\circ} \\ &= \frac{1.5^\circ \cdot 15}{180^\circ} \\ &= 12.5\,\% \end{aligned}$$

One eighth of the work is lost through the drop alone. This is why the drop is kept as small as safety allows. With the 20-tooth wheel from the previous example, the same drop would already cost 16.7 %.

Efficiency of the escapement

(7.18) MASTER

$$\eta = \frac{W_{\mathrm{U}}}{W_{\mathrm{S}}}$$

The efficiency states what proportion of the wheel's work reaches the balance. In the Swiss lever escapement it is about 30 % to 40 % in operation (guide value). The rest is lost through the drop, sliding friction on the impulse faces, the impacts and the acceleration of wheel and pallet fork.

Valid: Steady state, i.e. constant amplitude.

η	Efficiency (of the escapement)	1
W_{U}	Work delivered to the balance per beat (impulse minus unlocking)	$\mu\mathrm{J}$
W_{S}	Work of the escape wheel per beat	$\mu\mathrm{J}$

EXAMPLE The escape wheel delivers 0.251 µJ per beat; 0.088 µJ reaches the balance. What is the efficiency?

Given: $W_{\mathrm{U}} = 0.088\,\mu\mathrm{J}$; $W_{\mathrm{S}} = 0.251\,\mu\mathrm{J}$

$$\begin{aligned}\eta &= \frac{W_{\mathrm{U}}}{W_{\mathrm{S}}} \\ &= \frac{0.088\,\mu\mathrm{J}}{0.251\,\mu\mathrm{J}} \\ &\approx 0.35\end{aligned}$$

The efficiency is 0.35, i.e. 35 %. That is a usual value.

Amplitude from the energy balance

(7.19) APPROXIMATION ADVANCED

$$A \approx \sqrt{\frac{2 \cdot \eta \cdot M_{\mathrm{H}} \cdot Q}{z \cdot D}}$$

In the steady state, the energy that two beats supply to the balance equals its loss in one oscillation. This loss grows with the square of the amplitude. The amplitude therefore rises only with the square root of the torque: double the torque gives 1.4 times the amplitude, not double.

Valid: Q and η assumed constant; in reality both change somewhat with the amplitude. The quality factor is 2π times the oscillation energy divided by the loss per oscillation (Chapter 5). The formula gives A in radians.

A	Amplitude (in the steady state)	°
η	Efficiency (of the escapement)	1
M_{H}	Torque at the escape wheel	$\mu\mathrm{N{\cdot}m}$
Q	Quality factor (of the balance with balance spring)	1
z	Number of teeth (of the escape wheel)	1
D	Restoring torque coefficient (of the balance spring)	$\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

EXAMPLE A movement with 28 800 vibrations per hour has an escape wheel with 20 teeth that receives 1.6 µN·m. The restoring torque coefficient of the balance spring is 0.632 µN·m per radian, the quality factor 250, the efficiency of the escapement 0.35. What amplitude results?

Given: $\eta = 0.35$; $M_{\mathrm{H}} = 1.6\,\mu\mathrm{N{\cdot}m}$; $Q = 250$; $z = 20$; $D = 0.632\,\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}$

$$\begin{aligned}A &\approx \sqrt{\frac{2 \cdot \eta \cdot M_{\mathrm{H}} \cdot Q}{z \cdot D}} \\ &= \sqrt{\frac{2 \cdot 0.35 \cdot 1.6\,\mu\mathrm{N{\cdot}m} \cdot 250}{20 \cdot 0.632\,\frac{\mu\mathrm{N{\cdot}m}}{\mathrm{rad}}}} \\ &\approx 4{,}71\,\mathrm{rad} \\ &\approx 270^{\circ}\end{aligned}$$

The balance swings with about 270°. If the torque falls to half towards the end of the power reserve, the amplitude drops to about 190°.

7.4 The escapement on the timing machine

The timing machine listens to the escapement. Each beat of the lever escapement consists of three noises. At the first, the impulse pin strikes the fork and unlocking begins. At the second, the tooth strikes the impulse face and the fork strikes the impulse pin; impulse begins. At the third, the next tooth drops onto the locking face and the pallet fork strikes the banking. From the times between these noises the instrument calculates amplitude and beat error. The formulas, with examples and guide values, are given in Section 8.2; only the part that depends on the escapement follows here.

Amplitude. Between the first and the third noise the balance passes through its lift angle λ, half before and half after the rest position. The larger the amplitude, the faster the balance moves there and the shorter this impulse time. The timing machine measures the impulse time and, using the set lift angle, calculates back to the amplitude, formula (8.9). The lift angle is a design quantity of the escapement (Section 7.2). If it is set incorrectly on the instrument, the displayed amplitude is wrong in the same proportion, formula (8.10).

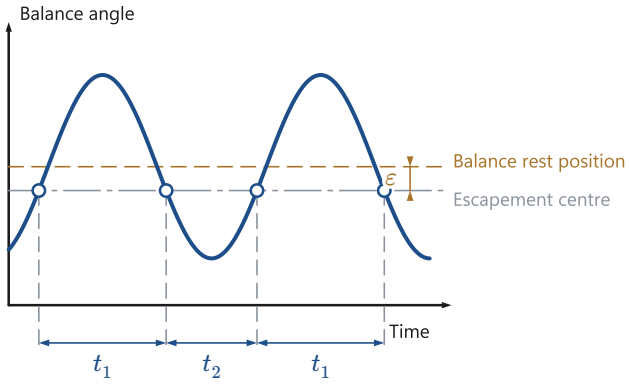


Fig. 7.9 Beat error: if the rest position of the balance lies at ε beside the centre of the escapement, long and short vibrations alternate (greatly exaggerated)

Beat error. If the rest position of the balance lies at an angle ε beside the centre of the escapement, the vibration to one side lasts longer than to the other (Fig. 7.9): the watch is out of beat. The timing machine shows half the time difference as the beat error in milliseconds, formula (8.7). How to calculate the angle ε from it is given in formula (8.8). The correction is made at the balance: the collet or the movable stud carrier is turned by ε until, with the balance at rest, the impulse pin lies on the line joining the pallet arbor and the balance arbor.

7.5 Clock escapements

Pendulum clocks use dead-beat and recoil anchor escapements. In the Graham escapement the locking faces are circular arcs about the anchor pivot: as long as a tooth rests on them, the wheel stands still. In the recoil escapement the locking faces are inclined; in the supplementary arc the pendulum pushes the wheel back a little, hence the name recoil. The pin-wheel escapement of large clocks carries pins on the wheel rim instead of teeth and, like the Graham escapement, is a dead-beat escapement.

Angular pitch, step, span, centre distance and pallet arm are calculated for all these escapements with the formulas from Sections 7.1 and 7.2.

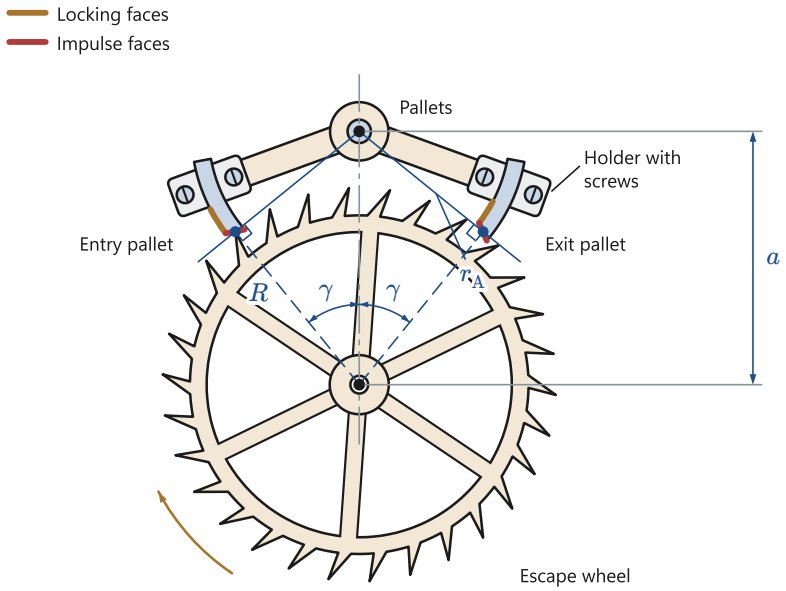


Fig. 7.10 Graham escapement with 30 teeth and a span of 6.5 pitches: the locking faces are circular arcs about the anchor pivot

Centre distance and pallet arm with tangential engagement

Number of teeth z	Span k in pitches	Span angle 2γ	Centre distance a in multiples of R	Pallet arm r_A in multiples of R
15	2.5	60°	1.155	0.577
30	6.5	78°	1.287	0.810
30	7.5	90°	1.414	1.000
30	8.5	102°	1.589	1.235
30	9.5	114°	1.836	1.540
30	10.5	126°	2.203	1.963

First row: Swiss lever escapement. The other rows apply to pendulum clocks with 30 teeth; the larger the span, the longer the pallet arm and the smaller the angle of the anchor for the same impulse face.

Pallet width of the Graham escapement

(7.20) MASTER

$$b_K = R \cdot \left(\frac{\pi}{z} - \gamma_F \right)$$

A pallet must fit alternately between two tooth tips. The pallet therefore gets half the angular pitch minus the drop. This is the width of the ring about the anchor pivot from which both pallets are cut.

Valid: pointed teeth; tangential engagement. Angles in radians: half the angular pitch is $\frac{\pi}{z}$, and $1^\circ = 0.01745\,\text{rad}$.

b_K	Width of the pallet, measured at the circumference of the wheel	mm
R	Radius (of the escape wheel; to the tooth tips)	mm
z	Number of teeth (of the escape wheel)	1
γ_F	Drop (insert as angle in radians)	rad

EXAMPLE An escape wheel with 30 teeth has a radius of 20 mm. The drop is to be 1°. How wide will the pallets be?

Given: $R = 20\,\text{mm}$; $z = 30$; $\gamma_F = 1^\circ$

$$\begin{aligned} b_K &= R \cdot \left(\frac{\pi}{z} - \gamma_F \right) \\ &= 20\,\text{mm} \cdot \left(\frac{\pi}{30} - 1^\circ \right) \\ &= 20\,\text{mm} \cdot (0{,}1047 - 0{,}0175) \\ &\approx 1.75\,\text{mm} \end{aligned}$$

The pallets will be 1.75 mm wide. On this wheel one degree of drop is 0.35 mm of clearance.

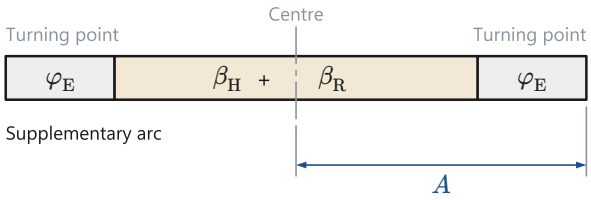


Fig. 7.11 Swing of the pendulum in the Graham escapement: impulse and lock in the middle, supplementary arc φ_E on the outside

Supplementary arc of the pendulum

(7.21) MASTER

$$\varphi_E = A - \frac{\beta_H + \beta_R}{2}$$

The anchor follows every movement of the pendulum. Impulse and lock lie in the middle of the swing, together $\beta_H + \beta_R$ wide. Whatever the pendulum swings beyond this on each side is the supplementary arc. During it the tooth slides on the locking face.

Valid: Anchor set centrally (clock in beat), crutch without play. The pendulum must swing further than $\frac{1}{2}(\beta_H + \beta_R)$, otherwise no tooth drops and the clock stops.

φ_E	Supplementary arc (on each side)	°
A	Amplitude (swing of the pendulum, from the centre to the point of reversal)	°
β_H	Impulse at the anchor	°
β_R	Lock (at the anchor)	°

Good to know: Guide values for Graham escapements with seconds pendulums: impulse at the anchor 1° to 2°, lock 0.5° to 1°. A large supplementary arc costs friction on the locking faces.

EXAMPLE A pendulum swings 1.75° to each side. The Graham anchor has 1.5° impulse and 0.5° lock. How large is the supplementary arc?

Given: $A = 1.75°$; $\beta_H = 1.5°$; $\beta_R = 0.5°$

$$\varphi_E = A - \frac{\beta_H + \beta_R}{2}$$
$$= 1.75° - \frac{1.5° + 0.5°}{2}$$
$$= 0.75°$$

On each side the pendulum swings 0.75° beyond impulse and lock. This reserve absorbs fluctuations in the driving force.

7.6 Other escapements

For the other escapements a few key quantities are sufficient. Angular pitch and beat rate are calculated for all of them as in Section 7.1; the table shows how the types differ.

Escapements at a glance

Escapement	Type	Impulses per oscillation	Step of the wheel per impulse
Swiss lever escapement	detached	2	half pitch
Pin pallet escapement	detached	2	half pitch
Chronometer escapement	detached	1	full pitch
Cylinder escapement	dead-beat	2	half pitch
Verge escapement	recoil	2	half pitch
Graham escapement	dead-beat	2	half pitch
Pin-wheel escapement (clock)	dead-beat	2	half pitch
Recoil escapement	recoil	2	half pitch

Detached: outside the impulse the oscillator swings without contact with the escapement. Dead-beat: during the supplementary arc a tooth rests against the oscillator or the anchor and the wheel stands still. Recoil: the wheel is turned back a little in the process.

The chronometer escapement drives the balance in only one direction of oscillation, but over the full pitch of the wheel. In the opposite direction the discharging pallet passes the passing spring (gold spring) without releasing the wheel. Formula (7.2) still applies to the beat rate, because here too one tooth passes per oscillation.

In the cylinder escapement the wheel works without a pallet fork directly on the balance staff, a hollow cylinder cut open at the side. The tooth rests alternately on the outside and the inside of the cylinder wall; the balance never swings freely.

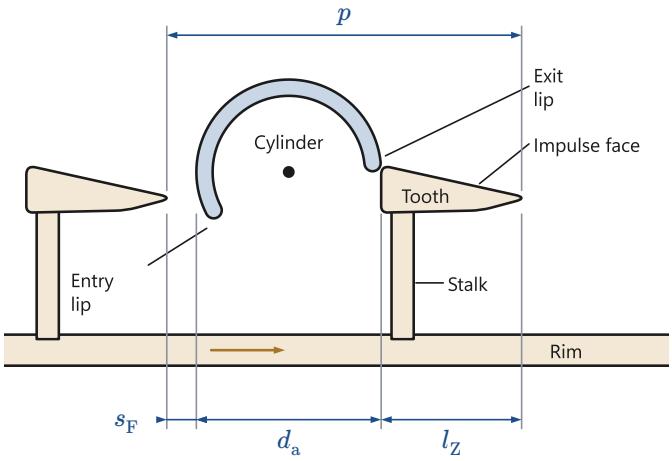


Fig. 7.12 Cylinder escapement with the pitch circle straightened, at the moment the right-hand tooth leaves the cylinder: the pitch p is made up of drop, cylinder diameter and tooth length

Cylinder escapement: pitch, tooth length and cylinder

(7.22) APPROXIMATION ADVANCED

$$\frac{2 \cdot \pi \cdot R}{z} \approx l_Z + d_a + s_F$$

REARRANGED $$d_a \approx \frac{2 \cdot \pi \cdot R}{z} - l_Z - s_F$$

The left-hand side is the pitch p of the wheel at its circumference. Between two teeth there must be room for the outside diameter of the cylinder plus the outside drop. Inside the cylinder there must be room for the tooth: the inside diameter exceeds the tooth length by the inside drop.

Valid: Pitch circle treated as a straight line (arc instead of chord).

R	Radius (of the escape wheel; cylinder wheel, to the tooth tips)	mm
z	Number of teeth (of the escape wheel)	1
l_Z	Length of a tooth	mm
d_a	Outside diameter of the cylinder	mm
s_F	Drop as travel (outside drop)	mm

EXAMPLE A cylinder wheel with 15 teeth has a radius of 2.50 mm, the teeth are 0.45 mm long, and the outside drop is to be 0.05 mm. What outside diameter may the cylinder have?

Given: $R = 2.50\,\text{mm}$; $z = 15$; $l_Z = 0.45\,\text{mm}$; $s_F = 0.05\,\text{mm}$

$$\begin{aligned} d_a &\approx \frac{2 \cdot \pi \cdot R}{z} - l_Z - s_F \\ &= \frac{2 \cdot \pi \cdot 2.50\,\text{mm}}{15} - 0.45\,\text{mm} - 0.05\,\text{mm} \\ &= 1{,}047\,\text{mm} - 0{,}45\,\text{mm} - 0{,}05\,\text{mm} \\ &\approx 0.55\,\text{mm} \end{aligned}$$

The cylinder may measure 0.55 mm on the outside. With a wall thickness of 0.04 mm, 0.47 mm remains inside, i.e. an inside drop of 0.02 mm.

8 Rate Testing and Measurement

Whether a watch keeps good time is decided by measurement. Two terms carry the whole chapter: the time error states how far the watch deviates from the correct time; the rate states how fast the time error changes. The time error is read off; the rate is calculated from it or measured in less than a minute on the timing machine.

The timing machine also gives amplitude and beat error. Measurements are taken in several positions and at different states of wind, because only the differences show where a fault lies. In this chapter, S stands for the time error and D for the difference between vertical and horizontal.

8.1 Time error and rate

The time error is a span of time in seconds; the rate is a change in time error per day. The same sign rule applies to both: plus means the watch is ahead or gaining.

Time error

(8.1) JOURNEYMAN

$$S = t_{\mathrm{U}} - t_{\mathrm{N}}$$

The time error is the reading of the watch minus the correct time. A radio-controlled clock or a time signal serves as the reference.

Valid: both times read at the same instant.

S	Time error (+ means: watch is ahead)	s
t_{U}	Reading of the watch	s
t_{N}	Reference time (correct time at the same instant)	s

EXAMPLE At the full-minute time signal, the seconds hand of the watch already stands at 12 s.

Given: $t_{\mathrm{U}} = 12\,\mathrm{s}$; $t_{\mathrm{N}} = 0\,\mathrm{s}$

$$\begin{aligned} S &= t_{\mathrm{U}} - t_{\mathrm{N}} \\ &= 12\,\mathrm{s} - 0\,\mathrm{s} \\ &= 12\,\mathrm{s} \end{aligned}$$

The watch is 12 s ahead; its time error is +12 s.

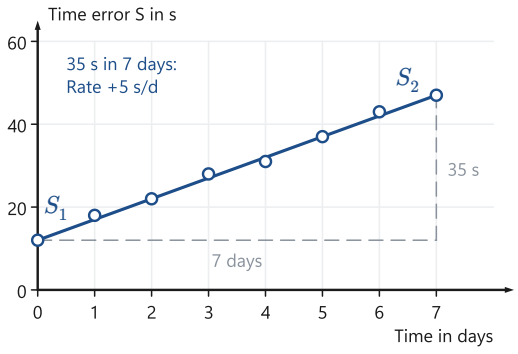


Fig. 8.1 Time error over time: the rate is the slope of the line through the read time errors

Rate from two time errors

(8.2) JOURNEYMAN

$$G = \frac{S_2 - S_1}{t_2 - t_1}$$

The rate is the change in time error per day. In the time-error/time diagram it is the slope of the line.

Valid: mean rate between the two readings; the watch is not set in between.

G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
S_2	Time error at the second reading	s
S_1	Time error at the first reading	s
t_2	Time of the second reading	d
t_1	Time of the first reading	d

Good to know: The longer the interval, the more accurate the rate: the reading error is spread over more days (Section 8.7).

EXAMPLE On day 0 the watch is 12 s ahead, on day 7 it is 47 s ahead. What is its rate?

Given: $S_2 = 47\,\mathrm{s}$; $S_1 = 12\,\mathrm{s}$; $t_2 = 7\,\mathrm{d}$; $t_1 = 0\,\mathrm{d}$

$$\begin{aligned} G &= \frac{S_2 - S_1}{t_2 - t_1} \\ &= \frac{47\,\mathrm{s} - 12\,\mathrm{s}}{7\,\mathrm{d} - 0\,\mathrm{d}} \\ &= 5.0\,\tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The watch gains 5 s per day.

Change in time error over a running period

(8.3) JOURNEYMAN

$$\Delta S = G \cdot t$$

REARRANGED $t = \dfrac{\Delta S}{G}$

At a constant rate, the time error grows uniformly. This is how a rate is extrapolated to a month or a year, or conversely how a figure in seconds per month is converted back to the day.

Valid: constant rate over the whole running period.

ΔS	Change in time error (+ means: watch has gained)	s
G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
t	Time (running period)	d

EXAMPLE A quartz watch gains 0.5 s per day. How much does it gain in a month of 30 days?

Given: $G = 0.5\,\frac{\mathrm{s}}{\mathrm{d}}$; $t = 30\,\mathrm{d}$

$$\begin{aligned} \Delta S &= G \cdot t \\ &= 0.5\,\tfrac{\mathrm{s}}{\mathrm{d}} \cdot 30\,\mathrm{d} \\ &= 15\,\mathrm{s} \end{aligned}$$

15 s per month, a usual value for a simple quartz watch. Per year it would be about 3 min.

Mean rate

(8.4) JOURNEYMAN

$$\bar{G} = \frac{\sum G_i}{n}$$

Over several days, the mean daily rate is also the total change in time error divided by the number of days. Averaged over several positions, it shows how the watch is regulated.

$\bar{G}$	Mean rate (over days or positions)	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum G_i$	Sum of the individual rates, with sign	$\frac{\mathrm{s}}{\mathrm{d}}$
n	Number (of individual rates)	1

Valid: all individual rates measured over equal periods; observe the signs.

EXAMPLE On five days the rates are found to be +6, +4, +7, +3 and +5 seconds per day. The sum is +25 seconds per day.

Given: $\sum G_i = 25\ \frac{\mathrm{s}}{\mathrm{d}}; \quad n = 5$

$$\begin{aligned}\bar{G} &= \frac{\sum G_i}{n} \\ &= \frac{25\ \frac{\mathrm{s}}{\mathrm{d}}}{5} \\ &= 5.0\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

On average +5 s per day. Over the five days the watch has gained 25 s.

Converting the rate

Rate in $\frac{\mathrm{s}}{\mathrm{d}}$	Seconds per month (30 days)	Seconds per year (365 days)	ppm
0.1	3	36.5	1.16
0.5	15	182.5	5.79
1	30	365	11.6
5	150	1825	57.9
10	300	3650	116
30	900	10 950	347

ppm means parts per million. 1 s per day is an error of 1 in 86 400, i.e. 11.6 ppm (Section 1.1).

Rate from the deviation of the beat rate

(8.5) JOURNEYMAN

$$G = \frac{n_{\text{ist}} - n_{\text{soll}}}{n_{\text{soll}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}}$$

Every watch counts oscillations. If the oscillator swings faster by a certain fraction, the watch gains by the same fraction. The formula with frequencies instead of beat rates is given in Section 5.4, for quartz in Section 10.5.

G	Rate (+ means: watch gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
n_{ist}	Measured beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$
n_{soll}	Nominal beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$

Valid: beat rate averaged over the measuring time.

EXAMPLE A movement with 28 800 vibrations per hour makes 28 803 vibrations in one hour.

Given: $n_{\text{ist}} = 28\,803\ \frac{\mathrm{A}}{\mathrm{h}}; \quad n_{\text{soll}} = 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}$

$$\begin{aligned}G &= \frac{n_{\text{ist}} - n_{\text{soll}}}{n_{\text{soll}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{28\,803\ \frac{\mathrm{A}}{\mathrm{h}} - 28\,800\ \frac{\mathrm{A}}{\mathrm{h}}}{28\,800\ \frac{\mathrm{A}}{\mathrm{h}}} \cdot 86\,400\ \frac{\mathrm{s}}{\mathrm{d}} \\ &= 9.0\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The watch gains 9 s per day. Three beats too many per hour amount to three eighths of a second on the dial, and 24 times as much per day.

Key point: The time error is a state; the rate is a change. For both, plus means: the watch is ahead.

8.2 Timing machine

The timing machine picks up the beats of the watch with a microphone and compares their sequence with a quartz frequency. From a measuring time of less than a minute it calculates rate, beat error and amplitude. All three values follow from measured times.

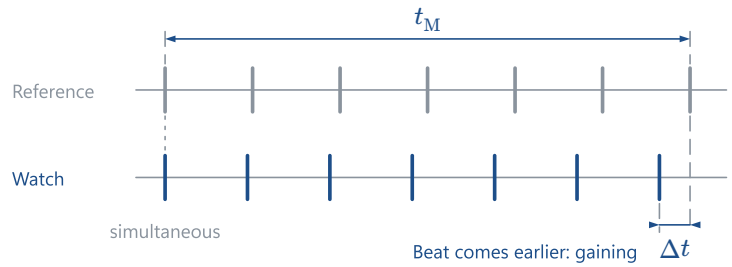


Fig. 8.2 Principle of the timing machine: during the measuring time t_M the beats of the watch shift by Δt relative to the reference frequency

Rate from the time shift of the beats

(8.6) MASTER

$$G = \frac{\Delta t}{t_M} \cdot 86\,400\ \tfrac{s}{d}$$

REARRANGED $$\Delta t = \frac{G \cdot t_M}{86\,400\ \frac{s}{d}}$$

G	Rate (+ means: watch gains)	$\frac{s}{d}$
Δt	Time shift of the beats relative to the reference frequency during the measuring time (+ means: beats arrive earlier)	ms
t_M	Measuring time	s

If each beat arrives slightly earlier than the reference frequency dictates, the lead accumulates. The rate follows from the lead and the measuring time. In the timing machine's diagram it is the slope of the dotted line.

Valid: uniform rate during the measuring time; averaged over tick and tock so that the beat error does not enter the rate.

Caution: The interval between beats varies from beat to beat, mainly due to the teeth of the escape wheel and the meshing of the gear train. A few seconds are therefore not enough for a statement to within 1 s per day; measuring times of 10 s to 60 s are usual, longer if the trace is unsteady.

EXAMPLE In 30 s of measuring time, the beats of a watch advance by 2.0 ms relative to the reference frequency.

Given: $\Delta t = 2.0\,ms$; $t_M = 30\,s$

$$G = \frac{\Delta t}{t_M} \cdot 86\,400\ \tfrac{s}{d}$$
$$= \frac{2.0\,ms}{30\,s} \cdot 86\,400\ \tfrac{s}{d}$$
$$= 5.76\ \tfrac{s}{d}$$

The watch gains 5.76 s per day. The timing machine displays +5.8 after rounding.



Fig. 8.3 Timing machine diagram: the slope of the dotted line shows the rate, the distance between two lines the beat error

Time shift of the beats at a rate of 1 $\frac{\mathrm{s}}{\mathrm{d}}$

Measuring time	Time shift
2 s	23 µs
4 s	46 µs
10 s	116 µs
30 s	347 µs
60 s	694 µs

For comparison: at 28 800 vibrations per hour, two beats are 125 000 µs apart.

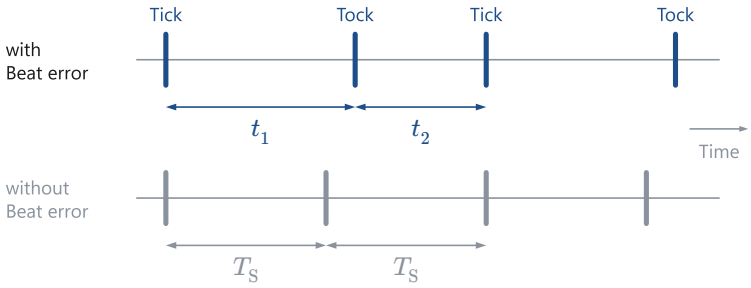


Fig. 8.4 Beat error: more time passes from tick to tock than from tock to tick; T_S is the beat duration without beat error

Beat error

(8.7) JOURNEYMAN

$$t_A = \frac{|t_1 - t_2|}{2}$$

If the balance at rest does not lie exactly in the centre of the escapement, tick and tock follow at unequal intervals. The beat error is the deviation of each interval from the mean, i.e. half the difference between the two intervals. Section 7.4 shows how beat error arises at the escapement.

Valid: times measured in each case from the first sound of one beat to the first sound of the next; displayed as an absolute value.

t_A	Beat error	ms
t_1	Duration of one vibration (tick to tock)	ms
t_2	Duration of the following vibration (tock to tick)	ms

Good to know: Guide values: with the movement fully wound and in the dial-up position, movement manufacturers usually allow at most 0.6 ms to 0.8 ms (ETA 2892A2 and ETA 7750: 0.6 ms; Sellita SW200 0.8 ms or 0.6 ms depending on grade). From about 1 ms onwards the watch should in any case be put in beat. The movement manufacturer's specification is decisive. A large beat error makes it harder for the watch to start by itself.

EXAMPLE On a movement with 28 800 vibrations per hour, the timing machine measures 125.6 ms from tick to tock and 124.4 ms from tock to tick.

Given: $t_1 = 125.6\,\text{ms}$; $t_2 = 124.4\,\text{ms}$

$$\begin{aligned} t_A &= \frac{|t_1 - t_2|}{2} \\ &= \frac{|125.6\,\text{ms} - 124.4\,\text{ms}|}{2} \\ &= 0.6\,\text{ms} \end{aligned}$$

0.6 ms: one interval is 0.6 ms longer, the other 0.6 ms shorter than the nominal beat duration of 125 ms. In the diagram, two lines run side by side.

Beat error as an angle

(8.8) APPROXIMATION ADVANCED

$$\varepsilon \approx \pi \cdot f \cdot A \cdot t_A$$

The same offset of the rest position appears as a larger beat error in milliseconds at a small amplitude, because the balance then passes through the centre more slowly. Beat errors are therefore compared at the same amplitude.

Valid: offset small compared with the amplitude; sinusoidal oscillation.

ε	Angle (between the rest position of the balance and the centre of the escapement)	°
f	Frequency (of the balance; full oscillations per second; 28 800 vibrations per hour are 4 Hz)	Hz
A	Amplitude (from the rest position to the turning point)	°
t_A	Beat error	ms

EXAMPLE A watch with 28 800 vibrations per hour shows 0.6 ms beat error at 270° amplitude.

Given: $f = 4\,\text{Hz}$; $A = 270°$; $t_A = 0.6\,\text{ms}$

$$\begin{aligned} \varepsilon &\approx \pi \cdot f \cdot A \cdot t_A \\ &= \pi \cdot 4\,\text{Hz} \cdot 270° \cdot 0.6\,\text{ms} \\ &\approx 2.0° \end{aligned}$$

The rest position of the balance lies about 2° off the centre. The collet or the movable stud carrier must be turned by this angle. At 180° amplitude the same watch showed 0.9 ms.

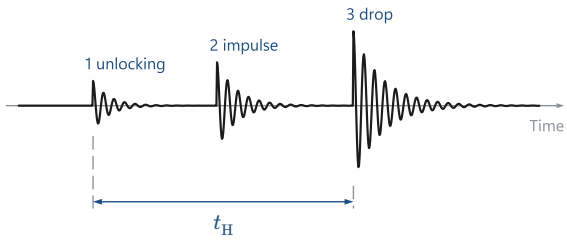


Fig. 8.5 Sound of one beat of the lever escapement: the lift time t_H extends from the first to the third pulse

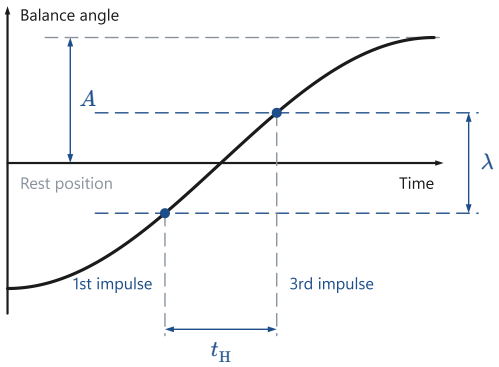


Fig. 8.6 The balance passes through the lift angle λ on both sides of the rest position in the lift time t_H (angle drawn enlarged)

Amplitude from lift angle and lift time

(8.9) MASTER

$$A = \frac{\lambda}{2 \cdot \sin\left(\pi \cdot f \cdot t_H\right)}$$

REARRANGED $$t_H = \frac{\arcsin\left(\frac{\lambda}{2 \cdot A}\right)}{\pi \cdot f}$$

A	Amplitude (from the rest position to the turning point)	°
λ	Lift angle (of the balance; manufacturer's specification, set on the timing machine)	°
f	Frequency (of the balance; full oscillations per second; 28 800 vibrations per hour are 4 Hz)	Hz
t_H	Lift time (from the first to the third sound of a beat)	ms

The greater its amplitude, the faster the balance passes through the lift angle. The timing machine measures the lift time between the first and the third sound of a beat and calculates the amplitude from it.

Valid: sinusoidal oscillation, lift angle symmetrical about the rest position. The product $\pi \cdot f \cdot t_H$ is an angle in radians; the calculator must be set to rad for the sine.

Good to know: Derivation: the balance oscillates according to $\varphi = A \cdot \sin(2\pi \cdot f \cdot t)$. The lift angle lies symmetrically about the rest position, so the balance reaches half the lift angle after half the lift time: $\frac{\lambda}{2} = A \cdot \sin(\pi \cdot f \cdot t_H)$.

Good to know: For usual amplitudes the approximation $A \approx \frac{\lambda}{2\pi \cdot f \cdot t_H}$ is sufficient. In the example it gives 268.7°, i.e. 0.4° too little.

EXAMPLE A movement with 28 800 vibrations per hour has a lift angle of 52°. The timing machine measures a lift time of 7.7 ms.

Given: $\lambda = 52°$; $f = 4\,\text{Hz}$; $t_H = 7.7\,\text{ms}$

$$\begin{aligned} A &= \frac{\lambda}{2 \cdot \sin\left(\pi \cdot f \cdot t_H\right)} \\ &= \frac{52°}{2 \cdot \sin\left(\pi \cdot 4\,\text{Hz} \cdot 7.7\,\text{ms}\right)} \\ &= \frac{52°}{2 \cdot \sin(0{,}0968)} \\ &\approx 269° \end{aligned}$$

The balance swings at about 269°. At 300° the lift time would have shrunk to 6.9 ms.

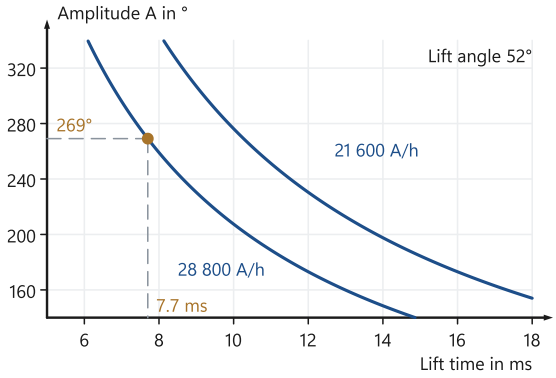


Fig. 8.7 Amplitude against lift time at a lift angle of 52°: the shorter the lift time, the greater the amplitude

Lift time in ms at a lift angle of 52°

Amplitude	28 800 A/h (4 Hz)	21 600 A/h (3 Hz)	18 000 A/h (2.5 Hz)
200°	10.4	13.8	16.6
220°	9.4	12.6	15.1
240°	8.6	11.5	13.8
260°	8.0	10.6	12.8
280°	7.4	9.9	11.8
300°	6.9	9.2	11.0
320°	6.5	8.6	10.4

Calculated with the rearranged formula. Between 280° and 300° the lift time at 4 Hz changes by only 0.5 ms: the timing machine must measure it to within hundredths of a millisecond.

Amplitude with incorrectly set lift angle

(8.10) MASTER

$$A = \frac{A_Z \cdot \lambda}{\lambda_Z}$$

A	Amplitude (true value)	°
A_Z	Amplitude displayed by the timing machine	°
λ	Lift angle (of the balance; true value of the movement)	°
λ_Z	Lift angle set on the timing machine	°

The timing machine only measures the lift time; the lift angle has to be entered. If it is set incorrectly, the displayed amplitude is wrong in the same proportion: 4 % too much lift angle gives 4 % too much amplitude. For the errors, $\frac{\Delta A}{A} = \frac{\Delta \lambda}{\lambda}$ applies.

Valid: follows from the amplitude formula: for the same lift time, the amplitude is proportional to the lift angle.

Caution: If the lift angle is unknown, check the amplitude at the balance. A balance with two arms shows the same picture at the turning points at 180° amplitude as at the rest position; at 270° the arms stand in the same place at both turning points, at right angles to the rest position. Set the lift angle on the timing machine so that the display matches.

EXAMPLE The timing machine is set to 52° and shows 280°. The movement, however, has a lift angle of 50°.

Given: $A_Z = 280°$; $\lambda = 50°$; $\lambda_Z = 52°$

$$A = \frac{A_Z \cdot \lambda}{\lambda_Z} = \frac{280° \cdot 50°}{52°} \approx 269°$$

The balance swings at only about 269°. Here each degree of error in the lift angle amounts to just over 5° of amplitude.

Usual lift angles (guide values)

Escapement and period of construction	Lift angle of the balance
Swiss lever escapement, more recent wristwatch movements	mostly 49° … 53°
Swiss lever escapement, older wristwatch and pocket watch movements	often smaller, about 38° … 46°
Co-Axial escapement	about 30° … 38°

The lift angle is a design parameter of the calibre. The manufacturer's data sheet is decisive; if it is missing, a visual check at the balance helps. Examples: ETA 2824-2 50°, ETA 2892A2 51°, ETA 7750 49°, Sellita SW200 50°, ETA 6497-1 44°.

Key point: The timing machine measures times. Rate, beat error and amplitude are calculated from them, and the amplitude is correct only with the right lift angle.

8.3 Positions and characteristic values

A watch does not run equally in every position: pivot friction, the centre of gravity of the balance and the balance spring act differently depending on the position. Testing is therefore carried out in up to six positions, once fully wound and once after 24 hours of running.

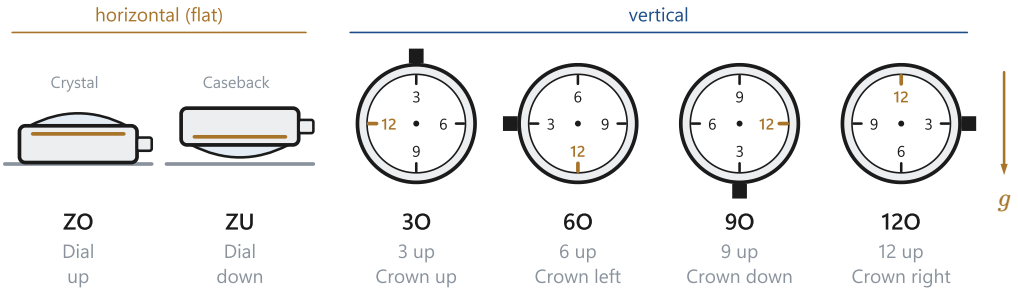


Fig. 8.8 The six test positions of a wristwatch with the crown at 3

The six test positions and their abbreviations

Position	German	International	Wristwatch crown
Dial up	ZO	CH	horizontal
Dial down	ZU	FH (also CB)	horizontal
3 up	3O	3H	up
6 up	6O	6H	left
9 up	9O	9H	down
12 up	12O	12H	right

The abbreviations state what is up. Internationally the French abbreviations are usual: H for haut (up), C for cadran (dial), F for fond (back), B for bas (down). In English the horizontal positions are called DU (dial up) and DD (dial down); the vertical ones crown up, crown down, crown left and crown right. The crown details apply to wristwatches with the crown at 3; on pocket watches it sits at 12.

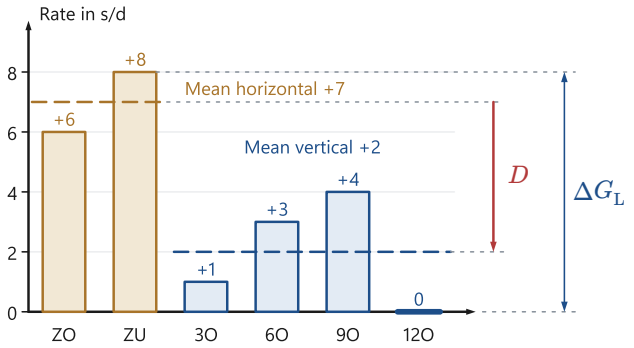


Fig. 8.9 Rates in six positions: greatest positional variation ΔG_L and vertical–horizontal difference D

Greatest positional variation

(8.11) JOURNEYMAN

$$\Delta G_\mathrm{L} = G_\mathrm{max} - G_\mathrm{min}$$

The greatest positional variation is the simplest measure of the quality of the adjustment: the smaller it is, the less the rate depends on how the watch is worn and laid down.

Valid: all positions measured at the same state of wind and the same temperature.

ΔG_L	Greatest positional variation	$\frac{\mathrm{s}}{\mathrm{d}}$
G_max	Highest rate of all positions (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_min	Lowest rate of all positions (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE Fully wound, a watch shows: ZO +6, ZU +8, 3O +1, 6O +3, 9O +4 and 12O 0 seconds per day.

Given: $G_\mathrm{max} = 8\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad G_\mathrm{min} = 0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G_\mathrm{L} &= G_\mathrm{max} - G_\mathrm{min} \\ &= 8\ \tfrac{\mathrm{s}}{\mathrm{d}} - 0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 8\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

8 s per day between the fastest position (dial down) and the slowest (12 up). The mean rate of the six positions is +3.7 s per day.

Difference between vertical and horizontal positions

(8.12) MASTER

$$D = G_{\mathrm{h}} - G_{\mathrm{f}}$$

The vertical positions are averaged and compared with the mean of the horizontal positions. A negative result means: the watch runs slower vertical than horizontal.

Valid: same state of wind in all positions. Chronometer testing forms D from only two positions, 6 up and dial up (Section 8.4).

D	Vertical–horizontal difference (+ means: rate higher vertical than horizontal)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{h}	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{f}	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$

Good to know: In the mean of the four vertical positions, the effect of poise error of the balance largely cancels out. What remains comes from the smaller amplitude and the different bearing of the pivots. Poise error shows in the differences between the vertical positions (Section 5.3).

EXAMPLE The watch from the previous example: mean of the four vertical positions +2.0, mean of the two horizontal positions +7.0 seconds per day.

Given: $G_{\mathrm{h}} = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_{\mathrm{f}} = 7.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} D &= G_{\mathrm{h}} - G_{\mathrm{f}} \\ &= 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} - 7.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -5.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

Vertical, the watch runs 5 s per day slower than horizontal. The amplitude is smaller in the vertical positions; the watch therefore loses at small amplitude, a sign of an isochronism error (Section 5.7).

What positional differences show (guide for fault finding)

Observation	Common cause
Dial up and dial down differ	Pivot ends or cap jewels, oil, balance spring not flat or rubbing
Vertical slower or faster than horizontal	Isochronism error: play of the balance spring between the regulator pins, shape of the balance spring, escapement
Vertical positions differ from one another	Poise error of the balance, balance spring not concentric
Amplitude much smaller vertical than horizontal	Friction at the balance pivots: pivots, hole jewels, oil

The table gives indications, not reliable diagnoses; often several causes act together.

Rate when worn from two groups of positions

(8.13) ADVANCED

$$G_{\mathrm{T}} = a \cdot G_{\mathrm{f}} + (1 - a) \cdot G_{\mathrm{h}}$$

On the wrist and on the bedside table the positions change. The rate when worn lies between the horizontal mean and the vertical mean, depending on the proportion of time.

Valid: simplified model with two groups of positions; temperature on the wrist, movement and state of wind are not taken into account.

G_{T}	Expected rate when worn	$\frac{\mathrm{s}}{\mathrm{d}}$
a	Proportion of time in horizontal positions	1
G_{f}	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{h}	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE The watch from the previous examples lies flat at night and at times during the day, 40 % of the time in total.

Given: $a = 0.4$; $G_{\mathrm{f}} = 7.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_{\mathrm{h}} = 2.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned} G_{\mathrm{T}} &= a \cdot G_{\mathrm{f}} + (1 - a) \cdot G_{\mathrm{h}} \\ &= 0.4 \cdot 7.0\ \tfrac{\mathrm{s}}{\mathrm{d}} + (1 - 0.4) \cdot 2.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 4.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

About +4 s per day. Laying the watch down flat at night makes it run faster here; laid down vertically it runs slower.

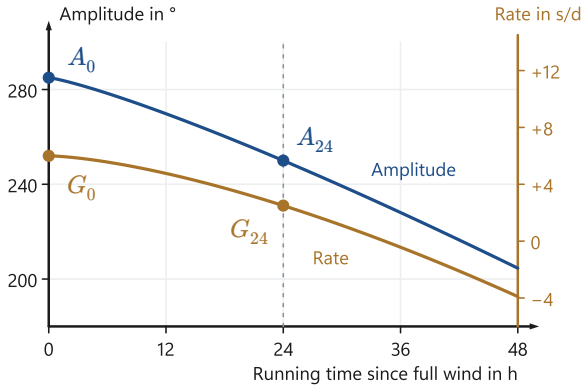


Fig. 8.10 Amplitude and rate over the running time (example): the values at 0 h and 24 h give the amplitude drop and the isochronism error

Isochronism error from 0 h and 24 h

(8.14) MASTER

$$\Delta G = G_{24} - G_0$$

As the mainspring runs down, the amplitude falls. If the rate changes in the process, the oscillator is not isochronous. The formula is the same as in Section 5.7, here with the two measuring times used in the workshop.

Valid: same position and same temperature for both measurements.

ΔG	Isochronism error (+ means: watch gains as the mainspring runs down)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{24}	Rate after 24 hours of running	$\frac{\mathrm{s}}{\mathrm{d}}$
G_0	Rate fully wound	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE Fully wound and dial up, a watch gains 6.0 s per day; after 24 hours of running in the same position, 2.5 s per day.

Given: $G_{24} = 2.5\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_0 = 6.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G &= G_{24} - G_0 \\ &= 2.5\ \tfrac{\mathrm{s}}{\mathrm{d}} - 6.0\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -3.5\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

After 24 hours the watch has become 3.5 s per day slower. If it is wound daily, its rate lies between the two values.

Amplitude drop in 24 hours

(8.15) JOURNEYMAN

$$\Delta A = A_0 - A_{24}$$

The amplitude drop shows how evenly the mainspring and gear train deliver the force over the day.

Valid: same position for both measurements, lift angle set correctly.

ΔA	Amplitude drop	°
A_0	Amplitude fully wound	°
A_{24}	Amplitude after 24 hours of running	°

EXAMPLE Dial up, the balance swings at 285° when fully wound and at 250° after 24 hours.

Given: $A_0 = 285°$; $A_{24} = 250°$

$$\begin{aligned}\Delta A &= A_0 - A_{24} \\ &= 285° - 250° \\ &= 35°\end{aligned}$$

35° in 24 hours. More important than the drop itself is that the amplitude after 24 hours is still sufficient in the vertical positions as well; movement manufacturers specify minimum values for this, often 190° to 200° in the 6-up position (ETA 2892A2 and ETA 7750: 200°).

Key point: Horizontal against vertical shows the influence of amplitude, vertical against vertical shows poise error, fully wound against 24 hours shows isochronism.

8.4 Chronometer testing

According to ISO 3159, a chronometer is a watch whose movement has been tested by an independent body over several days in different positions and at different temperatures; in Switzerland this is the COSC. The test lasts 15 days, with five positions and three temperatures. Seven criteria are calculated from the 15 daily rates. Other tests, such as that of METAS or to DIN 8319, apply to the finished watch and in some cases demand more.

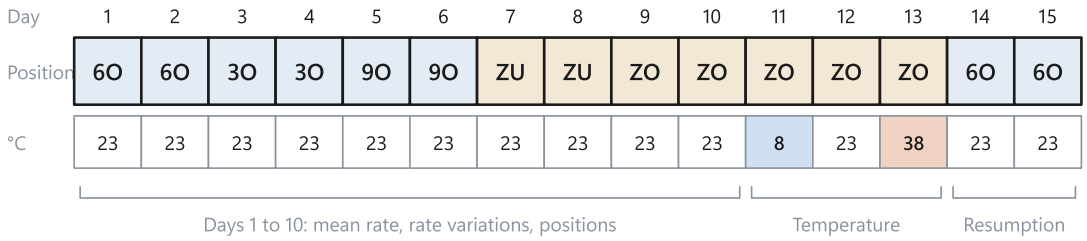


Fig. 8.11 Test programme of chronometer testing to ISO 3159: position and temperature on the 15 test days

The seven criteria of chronometer testing to ISO 3159 (watches over 20 mm in diameter)

No.	Criterion	Calculation from the daily rates G_1 to G_{15}	Limit to ISO 3159 in $\frac{\text{s}}{\text{d}}$
1	Mean daily rate	$\bar{G} = \frac{1}{10} \cdot (G_1 + G_2 + \cdots + G_{10})$	−4 … +6
2	Mean variation in rates	$\frac{1}{5} \cdot (\lvert G_2 - G_1 \rvert + \lvert G_4 - G_3 \rvert + \cdots + \lvert G_{10} - G_9 \rvert)$	at most 2
3	Greatest variation in rates	largest of the five absolute values from row 2	at most 5
4	Difference between horizontal and vertical positions D	$\frac{1}{2} \cdot (G_1 + G_2) - \frac{1}{2} \cdot (G_9 + G_{10})$	−6 … +8
5	Largest deviation from the mean daily rate	largest of the ten absolute values $\lvert G_i - \bar{G} \rvert$ of days 1 to 10	at most 10
6	Rate variation per °C C	$\frac{1}{30\,\text{K}} \cdot (G_{13} - G_{11})$	±0.6 per K
7	Resumption of rate	$G_{15} - \frac{1}{2} \cdot (G_1 + G_2)$	±5

Each of the seven limits must be met. For watches up to 20 mm in diameter the standard allows somewhat wider limits. The standard writes the daily rates as M_i; here they are written G_i, as throughout the book.

Mean variation in rates

(8.16) MASTER

$$\overline{\Delta G} = \frac{\sum |G_{2k} - G_{2k-1}|}{5}$$

$\overline{\Delta G}$	Mean variation in rates	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum \lvert G_{2k} - G_{2k-1} \rvert$	Sum of the five absolute values: difference between the two daily rates in each position	$\frac{\mathrm{s}}{\mathrm{d}}$

In each of the five positions, measurements are taken on two consecutive days. The criterion shows how well the watch repeats its rate from one day to the next.

Valid: to ISO 3159: days 1 to 10 at 23 °C; limit 2 s per day (watches over 20 mm).

EXAMPLE In the five positions, the two daily rates differ by 1.0, 0.8, 2.1, 0.7 and 1.4 seconds per day. The sum is 6.0 seconds per day.

Given: $\sum |G_{2k} - G_{2k-1}| = 6.0 \ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\overline{\Delta G} &= \frac{\sum |G_{2k} - G_{2k-1}|}{5} \\ &= \frac{6.0 \ \frac{\mathrm{s}}{\mathrm{d}}}{5} \\ &= 1.2 \ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

1.2 s per day; the limit of 2 s per day is met. The largest individual value, 2.1 s per day, is also below its limit of 5 s per day.

Temperature coefficient in chronometer testing (rate variation per °C)

(8.17) MASTER

$$C = \frac{G_{38} - G_8}{30\,\mathrm{K}}$$

C	Temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d \cdot K}}$
G_{38}	Rate at 38 °C (day 13)	$\frac{\mathrm{s}}{\mathrm{d}}$
G_8	Rate at 8 °C (day 11)	$\frac{\mathrm{s}}{\mathrm{d}}$

Both rates are measured in the same position, dial up. The general form for any test temperatures is given in Section 5.5.

Valid: to ISO 3159: limit ±0.6 s per day per kelvin (watches over 20 mm).

EXAMPLE A watch gains 2.4 s per day at 8 °C and 7.2 s per day at 38 °C.

Given: $G_{38} = 7.2 \ \frac{\mathrm{s}}{\mathrm{d}}; \quad G_8 = 2.4 \ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}C &= \frac{G_{38} - G_8}{30\,\mathrm{K}} \\ &= \frac{7.2 \ \frac{\mathrm{s}}{\mathrm{d}} - 2.4 \ \frac{\mathrm{s}}{\mathrm{d}}}{30\,\mathrm{K}} \\ &= 0.16 \ \frac{\mathrm{s}}{\mathrm{d \cdot K}}\end{aligned}$$

+0.16 s per day per kelvin: the watch gains more in the warmth. The limit of 0.6 is clearly met.

Resumption of rate

(8.18) MASTER

$$\Delta G_{\mathrm{W}} = G_{15} - \frac{G_1 + G_2}{2}$$

ΔG_{W}	Resumption of rate	$\frac{\mathrm{s}}{\mathrm{d}}$
G_{15}	Rate on the last test day	$\frac{\mathrm{s}}{\mathrm{d}}$
G_1	Rate on the first test day	$\frac{\mathrm{s}}{\mathrm{d}}$
G_2	Rate on the second test day	$\frac{\mathrm{s}}{\mathrm{d}}$

On the last day the watch is again in the position of the first two days. The criterion shows whether, after two weeks of testing with cold and heat, the watch returns to its initial rate.

Valid: to ISO 3159: 6-up position at 23 °C; limit ±5 s per day (watches over 20 mm).

EXAMPLE The watch from the example on the mean variation in rates gains 3.0 and 4.0 s per day on the first two days, and 4.5 s per day on the last day.

Given: $G_{15} = 4.5\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_1 = 3.0\ \frac{\mathrm{s}}{\mathrm{d}}$; $G_2 = 4.0\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}\Delta G_{\mathrm{W}} &= G_{15} - \frac{G_1 + G_2}{2} \\ &= 4.5\ \tfrac{\mathrm{s}}{\mathrm{d}} - \frac{3.0\ \frac{\mathrm{s}}{\mathrm{d}} + 4.0\ \frac{\mathrm{s}}{\mathrm{d}}}{2} \\ &= 1.0\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

+1.0 s per day, well within the limit of ±5 s per day. The watch has almost returned to its initial rate.

Key point: Chronometer means: tested for 15 days, in five positions and at three temperatures, with a mean daily rate between −4 and +6 s per day.

8.5 Water resistance and pressure

With every metre of depth, water presses harder on the crystal, caseback and gaskets. The pressure rating on a watch states the test overpressure. It applies to the new watch and to static pressure, in water or in air. The requirements for water-resistant watches are laid down in ISO 22810.

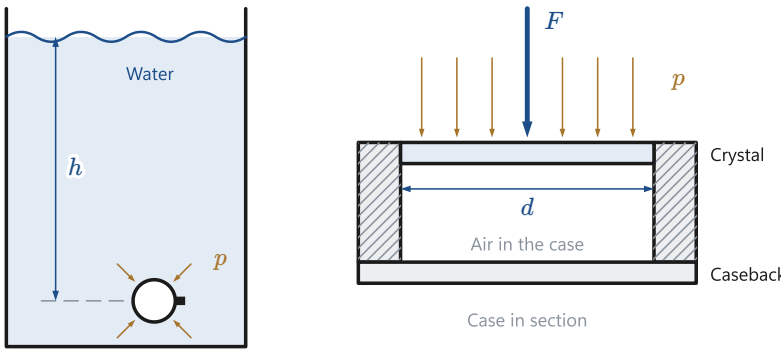


Fig. 8.12 Left: water pressure increases with depth h. Right: the pressure p loads the crystal of diameter d with the force F

Pressure of a water column

(8.19) JOURNEYMAN

$$p = \varrho \cdot g \cdot h$$

REARRANGED $h = \dfrac{p}{\varrho \cdot g}$

The pressure increases uniformly with depth, in fresh water by just under 1 bar per 10 m.

Valid: still water. Fresh water has $1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, sea water about $1025\ \frac{\mathrm{kg}}{\mathrm{m}^3}$.

p	Pressure (overpressure relative to the water surface)	bar
ϱ	Density (of the water)	$\frac{\mathrm{kg}}{\mathrm{m}^3}$
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	Height (of the water column above the watch, i.e. the diving depth)	m

EXAMPLE What overpressure acts at a depth of 100 m in fresh water?

Given: $\varrho = 1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}$; $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 100\,\mathrm{m}$

$$\begin{aligned} p &= \varrho \cdot g \cdot h \\ &= 1000\ \tfrac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9.81\ \tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot 100\,\mathrm{m} \\ &= 981\,000\,\mathrm{Pa} \\ &= 9.81\,\mathrm{bar} \end{aligned}$$

9.81 bar, rounded 10 bar. Hence the rule of thumb: 10 m of water column correspond to 1 bar.

Pressure force on crystal or caseback

(8.20) JOURNEYMAN

$$F = \frac{p \cdot \pi \cdot d^2}{4}$$

The force is pressure times area (Section 2.2), here written out with the area of a circle. It increases with the square of the diameter: large watches need thicker crystals and stiffer casebacks.

Valid: pressure uniform over a circular area; $1\,\mathrm{bar} = 0.1\ \frac{\mathrm{N}}{\mathrm{mm}^2}$.

F	Force (on the loaded area)	N
p	Pressure (overpressure outside relative to inside)	bar
d	Diameter (of the loaded area, here of the crystal)	mm

EXAMPLE A diver's watch is tested at 30 bar. The crystal is 28 mm in diameter. What force acts on it?

Given: $p = 30\,\mathrm{bar}$; $d = 28\,\mathrm{mm}$

$$\begin{aligned} F &= \frac{p \cdot \pi \cdot d^2}{4} \\ &= \frac{30\,\mathrm{bar} \cdot \pi \cdot (28\,\mathrm{mm})^2}{4} \\ &= 3\,\frac{\mathrm{N}}{\mathrm{mm}^2} \cdot \frac{\pi \cdot 784\,\mathrm{mm}^2}{4} \\ &\approx 1847\,\mathrm{N} \end{aligned}$$

About 1850 N, as much as the weight of 188 kg. This force presses the crystal into its gasket and bows it inwards. The example for 10 bar is given at formula (2.11).

Testing is usually done dry: the tester places the watch in a chamber under overpressure or vacuum and senses how the case deforms. A tight case is compressed or bulged and stays so as long as the pressure is applied. If it leaks, the pressure inside and outside equalises and the deformation recedes. In the wet test, the watch is first pressurised above the water, then immersed and the pressure released; air bubbles rise from a leaking watch.

Dynamic pressure during movement in water

(8.21) ADVANCED

$$p_{\text{dyn}} = \frac{\varrho \cdot v^2}{2}$$

p_{dyn}	Dynamic pressure (additional pressure due to the flow)	bar
ϱ	Density (of the water)	$\frac{\text{kg}}{\text{m}^3}$
v	Velocity (of the watch relative to the water)	$\frac{\text{m}}{\text{s}}$

When the watch moves through the water, the dynamic pressure adds to the pressure of the depth. It remains small.

Valid: uniform flow; the water is brought completely to rest at the watch. This is the greatest pressure a uniform movement produces; the impact on hitting the water surface is not covered by it.

EXAMPLE In a very fast arm movement, the wrist travels through the water at 5 m per second.

Given: $\varrho = 1000\ \frac{\text{kg}}{\text{m}^3}$; $v = 5\ \frac{\text{m}}{\text{s}}$

$$\begin{aligned} p_{\text{dyn}} &= \frac{\varrho \cdot v^2}{2} \\ &= \frac{1000\ \frac{\text{kg}}{\text{m}^3} \cdot \left(5\ \frac{\text{m}}{\text{s}}\right)^2}{2} \\ &= 12\,500\,\text{Pa} \\ &= 0.125\,\text{bar} \end{aligned}$$

0.125 bar, equivalent to 1.3 m more depth. Swimming movements therefore hardly increase the pressure. If a watch leaks in the water, the cause is usually aged gaskets, changes in temperature, or the crown and pushers having been operated under water.

Key point: 10 m of water column is about 1 bar. The force on the crystal is pressure times area.

8.6 Magnetism and shock

Magnetic fields and shocks are the most frequent disturbances in everyday life. Standards specify what a watch must withstand to be called antimagnetic or shock-resistant: ISO 764 requires a field of $4800\ \frac{\text{A}}{\text{m}}$, ISO 1413 a shock equivalent to a fall from a height of 1 m.

Flux density and field strength in air

(8.22) MASTER

$$B = \mu_0 \cdot H$$

B	Magnetic flux density ($1\ \frac{\text{V·s}}{\text{m}^2} = 1\,\text{T} = 10\,000\,\text{G}$)	$\frac{\text{V·s}}{\text{m}^2}$
μ_0	Magnetic constant	$\frac{\text{V·s}}{\text{A·m}}$
H	Magnetic field strength	$\frac{\text{A}}{\text{m}}$

The standards state the field strength H; magnets and measuring instruments mostly give the flux density B in millitesla or gauss. In air the two are firmly linked: 1 mT corresponds to $796\ \frac{\text{A}}{\text{m}}$ and 10 gauss.

Valid: in air and in non-magnetic materials.

EXAMPLE What flux density corresponds to the field strength of $4800\ \frac{\text{A}}{\text{m}}$ that an antimagnetic watch must withstand according to ISO 764?

Given: $\mu_0 = 1.2566 \cdot 10^{-6}\ \frac{\text{V·s}}{\text{A·m}}$; $H = 4800\ \frac{\text{A}}{\text{m}}$

$$\begin{aligned} B &= \mu_0 \cdot H \\ &= 1.2566 \cdot 10^{-6}\ \frac{\text{V·s}}{\text{A·m}} \cdot 4800\ \frac{\text{A}}{\text{m}} \\ &\approx 6.03 \cdot 10^{-3}\ \frac{\text{V·s}}{\text{m}^2} \end{aligned}$$

6.03 mT or 60 gauss. A small permanent magnet, for example in the clasp of a bag or in a loudspeaker, is many times stronger at its surface.

Converting magnetic fields in air

Field strength in $\frac{\mathrm{A}}{\mathrm{m}}$	Flux density in mT	Flux density in gauss	Example
40	0.05	0.5	Earth's magnetic field (order of magnitude)
796	1	10	Conversion value
4800	6.03	60.3	Test field for antimagnetic watches to ISO 764
79 600	100	1000	close to small permanent magnets (order of magnitude)

The gauss is an older unit: 1 G = 0.1 mT. Apart from the standard value, the examples are orders of magnitude.

Impact velocity in a fall

(8.23) MASTER

$$v = \sqrt{2 \cdot g \cdot h}$$

v	Velocity (on impact)	$\frac{\mathrm{m}}{\mathrm{s}}$
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
h	Height (height of fall)	m

The shock test to ISO 1413 simulates a watch falling from a height of 1 m onto a wooden floor. The formula gives the velocity at which the watch hits the floor.

Valid: free fall without air resistance.

EXAMPLE A watch falls 1 m from the workbench onto the floor.

Given: $g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $h = 1\,\mathrm{m}$

$$v = \sqrt{2 \cdot g \cdot h}$$
$$= \sqrt{2 \cdot 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot 1\,\mathrm{m}}$$
$$\approx 4.43\ \frac{\mathrm{m}}{\mathrm{s}}$$

$4.43\ \frac{\mathrm{m}}{\mathrm{s}}$, i.e. about 16 km per hour. In the shock test, the hammer strikes the watch at this velocity.

8.7 Reading error and scatter

Every measurement scatters. The mean, range and standard deviation of a series of measurements are given in Section 12.1; the example there is a series of rate values. For the timing machine this means, for example: if ten measurements in the same position scatter with a standard deviation of 0.6 s per day, their mean is reliable to about 0.2 s per day according to formula (12.4). For the rate from two read time errors, the reading error must be added.

Error of the rate due to reading

(8.24) MASTER

$$u_G = \frac{2 \cdot u_S}{t}$$

Two readings enter into the rate. In the worst case their errors add up; divided by the running period, this gives the maximum error of the rate.

Valid: maximum error; the watch runs undisturbed between the readings.

u_G	Maximum error of the rate due to reading	$\frac{\mathrm{s}}{\mathrm{d}}$
u_S	Maximum reading error of one time error	s
t	Time (running period between the two readings)	d

EXAMPLE The time error can be read on the seconds hand to within 0.5 s. There are 10 days between the two readings.

Given: $u_S = 0.5\,\mathrm{s}$; $t = 10\,\mathrm{d}$

$$\begin{aligned} u_G &= \frac{2 \cdot u_S}{t} \\ &= \frac{2 \cdot 0.5\,\mathrm{s}}{10\,\mathrm{d}} \\ &= 0.10\,\tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

After ten days the rate is reliable to 0.1 s per day, after one day only to 1 s per day. When an accurate result is needed more quickly, the timing machine is used.

Key point: Measuring for longer achieves more than reading more precisely: doubling the running period halves the reading error in the rate.

9 Manufacturing and Materials

Making parts oneself calls for different calculations from regulating: spindle speed from cutting speed, tapers and threads, fit dimensions and bearing clearance, divisions on the dividing head, blank lengths for bent parts. Added to these are the calculations for precious metals, tempering and electroplating.

In this chapter some symbols have the meaning usual in manufacturing: f is the feed, T the tolerance, D and d are diameters, s is a thickness, α a taper or bending angle, m a mass, M the molar mass and A an area; depending on the section, z is the number of teeth, the number of divisions or the valency, and i the number of passes or the ratio of the dividing head, which works towards the slow side and is therefore a reduction in the sense of Chapter 4. The tables contain guide values for a start; the material data sheet and a trial on an offcut are what count.

9.1 Cutting data

In turning, drilling and milling, the cutting edge removes a chip. The cutting speed states how fast it may do so; spindle speed follows from it and the diameter. The feed determines surface finish and machining time.

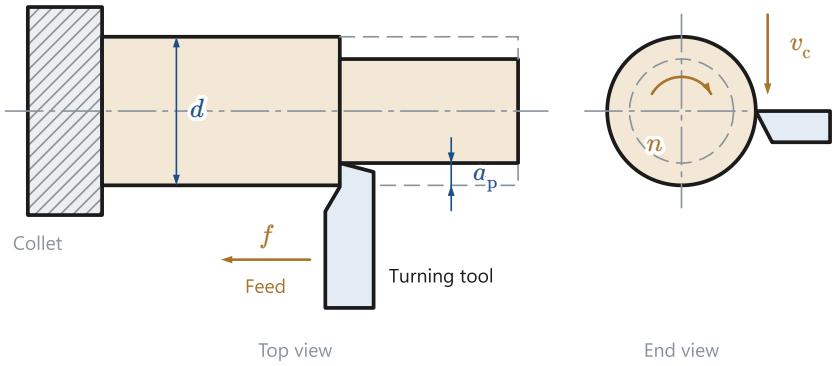


Fig. 9.1 Longitudinal turning: cutting speed v_c at the circumference, feed f per revolution, depth of cut a_p

Cutting speed

(9.1) JOURNEYMAN

$$v_c = \pi \cdot d \cdot n$$

REARRANGED $n = \dfrac{v_c}{\pi \cdot d}$

The cutting speed is the peripheral speed at the cutting edge. It depends on the workpiece material and the tool material (guide values in the table); the spindle speed is chosen accordingly. The general form is formula (2.6).

Valid: Turning, drilling, milling. In facing and on tapers the diameter changes, and with it the cutting speed; the calculation uses the largest diameter.

v_c	Cutting speed	$\frac{\text{m}}{\text{min}}$
d	Diameter (at the cutting point: workpiece in turning, tool in drilling and milling)	mm
n	Spindle speed	$\frac{1}{\text{min}}$

EXAMPLE A brass bar of 8 mm diameter runs at 3000 revolutions per minute. What is the cutting speed at the circumference?

Given: $d = 8\,\text{mm}$; $n = 3000\,\frac{1}{\text{min}}$

$$\begin{aligned} v_c &= \pi \cdot d \cdot n \\ &= \pi \cdot 8\,\text{mm} \cdot 3000\,\tfrac{1}{\text{min}} \\ &\approx 75\,398\,\frac{\text{mm}}{\text{min}} \\ &\approx 75.4\,\tfrac{\text{m}}{\text{min}} \end{aligned}$$

About 75 m per minute. That suits brass with a high-speed steel turning tool.

Spindle speed from cutting speed (workshop formula)

(9.2) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$n = \frac{1000 \cdot v_c}{\pi \cdot d}$$

The same formula as formula (9.1), rearranged for spindle speed and tailored to the usual units. The factor 1000 converts metres to millimetres.

Valid: v_c in m per minute, d in mm, n in revolutions per minute.

n	Spindle speed (numerical value in revolutions per minute)	$\frac{1}{\text{min}}$
v_c	Cutting speed (numerical value in m per minute)	$\frac{\text{m}}{\text{min}}$
d	Diameter (numerical value in mm)	mm

EXAMPLE A piece of silver steel of 3 mm diameter is to be turned with a high-speed steel tool at 25 m per minute. Which spindle speed is set?

Given: $v_c = 25\,\frac{\text{m}}{\text{min}}$; $d = 3\,\text{mm}$

$$\begin{aligned} n &= \frac{1000 \cdot v_c}{\pi \cdot d} \\ &= \frac{1000 \cdot 25}{\pi \cdot 3} \\ &\approx 2653\,\tfrac{1}{\text{min}} \end{aligned}$$

About 2650 revolutions per minute. On the machine the next lower step is chosen.

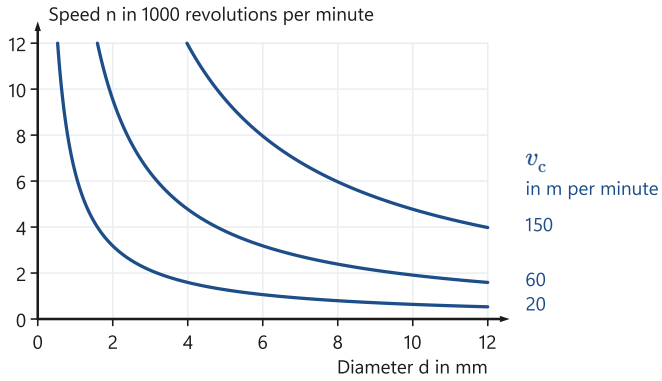


Fig. 9.2 Spindle speed against diameter for three cutting speeds: the smaller the diameter, the higher the spindle speed

Cutting speed in turning (guide values in m per minute)

Material	High-speed steel (HSS)	Carbide
Brass, leaded (CuZn39Pb3)	60 … 120	150 … 400
Nickel silver, leaded (CuNi18Zn19Pb1)	40 … 80	100 … 200
Free-cutting steel (11SMnPb30)	40 … 70	120 … 250
Tool steel, soft-annealed (silver steel 115CrV3)	20 … 30	60 … 120
Steel, hardened and tempered blue	–	25 … 40
Stainless steel, austenitic (1.4404, 1.4435)	10 … 20	60 … 120
Titanium (Grade 2 and Grade 5)	8 … 15	30 … 60

Guide values for turning with a sharp cutting edge. On the watchmaker's lathe and with the hand graver, start at the lower value; for reaming and thread cutting use at most half. High-speed steel cuts hardened, blue-tempered steel only as a hand graver at low speed and with frequent resharpening; on the machine carbide is used. Stainless steel and titanium need coolant and a steady feed.

Feed rate in turning and drilling

(9.3) JOURNEYMAN

$$v_\mathrm{f} = f \cdot n$$

The feed per revolution determines the surface finish: smaller feed, finer turning marks. The feed rate states how fast the slide travels.

v_f	Feed rate	$\frac{\mathrm{mm}}{\mathrm{min}}$
f	Feed per revolution	mm
n	Spindle speed	$\frac{1}{\mathrm{min}}$

EXAMPLE The longitudinal slide advances 0.02 mm per revolution, the spindle runs at 2500 revolutions per minute.

Given: $f = 0.02\,\mathrm{mm}$; $n = 2500\,\frac{1}{\mathrm{min}}$

$$\begin{aligned} v_\mathrm{f} &= f \cdot n \\ &= 0.02\,\mathrm{mm} \cdot 2500\,\tfrac{1}{\mathrm{min}} \\ &= 50\,\tfrac{\mathrm{mm}}{\mathrm{min}} \end{aligned}$$

The turning tool travels 50 mm per minute.

Feed rate in milling

(9.4) JOURNEYMAN

$$v_\mathrm{f} = f_\mathrm{z} \cdot z \cdot n$$

Each cutting edge removes a chip of thickness f_z. If the feed per tooth is too small, the cutter merely rubs and goes blunt; if it is too large, the cutter breaks.

Valid: As a guide for small cutters: feed per tooth about 0.5 % to 1 % of the cutter diameter.

v_f	Feed rate	$\frac{\mathrm{mm}}{\mathrm{min}}$
f_z	Feed per tooth	mm
z	Number of teeth (cutting edges of the milling cutter)	1
n	Spindle speed	$\frac{1}{\mathrm{min}}$

EXAMPLE A carbide milling cutter of 1 mm diameter with 2 cutting edges runs in brass at 20 000 revolutions per minute. It is to remove 0.005 mm per tooth.

Given: $f_\mathrm{z} = 0.005\,\mathrm{mm}; \quad z = 2; \quad n = 20\,000\,\frac{1}{\mathrm{min}}$

$$\begin{aligned} v_\mathrm{f} &= f_\mathrm{z} \cdot z \cdot n \\ &= 0.005\,\mathrm{mm} \cdot 2 \cdot 20\,000\,\tfrac{1}{\mathrm{min}} \\ &= 200\,\tfrac{\mathrm{mm}}{\mathrm{min}} \end{aligned}$$

The table travels at 200 mm per minute. The cutting speed is then 63 m per minute according to formula (9.1).

Machining time in turning and drilling

(9.5) JOURNEYMAN

$$t_\mathrm{h} = \frac{L \cdot i}{f \cdot n}$$

The machining time is the feed travel of all passes divided by the feed rate. It is the basis for costing turned parts.

Valid: Same spindle speed and same feed in all passes.

t_h	Machining time (time during which the tool is feeding)	min
L	Length (feed travel: workpiece length with approach and overrun)	mm
i	Number (of passes)	1
f	Feed per revolution	mm
n	Spindle speed	$\frac{1}{\mathrm{min}}$

EXAMPLE A pivot is turned over a length of 10 mm in 3 passes; with approach and overrun the feed travel is 12 mm. Feed 0.02 mm, spindle speed 2500 revolutions per minute.

Given: $L = 12\,\mathrm{mm}; \quad i = 3; \quad f = 0.02\,\mathrm{mm}; \quad n = 2500\,\frac{1}{\mathrm{min}}$

$$\begin{aligned} t_\mathrm{h} &= \frac{L \cdot i}{f \cdot n} \\ &= \frac{12\,\mathrm{mm} \cdot 3}{0.02\,\mathrm{mm} \cdot 2500\,\frac{1}{\mathrm{min}}} \\ &= 0.72\,\mathrm{min} \end{aligned}$$

The machine cuts for 0.72 minutes, that is 43 seconds. Setting up, measuring and return travel come on top.

Metal removal rate in turning

(9.6) ADVANCED

$$Q = a_\mathrm{p} \cdot f \cdot v_\mathrm{c}$$

Depth of cut times feed is the cross-section of the chip, the cutting speed its length per unit time. The metal removal rate is used to compare cutting data and to estimate the drive power required.

Valid: Longitudinal turning; cutting speed at the mean diameter of the pass.

Q	Metal removal rate (volume removed per unit time)	$\frac{\mathrm{mm}^3}{\mathrm{min}}$
a_p	Depth of cut (half the diameter reduction per pass)	mm
f	Feed per revolution	mm
v_c	Cutting speed	$\frac{\mathrm{m}}{\mathrm{min}}$

EXAMPLE Brass is rough-turned with 0.5 mm depth of cut, 0.05 mm feed and 75 m per minute.

Given: $a_\mathrm{p} = 0.5\,\mathrm{mm}$; $f = 0.05\,\mathrm{mm}$; $v_\mathrm{c} = 75\ \frac{\mathrm{m}}{\mathrm{min}}$

$$\begin{aligned} Q &= a_\mathrm{p} \cdot f \cdot v_\mathrm{c} \\ &= 0.5\,\mathrm{mm} \cdot 0.05\,\mathrm{mm} \cdot 75\ \tfrac{\mathrm{m}}{\mathrm{min}} \\ &= 0{,}025\,\mathrm{mm}^2 \cdot 75\,000\ \frac{\mathrm{mm}}{\mathrm{min}} \\ &= 1875\ \tfrac{\mathrm{mm}^3}{\mathrm{min}} \end{aligned}$$

Just under 1.9 cm³ of brass is machined per minute.

Key point: Small diameter, high spindle speed: half the diameter requires twice the spindle speed.

9.2 Taper and slope

A taper is defined by two diameters and the length, or by the taper ratio. For turning it, the half taper angle or the tailstock offset is needed.

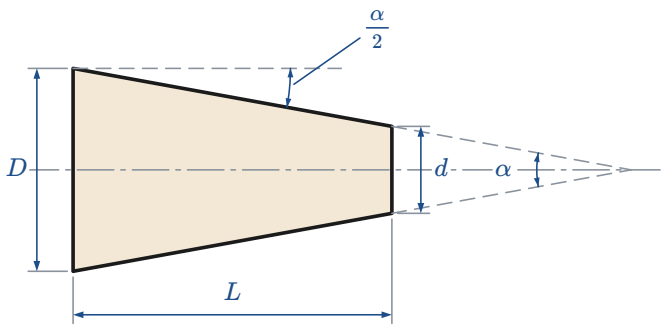


Fig. 9.3 Taper: diameters D and d, length L, taper angle α and setting angle $\frac{\alpha}{2}$ (drawn steeper for clarity)

Taper ratio (conicity)

(9.7) JOURNEYMAN

$$C = \frac{D - d}{L}$$

REARRANGED $d = D - C \cdot L$

The taper ratio states by how much the diameter decreases per unit length. 1 : 50 means: over 50 mm of length the taper becomes 1 mm thinner.

C	Taper ratio (written as $1 : x$ with $x = \frac{1}{C}$)	1
D	Diameter (large taper diameter)	mm
d	Diameter (small taper diameter)	mm
L	Length (taper length, measured along the axis)	mm

Good to know: The slope refers to the centre line and is half the taper ratio: a 1 : 50 taper has a slope of 1 : 100.

EXAMPLE A taper pin is 20 mm long and measures 2.0 mm at the thin end and 2.4 mm at the thick end.

Given: $D = 2.4\,\mathrm{mm}$; $d = 2.0\,\mathrm{mm}$; $L = 20\,\mathrm{mm}$

$$\begin{aligned} C &= \frac{D - d}{L} \\ &= \frac{2.4\,\mathrm{mm} - 2.0\,\mathrm{mm}}{20\,\mathrm{mm}} \\ &= 0.02 \end{aligned}$$

The taper ratio is 0.02, i.e. 1 : 50. Taper pins are standardised this way.

Taper angle and setting angle

(9.8) JOURNEYMAN

$$\tan\left(\frac{\alpha}{2}\right) = \frac{D - d}{2 \cdot L}$$

REARRANGED $\alpha = 2 \cdot \arctan\left(\frac{D - d}{2 \cdot L}\right)$

When turning a taper with the top slide, the slide is swivelled by half the taper angle. The half taper angle is therefore called the setting angle.

α	Taper angle (full angle at the apex of the cone; $\frac{\alpha}{2}$ is the setting angle)	°
D	Diameter (large taper diameter)	mm
d	Diameter (small taper diameter)	mm
L	Length (taper length, measured along the axis)	mm

Good to know: For slender tapers it is sufficient to use: setting angle in degrees ≈ $28.65 \cdot C$. A 1 : 50 taper thus has a setting angle of 0.573°.

EXAMPLE The tapered seat of an arbor narrows from 1.2 mm to 1.0 mm over a length of 5 mm. To what angle is the top slide swivelled?

Given: $D = 1.2\,\mathrm{mm}$; $d = 1.0\,\mathrm{mm}$; $L = 5\,\mathrm{mm}$

$$\begin{aligned} \alpha &= 2 \cdot \arctan\left(\frac{D - d}{2 \cdot L}\right) \\ &= 2 \cdot \arctan\left(\frac{1.2\,\mathrm{mm} - 1.0\,\mathrm{mm}}{2 \cdot 5\,\mathrm{mm}}\right) \\ &= 2 \cdot \arctan 0{,}02 \\ &\approx 2.29^\circ \end{aligned}$$

The taper angle is 2.29°. The top slide is set to half of this, i.e. about 1.15°.

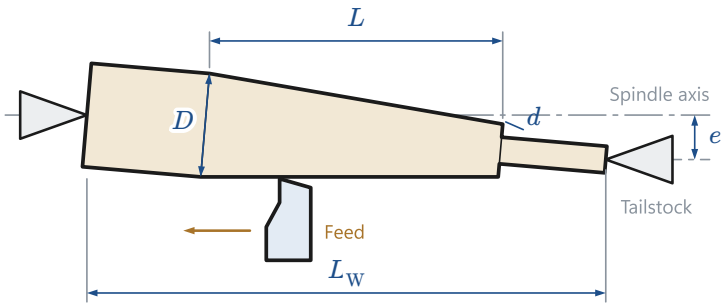


Fig. 9.4 Taper turning between centres: the tailstock is offset by e, the turning tool runs parallel to the spindle axis (plan view, exaggerated)

Tailstock offset

(9.9) APPROXIMATION JOURNEYMAN

$$e \approx \frac{L_W \cdot (D - d)}{2 \cdot L}$$

If the tailstock centre is offset sideways, the workpiece lies at an angle to the spindle axis, and the turning tool travelling lengthwise turns a taper. If the taper extends over the whole workpiece length, the offset is half the difference in diameter.

Valid: Turning between centres, only for slender tapers. As a guide, the offset should be at most one fiftieth of the workpiece length.

e	Tailstock offset (lateral displacement of the tailstock centre)	mm
L_W	Length (workpiece between centres)	mm
D	Diameter (large taper diameter)	mm
d	Diameter (small taper diameter)	mm
L	Length (taper length, measured along the axis)	mm

Caution: With the tailstock offset, the centre holes sit askew on the centres. Therefore choose only small offsets, measure after the first pass and readjust.

EXAMPLE A workpiece 60 mm long is to receive a 1 : 50 taper over 30 mm of its length, from 4.0 mm to 3.4 mm.

Given: $L_W = 60\,\text{mm}$; $D = 4.0\,\text{mm}$; $d = 3.4\,\text{mm}$; $L = 30\,\text{mm}$

$$\begin{aligned} e &\approx \frac{L_W \cdot (D - d)}{2 \cdot L} \\ &= \frac{60\,\text{mm} \cdot (4.0\,\text{mm} - 3.4\,\text{mm})}{2 \cdot 30\,\text{mm}} \\ &= 0.60\,\text{mm} \end{aligned}$$

The tailstock is offset by 0.60 mm towards the turning tool; the thin end of the taper is then at the tailstock. The offset is one hundredth of the workpiece length and therefore permissible.

9.3 Threads

Today's watch screws have metric V-threads with a 60° flank angle. All dimensions of the profile follow from the nominal diameter d and the pitch P.

Pitch from threads per inch (inch threads)

(9.10) JOURNEYMAN

$$P = \frac{1}{n_{\mathrm{G}}}$$

P	Thread pitch	mm
n_{G}	Number of threads (threads per unit length; for inch threads per inch)	$\frac{1}{\mathrm{in}}$

The pitch is the distance a screw advances in one revolution. For metric threads it is part of the designation; for inch threads the number of threads per inch is given instead.

Valid: Single-start threads.

EXAMPLE A thread on an English watch has 40 threads per inch.

Given: $n_{\mathrm{G}} = 40\ \frac{1}{\mathrm{in}}$

$$\begin{aligned} P &= \frac{1}{n_{\mathrm{G}}} \\ &= \frac{1}{40\ \frac{1}{\mathrm{in}}} \\ &= \frac{25{,}4\,\mathrm{mm}}{40} \\ &= 0.635\,\mathrm{mm} \end{aligned}$$

The pitch is 0.635 mm, since one inch is 25.4 mm.

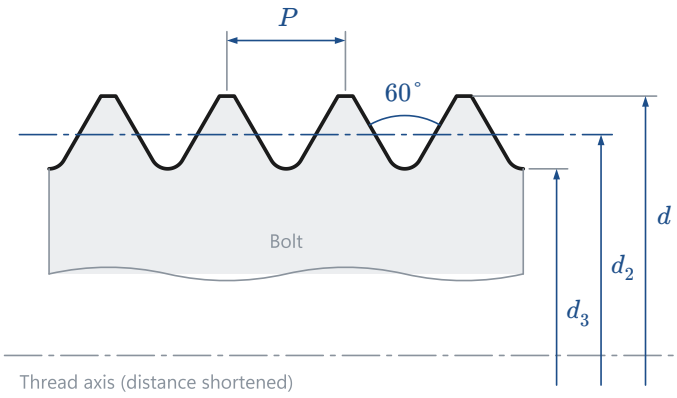


Fig. 9.5 Metric ISO thread, profile of the bolt: pitch P, major diameter d, pitch diameter d_2, minor diameter d_3

Pitch diameter

(9.11) MASTER

$$d_2 = d - 0.6495 \cdot P$$

d_2	Pitch diameter	mm
d	Diameter (nominal diameter of the thread, outside on the bolt)	mm
P	Thread pitch	mm

At the pitch diameter, thread tooth and groove are equally wide. It decides how screw and nut fit together and is checked with gauges or over measuring wires.

Valid: Metric ISO thread with 60° flank angle, nominal size without tolerance. The factor is $\frac{3}{8} \cdot \sqrt{3}$.

EXAMPLE Thread M 1.6 with a pitch of 0.35 mm.

Given: $d = 1.6\,\mathrm{mm}; \quad P = 0.35\,\mathrm{mm}$

$$\begin{aligned} d_2 &= d - 0.6495 \cdot P \\ &= 1.6\,\mathrm{mm} - 0.6495 \cdot 0.35\,\mathrm{mm} \\ &\approx 1.373\,\mathrm{mm} \end{aligned}$$

The pitch diameter is 1.373 mm.

Minor diameter of the external thread

(9.12) MASTER

$$d_3 = d - 1.2269 \cdot P$$

The minor diameter is the load-bearing cross-section of the screw. An overtightened screw shears off there.

Valid: Metric ISO thread from M 1 upwards (thread depth $0.6134 \cdot P$). For the miniature threads S 0.3 to S 1.4 the thread depth is smaller: $d_3 = d - 1.12 \cdot P$.

d_3	Minor diameter (at the thread root of the bolt)	mm
d	Diameter (nominal diameter of the thread, outside on the bolt)	mm
P	Thread pitch	mm

EXAMPLE Thread M 1.6 with a pitch of 0.35 mm.

Given: $d = 1.6\,\mathrm{mm}$; $P = 0.35\,\mathrm{mm}$

$$\begin{aligned} d_3 &= d - 1.2269 \cdot P \\ &= 1.6\,\mathrm{mm} - 1.2269 \cdot 0.35\,\mathrm{mm} \\ &\approx 1.171\,\mathrm{mm} \end{aligned}$$

At the thread root the screw is still 1.171 mm thick.

Tapping drill diameter

(9.13) APPROXIMATION JOURNEYMAN

$$d_\mathrm{K} \approx d - P$$

The hole is drilled to this diameter before the tap cuts the profile.

Valid: Metric threads with 60° flank angle. In tough materials such as steel and nickel silver, rather choose one drill size larger so that the tap does not jam.

d_K	Tapping drill diameter (hole drilled before thread cutting)	mm
d	Diameter (nominal diameter of the thread, outside on the bolt)	mm
P	Thread pitch	mm

EXAMPLE A thread is to be cut for an M 1.2 screw with a pitch of 0.25 mm.

Given: $d = 1.2\,\mathrm{mm}$; $P = 0.25\,\mathrm{mm}$

$$\begin{aligned} d_\mathrm{K} &\approx d - P \\ &= 1.2\,\mathrm{mm} - 0.25\,\mathrm{mm} \\ &= 0.95\,\mathrm{mm} \end{aligned}$$

The hole is drilled with 0.95 mm.

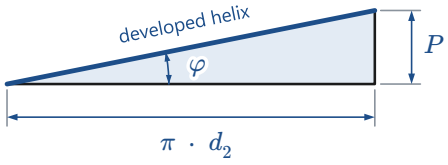


Fig. 9.6 One thread turn unrolled: pitch P over the circumference at the pitch diameter

Lead angle

(9.14) MASTER

$$\tan \varphi = \frac{P}{\pi \cdot d_2}$$

REARRANGED $\varphi = \arctan\left(\frac{P}{\pi \cdot d_2}\right)$

φ	Lead angle (at the pitch diameter)	°
P	Thread pitch	mm
d_2	Pitch diameter	mm

Unrolling one turn of a thread gives a wedge: height P, base the circumference at the pitch diameter. If the lead angle is smaller than the angle of friction (Chapter 2), the screw does not loosen by itself.

Valid: Single-start threads.

EXAMPLE Thread M 1.6: pitch 0.35 mm, pitch diameter 1.373 mm.

Given: $P = 0.35\,\text{mm}$; $d_2 = 1.373\,\text{mm}$

$$\begin{aligned}\varphi &= \arctan\left(\frac{P}{\pi \cdot d_2}\right)\\ &= \arctan\left(\frac{0.35\,\text{mm}}{\pi \cdot 1.373\,\text{mm}}\right)\\ &\approx \arctan 0{,}0811\\ &\approx 4.64^\circ\end{aligned}$$

The lead angle is 4.64°. It lies below the angle of friction, so the screw holds by itself.

Metric watch and fine threads (nominal sizes in mm)

Thread	Pitch P	Pitch diameter d_2	Minor diameter d_3	Tapping drill
S 0.3	0.08	0.248	0.210	0.22
S 0.4	0.1	0.335	0.288	0.30
S 0.5	0.125	0.419	0.360	0.38
S 0.6	0.15	0.503	0.432	0.45
S 0.7	0.175	0.586	0.504	0.53
S 0.8	0.2	0.670	0.576	0.60
S 0.9	0.225	0.754	0.648	0.68
M 1	0.25	0.838	0.693	0.75
M 1.2	0.25	1.038	0.893	0.95
M 1.4	0.3	1.205	1.032	1.10
M 1.6	0.35	1.373	1.171	1.25
M 2	0.4	1.740	1.509	1.60

S: ISO miniature thread (minor diameter calculated with $d - 1.12 \cdot P$). M: ISO metric coarse thread. Tapping drill according to formula (9.13), rounded to 0.01 mm. Screws in older movements often deviate from the standard; if in doubt, measure diameter and pitch.

Key point: Tapping hole equals nominal diameter minus pitch.

9.4 Fits and tolerances

No part can be made exactly to size. The drawing therefore specifies two limits of size between which the actual size must lie. Whether two parts fit together with clearance or with interference is decided by the position of their tolerance zones.

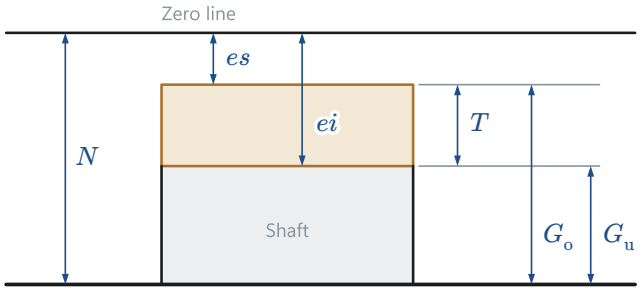


Fig. 9.7 Limits of size of a shaft: nominal size N, deviations es and ei, upper limit of size G_o, lower limit of size G_u, tolerance T (tolerance zone greatly enlarged)

Upper and lower limit of size

(9.15) JOURNEYMAN

$$G_o = N + es$$

The deviations are added to the nominal size with their signs. Correspondingly, the lower limit of size is $G_u = N + ei$ with the lower deviation ei.

Valid: Shafts and all external dimensions; for holes and internal dimensions with ES and EI.

G_o	Upper limit of size	mm
N	Nominal size	mm
es	Upper deviation (+ means: above the nominal size; for holes ES)	mm

EXAMPLE A shaft has the nominal size 2 mm and the deviations −0.006 mm and −0.016 mm (tolerance zone f7).

Given: $N = 2\,\text{mm}$; $es = -0.006\,\text{mm}$

$$\begin{aligned} G_o &= N + es \\ &= 2\,\text{mm} + (-0.006\,\text{mm}) \\ &= 1.994\,\text{mm} \end{aligned}$$

The upper limit of size is 1.994 mm. With the lower deviation, the lower limit of size is 1.984 mm.

Tolerance

(9.16) JOURNEYMAN

$$T = es - ei$$

The tolerance is the distance between the two limits of size: $T = G_o - G_u$. It has no sign.

T	Tolerance (permissible difference between upper and lower limit of size)	mm
es	Upper deviation (+ means: above the nominal size; for holes ES)	mm
ei	Lower deviation (+ means: above the nominal size; for holes EI)	mm

EXAMPLE Shaft with the deviations −0.006 mm and −0.016 mm.

Given: $es = -0.006\,\text{mm}$; $ei = -0.016\,\text{mm}$

$$\begin{aligned} T &= es - ei \\ &= -0.006\,\text{mm} - (-0.016\,\text{mm}) \\ &= 0.010\,\text{mm} \end{aligned}$$

The tolerance is 0.010 mm: the diameter may vary by this much from part to part.

How two parts fit together depends on the position of the tolerance zones. **Clearance fit:** the hole is always larger than the shaft, as with a pivot in its bearing. **Interference fit:** the shaft is always larger than the hole, as

with a jewel in the mainplate or a wheel on its arbor. **Transition fit:** depending on the actual sizes, there is a small clearance or a small interference, as with a dowel pin.

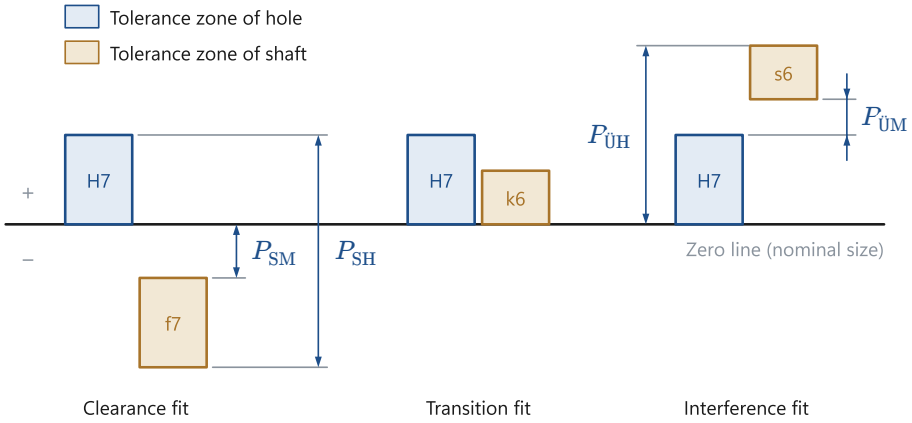


Fig. 9.8 Tolerance zones of hole and shaft relative to the zero line, for three fits with the hole H7

Maximum clearance

(9.17) JOURNEYMAN

$$P_{SH} = G_{oB} - G_{uW}$$

The largest clearance arises when the largest permissible hole meets the smallest permissible shaft.

P_{SH}	Maximum clearance	mm
G_{oB}	Upper limit of size (hole)	mm
G_{uW}	Lower limit of size (shaft)	mm

EXAMPLE Hole 2 mm with the deviations 0 and +0.010 mm (H7), shaft 2 mm with −0.006 mm and −0.016 mm (f7).

Given: $G_{oB} = 2.010\,\text{mm}$; $G_{uW} = 1.984\,\text{mm}$

$$\begin{aligned} P_{SH} &= G_{oB} - G_{uW} \\ &= 2.010\,\text{mm} - 1.984\,\text{mm} \\ &= 0.026\,\text{mm} \end{aligned}$$

Largest hole and smallest shaft give 0.026 mm clearance.

Minimum clearance

(9.18) JOURNEYMAN

$$P_{SM} = G_{uB} - G_{oW}$$

The smallest clearance arises when the smallest hole meets the largest shaft. If it remains positive, every pair can be assembled.

P_{SM}	Minimum clearance (a negative value is an interference)	mm
G_{uB}	Lower limit of size (hole)	mm
G_{oW}	Upper limit of size (shaft)	mm

EXAMPLE The same fit: hole 2.000 mm to 2.010 mm, shaft 1.984 mm to 1.994 mm.

Given: $G_{uB} = 2.000\,\text{mm}$; $G_{oW} = 1.994\,\text{mm}$

$$\begin{aligned} P_{SM} &= G_{uB} - G_{oW} \\ &= 2.000\,\text{mm} - 1.994\,\text{mm} \\ &= 0.006\,\text{mm} \end{aligned}$$

Even in the worst case 0.006 mm clearance remains: it is a clearance fit.

Maximum interference

(9.19) JOURNEYMAN

$$P_{\mathrm{Imax}} = G_{\mathrm{oW}} - G_{\mathrm{uB}}$$

P_{Imax}	Maximum interference	mm
G_{oW}	Upper limit of size (shaft)	mm
G_{uB}	Lower limit of size (hole)	mm

In an interference fit the inner part is larger than the hole. The difference is called interference; it is taken up elastically during pressing in and holds the part firmly.

Valid: The inner part (here the jewel) counts as the shaft.

EXAMPLE A jewel of 1.200 mm to 1.204 mm diameter is pressed into a hole reamed to 1.190 mm to 1.194 mm.

Given: $G_{\mathrm{oW}} = 1.204\,\mathrm{mm}$; $G_{\mathrm{uB}} = 1.190\,\mathrm{mm}$

$$\begin{aligned} P_{\mathrm{Imax}} &= G_{\mathrm{oW}} - G_{\mathrm{uB}} \\ &= 1.204\,\mathrm{mm} - 1.190\,\mathrm{mm} \\ &= 0.014\,\mathrm{mm} \end{aligned}$$

The largest jewel is 0.014 mm thicker than the smallest hole.

Minimum interference

(9.20) JOURNEYMAN

$$P_{\mathrm{Imin}} = G_{\mathrm{uW}} - G_{\mathrm{oB}}$$

P_{Imin}	Minimum interference	mm
G_{uW}	Lower limit of size (shaft)	mm
G_{oB}	Upper limit of size (hole)	mm

The minimum interference decides whether the seat holds in every case.

Good to know: Jewels are pressed in with about 0.01 mm interference: the hole is reamed smaller than the jewel by this amount. With too little interference the jewel creeps; with too much it can crack or distort the mainplate.

EXAMPLE The same jewel: jewel 1.200 mm to 1.204 mm, hole 1.190 mm to 1.194 mm.

Given: $G_{\mathrm{uW}} = 1.200\,\mathrm{mm}$; $G_{\mathrm{oB}} = 1.194\,\mathrm{mm}$

$$\begin{aligned} P_{\mathrm{Imin}} &= G_{\mathrm{uW}} - G_{\mathrm{oB}} \\ &= 1.200\,\mathrm{mm} - 1.194\,\mathrm{mm} \\ &= 0.006\,\mathrm{mm} \end{aligned}$$

Even the smallest jewel still sits in the largest hole with 0.006 mm interference.

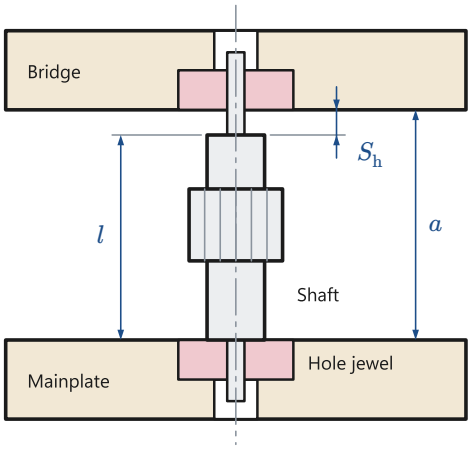


Fig. 9.9 End shake S_h: distance a between the bearing faces minus length l between the shoulders (play exaggerated)

End shake of an arbor

(9.21) JOURNEYMAN

$$S_\mathrm{h} = a - l$$

Every arbor needs some play in the axial direction, otherwise it jams between the bearings. In bearings with cap jewels, the pivot ends and the distance between the two cap jewels count instead of the shoulders.

S_h	End shake (play of the arbor in the axial direction)	mm
a	Distance (between the bearing faces; inner faces of the two hole jewels or bearings)	mm
l	Length (arbor between the two shoulders)	mm

EXAMPLE The hole jewels of a third wheel are 2.45 mm apart; the arbor measures 2.42 mm between the shoulders.

Given: $a = 2.45\,\mathrm{mm}$; $l = 2.42\,\mathrm{mm}$

$$\begin{aligned} S_\mathrm{h} &= a - l \\ &= 2.45\,\mathrm{mm} - 2.42\,\mathrm{mm} \\ &= 0.03\,\mathrm{mm} \end{aligned}$$

The arbor has 0.03 mm end shake, a usual value for the gear train of a wristwatch.

Maximum end shake

(9.22) MASTER

$$S_\mathrm{h,max} = a_\mathrm{max} - l_\mathrm{min}$$

The largest play results from the largest distance and the shortest arbor.

$S_\mathrm{h,max}$	End shake (maximum value)	mm
a_max	Distance between the bearing faces (upper limit of size)	mm
l_min	Length (arbor between the shoulders, lower limit of size)	mm

EXAMPLE Distance between the hole jewels 2.45 mm ± 0.01 mm, arbor length between the shoulders 2.42 mm ± 0.005 mm.

Given: $a_\mathrm{max} = 2.46\,\mathrm{mm}$; $l_\mathrm{min} = 2.415\,\mathrm{mm}$

$$\begin{aligned} S_\mathrm{h,max} &= a_\mathrm{max} - l_\mathrm{min} \\ &= 2.46\,\mathrm{mm} - 2.415\,\mathrm{mm} \\ &= 0.045\,\mathrm{mm} \end{aligned}$$

In the worst case the arbor has 0.045 mm play.

Minimum end shake

(9.23) MASTER

$$S_\mathrm{h,min} = a_\mathrm{min} - l_\mathrm{max}$$

The smallest play results from the smallest distance and the longest arbor. It must remain greater than zero.

$S_\mathrm{h,min}$	End shake (minimum value)	mm
a_min	Distance between the bearing faces (lower limit of size)	mm
l_max	Length (arbor between the shoulders, upper limit of size)	mm

EXAMPLE The same dimensions: distance 2.45 mm ± 0.01 mm, arbor length 2.42 mm ± 0.005 mm.

Given: $a_\mathrm{min} = 2.44\,\mathrm{mm}$; $l_\mathrm{max} = 2.425\,\mathrm{mm}$

$$\begin{aligned} S_\mathrm{h,min} &= a_\mathrm{min} - l_\mathrm{max} \\ &= 2.44\,\mathrm{mm} - 2.425\,\mathrm{mm} \\ &= 0.015\,\mathrm{mm} \end{aligned}$$

At least 0.015 mm play always remains. The arbor therefore never jams.

Tolerance of end shake (tolerance chain)

(9.24) MASTER

$$T_{\text{Luft}} = T_a + T_l$$

In a dimension chain the tolerances of all dimensions involved add up. The play therefore varies more than any single dimension.

Valid: Arithmetic worst-case calculation; applies equally to chains of more than two dimensions.

T_{Luft}	Tolerance (end shake)	mm
T_a	Tolerance (distance between the bearing faces)	mm
T_l	Tolerance (arbor length)	mm

Good to know: If the play must be held closer than the sum of the tolerances allows, it is adjusted during assembly: the pressed-in jewel is moved until the end shake is correct.

EXAMPLE The distance between the hole jewels is toleranced to ± 0.01 mm (tolerance 0.02 mm), the arbor length to ± 0.005 mm (tolerance 0.01 mm).

Given: $T_a = 0.02\,\mathrm{mm}; \quad T_l = 0.01\,\mathrm{mm}$

$$\begin{aligned} T_{\text{Luft}} &= T_a + T_l \\ &= 0.02\,\mathrm{mm} + 0.01\,\mathrm{mm} \\ &= 0.03\,\mathrm{mm} \end{aligned}$$

The end shake varies by 0.03 mm, namely between 0.015 mm and 0.045 mm.

9.5 Indexing

To mill wheels, pinions and divisions, the workpiece is turned on by a fixed angle after each cut. In direct indexing, the spindle itself locks into an index plate. In indirect indexing, a crank turns the spindle through a worm gear, usually with a ratio of 40 : 1.

Indexing angle

(9.25) JOURNEYMAN

$$\tau = \frac{360^\circ}{z}$$

The full circumference is divided into z equal parts.

τ	Indexing angle (angle through which the workpiece turns per indexing step)	°
z	Number of divisions (number of equal parts around the circumference, for example the number of teeth)	1

EXAMPLE A pinion with 12 leaves is milled.

Given: $z = 12$

$$\begin{aligned} \tau &= \frac{360^\circ}{z} \\ &= \frac{360^\circ}{12} \\ &\approx 30^\circ \end{aligned}$$

After each leaf the pinion is turned on by 30°. For the escape wheel the same angle is called the pitch, formula (7.1).

Direct indexing

(9.26) JOURNEYMAN

$$n_{\mathrm{L}} = \frac{N_{\mathrm{L}}}{z}$$

In direct indexing, the index plate sits on the spindle that carries the workpiece, and the index pin engages directly in the hole circle.

Valid: The number of holes must be divisible by the number of divisions.

n_{L}	Hole spaces per indexing step	1
N_{L}	Number of holes in the hole circle	1
z	Number of divisions (number of equal parts around the circumference, for example the number of teeth)	1

EXAMPLE The index plate on the spindle has a hole circle with 60 holes. A pinion with 12 leaves is to be milled.

Given: $N_{\mathrm{L}} = 60; \quad z = 12$

$$\begin{aligned} n_{\mathrm{L}} &= \frac{N_{\mathrm{L}}}{z} \\ &= \frac{60}{12} \\ &= 5 \end{aligned}$$

After each leaf the plate is moved on by 5 hole spaces.

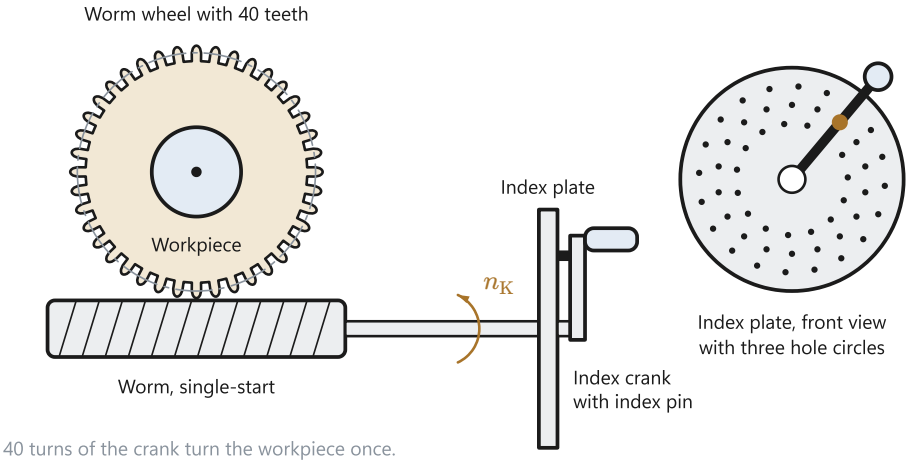


Fig. 9.10 Dividing head: the index crank turns the workpiece through the worm and worm wheel; the index pin engages in a hole circle of the index plate

Indirect indexing with the dividing head

(9.27) MASTER

$$n_{\mathrm{K}} = \frac{i}{z}$$

The crank turns the workpiece through the worm and worm wheel. The fraction of a turn is counted off on a hole circle: reduce or expand the fraction so that an available number of holes is in the denominator. The numerator is then the number of hole spaces.

Valid: A suitable hole circle must be available for the fraction; otherwise only differential indexing or a different index plate will help.

n_{K}	Crank turns per indexing step	1
i	Ratio (of the dividing head, reducing: crank turns per revolution of the workpiece; equal to the number of teeth of the worm wheel with a single-start worm, usually 40)	1
z	Number of divisions (number of equal parts around the circumference, for example the number of teeth)	1

Caution: Hole spaces are counted, not holes. The hole in which the index pin sits does not count; the sector arms are set accordingly.

EXAMPLE An escape wheel with 15 teeth is milled on a dividing head with a ratio of 40.

Given: $i = 40$; $z = 15$

$$\begin{aligned} n_{\mathrm{K}} &= \frac{i}{z} \\ &= \frac{40}{15} \\ &= 2\,\tfrac{2}{3} = 2\,\tfrac{10}{15} \\ &\approx 2.667 \end{aligned}$$

Per tooth, two full crank turns and 10 hole spaces on the 15-hole circle. Equivalent are 12 hole spaces on the 18-hole circle.

Angular indexing with the dividing head

(9.28) MASTER

$$n_{\mathrm{K}} = \frac{i \cdot \tau}{360^\circ}$$

If an angle is given instead of the number of divisions, it is converted into crank turns. With a ratio of 40, one crank turn rotates the workpiece by 9°.

n_{K}	Crank turns per indexing step	1
i	Ratio (of the dividing head, reducing: crank turns per revolution of the workpiece; equal to the number of teeth of the worm wheel with a single-start worm, usually 40)	1
τ	Indexing angle (angle through which the workpiece turns per indexing step)	°

EXAMPLE On a dividing head with a ratio of 40, a minute division is to be marked out, i.e. lines spaced 6° apart.

Given: $i = 40$; $\tau = 6^\circ$

$$\begin{aligned} n_{\mathrm{K}} &= \frac{i \cdot \tau}{360^\circ} \\ &= \frac{40 \cdot 6^\circ}{360^\circ} \\ &= \tfrac{2}{3} = \tfrac{12}{18} \\ &\approx 0.667 \end{aligned}$$

Two thirds of a turn, i.e. 12 hole spaces on the 18-hole circle.

Key point: Dividing head with ratio 40: crank turns equal 40 divided by the number of divisions.

9.6 Bending and blank length

In bending, the outer side is stretched and the inner side compressed. In between lies the neutral axis, which keeps its length. Its length is the blank length.

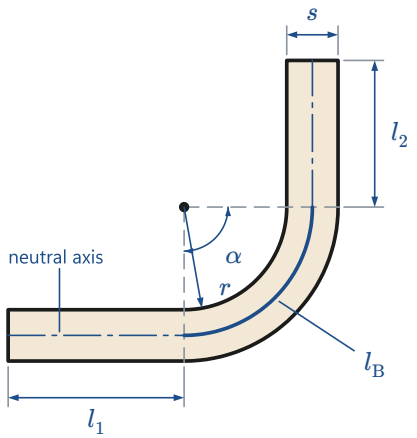


Fig. 9.11 Bent part: the neutral axis lies at the radius $r + \frac{s}{2}$

Developed length of a bend

(9.29) JOURNEYMAN

$$l_\mathrm{B} = \frac{\pi \cdot \left(r + \frac{s}{2}\right) \cdot \alpha}{180^\circ}$$

The neutral axis lies in the middle of the sheet and keeps its length during bending. The blank length of the whole part is the sum of the straight legs and all bends: $L = l_1 + l_\mathrm{B} + l_2$.

Valid: Bend radius at least about five times the thickness. For tighter radii, calculate with the correction factor from formula (9.30).

l_B	Length (arc of the neutral axis)	mm
r	Bend radius (measured on the inside)	mm
s	Thickness (sheet, strip or wire)	mm
α	Bend angle (angle swept by the bend)	°

EXAMPLE A spring sheet of 0.5 mm thickness is bent through 90° with an inside radius of 3 mm.

Given: $r = 3\,\mathrm{mm}$; $s = 0.5\,\mathrm{mm}$; $\alpha = 90^\circ$

$$l_\mathrm{B} = \frac{\pi \cdot \left(r + \frac{s}{2}\right) \cdot \alpha}{180^\circ}$$

$$= \frac{\pi \cdot \left(3\,\mathrm{mm} + \frac{0.5\,\mathrm{mm}}{2}\right) \cdot 90^\circ}{180^\circ}$$

$$\approx 5.11\,\mathrm{mm}$$

The bend requires 5.11 mm of material.

Correction factor for tight bend radii

(9.30) ADVANCED

$$k = 0.65 + 0.5 \cdot \lg\left(\frac{r}{s}\right)$$

k	Correction factor (position of the neutral axis)	1
r	Bend radius (measured on the inside)	mm
s	Thickness (sheet, strip or wire)	mm

With a tight radius the neutral axis shifts towards the inside. In formula (9.29), $\frac{s}{2}$ is then replaced by $k \cdot \frac{s}{2}$: $l_\mathrm{B} = \pi \cdot \left(r + k \cdot \frac{s}{2}\right) \cdot \frac{\alpha}{180°}$.

Valid: Usual rule for sheet-metal bent parts; ratio of r to s between 0.65 and 5. Above that, $k = 1$. For precise parts, bend a test piece.

EXAMPLE A sheet of 0.5 mm thickness is bent with an inside radius of 1 mm.

Given: $r = 1\,\mathrm{mm}$; $s = 0.5\,\mathrm{mm}$

$$k = 0.65 + 0.5 \cdot \lg\left(\frac{r}{s}\right)$$
$$= 0.65 + 0.5 \cdot \lg\left(\frac{1\,\mathrm{mm}}{0.5\,\mathrm{mm}}\right)$$
$$\approx 0.80$$

With $k = 0.80$, a 90° bend is 1.88 mm long instead of 1.96 mm.

Blank length for a ring

(9.31) JOURNEYMAN

$$L = \pi \cdot (d_\mathrm{i} + s)$$

L	Length (blank, developed)	mm
d_i	Diameter (inside diameter of the ring)	mm
s	Thickness (sheet, strip or wire)	mm

For a closed ring, such as a bezel or spring ring, the circumference at the mean diameter counts. Calculated from the outside diameter: $L = \pi \cdot (d_\mathrm{a} - s)$.

Valid: Butt-jointed ring; allowance for cleaning up the soldered joint according to experience.

EXAMPLE A bezel with 30 mm inside diameter is to be bent from strip 0.8 mm thick.

Given: $d_\mathrm{i} = 30\,\mathrm{mm}$; $s = 0.8\,\mathrm{mm}$

$$L = \pi \cdot (d_\mathrm{i} + s)$$
$$= \pi \cdot (30\,\mathrm{mm} + 0.8\,\mathrm{mm})$$
$$\approx 96.8\,\mathrm{mm}$$

The strip is cut to a length of 96.8 mm.

9.7 Precious metals

Precious metals are almost always worked as alloys. The fineness states what proportion of the mass is pure precious metal, in parts per thousand (‰) or, for gold, also in carats. The hallmark gives the fineness in parts per thousand: 750 means 750 ‰ fine gold. Whether a piece can match the stamped alloy is shown by its density; how to determine it by weighing in air and in water is described in Section 1.3, formula (1.29).

Fine weight

(9.32) JOURNEYMAN

$$m_\mathrm{f} = m \cdot w$$

REARRANGED $w = \dfrac{m_\mathrm{f}}{m}$

The fineness is the proportion of pure precious metal in the mass of the alloy. The material value is calculated from the fine weight and the day's price.

m_f	Fine weight (mass of the pure precious metal)	g
m	Mass (alloy)	g
w	Fineness (mass fraction of the precious metal)	‰

EXAMPLE A gold case stamped 750 weighs 42 g without movement and crystal.

Given: $m = 42\,\mathrm{g}$; $w = 750\,‰$

$$\begin{aligned} m_\mathrm{f} &= m \cdot w \\ &= 42\,\mathrm{g} \cdot 750\,‰ \\ &= 31.5\,\mathrm{g} \end{aligned}$$

The case contains 31.5 g of fine gold.

Fineness in carats

(9.33) JOURNEYMAN

$$K = 24 \cdot w$$

The carat divides the alloy into 24 parts: 18 carat means 18 parts gold and 6 parts alloying metals.

Valid: Gold alloys. Not to be confused with the carat of gemstones, a mass of 0.2 g.

K	Carat number (24 carat is pure gold)	1
w	Fineness (mass fraction of the precious metal)	‰

EXAMPLE Which carat number corresponds to gold stamped 750?

Given: $w = 750\,‰$

$$\begin{aligned} K &= 24 \cdot w \\ &= 24 \cdot 750\,‰ \\ &= 18 \end{aligned}$$

750 gold is 18 carat. 585 gold works out at 14.04 carat and is called 14 carat.

Common finenesses

Metal	Fineness in ‰	Designation
Gold	333	8 carat
Gold	375	9 carat
Gold	585	14 carat
Gold	750	18 carat
Gold	916	22 carat
Gold	999	24 carat, fine gold
Silver	800	
Silver	925	Sterling silver
Silver	999	Fine silver
Platinum	950	

Precious metal prices are quoted per troy ounce: 1 troy ounce = 31.1035 g. The troy ounce is heavier than the avoirdupois ounce of 28.35 g.

Fineness of a mixture

(9.34) JOURNEYMAN

$$w = \frac{m_1 \cdot w_1 + m_2 \cdot w_2}{m_1 + m_2}$$

When melted together, the fine weight is preserved: $m_1 \cdot w_1 + m_2 \cdot w_2 = (m_1 + m_2) \cdot w$.

Valid: Without melting loss. For a pure alloying metal such as copper the fineness is zero, for fine metal 1000 ‰.

w	Fineness (mixture)	‰
m_1	Mass (alloy 1)	g
w_1	Fineness (alloy 1)	‰
m_2	Mass (alloy 2)	g
w_2	Fineness (alloy 2)	‰

EXAMPLE 20 g of 750 gold and 10 g of 333 gold are melted together.

Given: $m_1 = 20\,\text{g}$; $w_1 = 750\,‰$; $m_2 = 10\,\text{g}$; $w_2 = 333\,‰$

$$\begin{aligned} w &= \frac{m_1 \cdot w_1 + m_2 \cdot w_2}{m_1 + m_2} \\ &= \frac{20\,\text{g} \cdot 750\,‰ + 10\,\text{g} \cdot 333\,‰}{20\,\text{g} + 10\,\text{g}} \\ &\approx 611\,‰ \end{aligned}$$

The mixture has a fineness of 611 ‰. It may only be stamped 585.

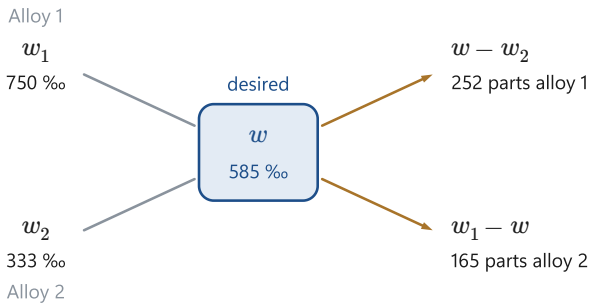


Fig. 9.12 Alligation cross with the example for formula (9.35)

Alligation cross

(9.35) MASTER

$$\frac{m_1}{m_2} = \frac{w - w_2}{w_1 - w}$$

REARRANGED $$m_1 = \frac{m_2 \cdot (w - w_2)}{w_1 - w}$$

m_1	Mass (alloy 1)	g
m_2	Mass (alloy 2)	g
w	Fineness (desired mixture)	‰
w_1	Fineness (alloy 1)	‰
w_2	Fineness (alloy 2)	‰

The alligation cross gives the mass ratio of two alloys for a desired fineness: the differences from the target value are read off crosswise.

Valid: The desired fineness must lie between the two starting values.

EXAMPLE 585 gold is to be made from 750 gold and 333 gold. 10 g of 333 gold are available. How much 750 gold is needed?

Given: $m_2 = 10\,\mathrm{g}$; $w = 585\,‰$; $w_2 = 333\,‰$; $w_1 = 750\,‰$

$$\begin{aligned} m_1 &= \frac{m_2 \cdot (w - w_2)}{w_1 - w} \\ &= \frac{10\,\mathrm{g} \cdot (585\,‰ - 333\,‰)}{750\,‰ - 585\,‰} \\ &= 10\,\mathrm{g} \cdot \frac{252}{165} \\ &\approx 15.3\,\mathrm{g} \end{aligned}$$

10 g of 333 gold require 15.3 g of 750 gold. The cross shows the same ratio: 252 parts to 165 parts.

Raising the fineness with fine metal

(9.36) MASTER

$$m_{\mathrm{zu}} = \frac{m_1 \cdot (w_2 - w_1)}{1000\,‰ - w_2}$$

m_{zu}	Mass (fine metal added)	g
m_1	Mass (existing alloy)	g
w_1	Fineness (before)	‰
w_2	Fineness (after)	‰

Special case of the mixture calculation: the second alloy is pure precious metal.

Valid: Without melting loss; fine metal taken as 1000 ‰.

EXAMPLE 20 g of 585 gold are to be brought up to 750.

Given: $m_1 = 20\,\mathrm{g}$; $w_1 = 585\,‰$; $w_2 = 750\,‰$

$$\begin{aligned} m_{\mathrm{zu}} &= \frac{m_1 \cdot (w_2 - w_1)}{1000\,‰ - w_2} \\ &= \frac{20\,\mathrm{g} \cdot (750\,‰ - 585\,‰)}{1000\,‰ - 750\,‰} \\ &= 13.2\,\mathrm{g} \end{aligned}$$

13.2 g of fine gold must be added; the result is 33.2 g of 750 gold.

Lowering the fineness with alloying metal

(9.37) MASTER

$$m_{\mathrm{zu}} = \frac{m_1 \cdot (w_1 - w_2)}{w_2}$$

Special case of the mixture calculation: the added metal contains no precious metal.

Valid: Without melting loss.

m_{zu}	Mass (added metal without precious metal content)	g
m_1	Mass (existing alloy)	g
w_1	Fineness (before)	‰
w_2	Fineness (after)	‰

EXAMPLE 20 g of 750 gold are to be brought down to 585.

Given: $m_1 = 20\,\mathrm{g}$; $w_1 = 750\,‰$; $w_2 = 585\,‰$

$$\begin{aligned} m_{\mathrm{zu}} &= \frac{m_1 \cdot (w_1 - w_2)}{w_2} \\ &= \frac{20\,\mathrm{g} \cdot (750\,‰ - 585\,‰)}{585\,‰} \\ &\approx 5.64\,\mathrm{g} \end{aligned}$$

5.64 g of alloying metal are added; for yellow gold, silver and copper in the ratio for the desired colour.

9.8 Hardness and heat treatment

Hardened steel is glass-hard and brittle. In tempering, i.e. reheating to 200 °C to 350 °C, it gives up some hardness and becomes tougher. On brightly polished steel a thin oxide layer indicates the temperature: the temper colour.

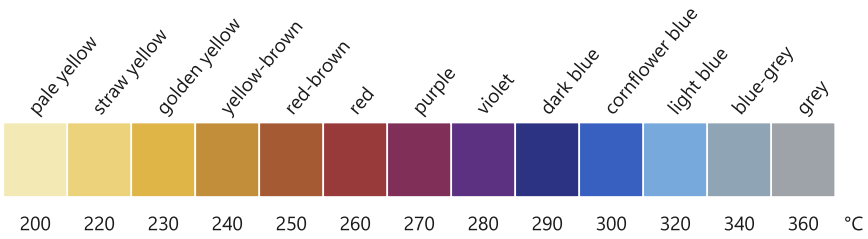


Fig. 9.13 Temper colours of bright unalloyed steel (colour reproduction in print only approximate)

Temper colours of unalloyed steel (guide values)

Temper colour	Temperature in °C	Examples
Pale yellow	200	Gravers, scrapers
Straw yellow	220	Turning tools, milling cutters, drills
Golden yellow	230	Taps, dies
Yellowish brown	240	Stamps, punches
Brownish red	250	Centre punches, drifts
Red	260	Chisels
Purple	270	Screwdriver blades
Violet	280	Screwdriver blades, arbors and pinions
Dark blue	290	Arbors, pinions, springs
Cornflower blue	300	Springs; blued screws and hands
Light blue	320	Soft springs
Bluish grey	340	
Grey	360	Hardly any hardness left

The colours apply to bright, unalloyed steel heated briskly. If the temperature is held longer, the colour continues to run although the steel gets no hotter; alloyed steels colour only at higher temperatures. The examples are workshop experience values.

Hardening temperatures of tool steels (guide values)

Steel	Hardening temperature in °C	Quench in
Unalloyed tool steel with about 1 % carbon (C105U)	770 … 800	Water
Silver steel 115CrV3 (1.2210)	780 … 810	Water
Silver steel 115CrV3 (1.2210), thin parts	810 … 840	Oil
Oil-hardening steel 100MnCrW4 (1.2510)	780 … 820	Oil
Oil-hardening steel 90MnCrV8 (1.2842)	790 … 820	Oil

The manufacturer's data sheet is what counts. At about 800 °C the steel glows cherry red and is no longer attracted by a magnet. Temper immediately after quenching.

Vickers hardness and Rockwell hardness of steel (rough conversion)

HV	HRC
240	20
300	30
350	35 … 36
400	41
450	45
500	49
550	52
600	55
650	58
700	60
800	64
900	67

Values from the standardised conversion table for unalloyed and low-alloy steels, rounded to whole HRC. A conversion is only ever a guide; it does not apply to other materials. Hardened tool steel reaches 62 to 66 HRC, tempered blue still about 55 HRC.

9.9 Electroplating

In electroplating, direct current deposits metal from an electrolyte onto the workpiece. The deposited mass follows from current and time, the coating thickness from mass, area and density.

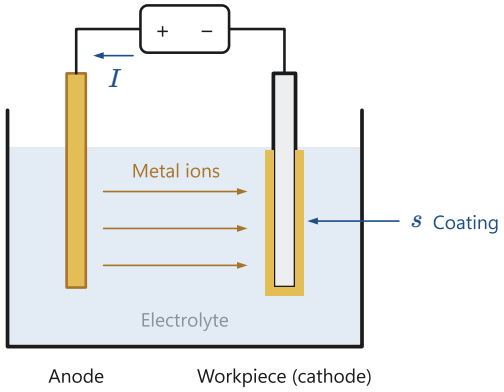


Fig. 9.14 Electroplating bath: the workpiece is the cathode; the coating of thickness s grows on it

Deposited mass (Faraday's law)

(9.38) ADVANCED

$$m = \frac{\eta \cdot M \cdot I \cdot t}{z \cdot F}$$

The deposited mass grows with the charge that has flowed, i.e. with current times time. The higher the valency of the ions, the more charge is needed per atom.

Valid: Direct current at constant current. The current efficiency depends on the bath: close to 1 for silver and copper baths, considerably lower for acid gold baths and for rhodium.

m	Mass (deposited metal)	mg
η	Current efficiency (fraction of the current that deposits metal)	1
M	Molar mass of the deposited metal	$\frac{\mathrm{g}}{\mathrm{mol}}$
I	Current	A
t	Time (duration of deposition)	s
z	Valency (charge number of the metal ions in the bath)	1
F	Faraday constant ($96\,485\ \frac{\mathrm{C}}{\mathrm{mol}}$)	$\frac{\mathrm{C}}{\mathrm{mol}}$

EXAMPLE A case is gold-plated for 10 minutes at 0.05 A. In the bath gold is monovalent; the current efficiency is assumed to be 90 %.

Given: $\eta = 0.9$; $M = 196.97\ \frac{\mathrm{g}}{\mathrm{mol}}$; $I = 0.05\ \mathrm{A}$; $t = 600\ \mathrm{s}$; $z = 1$; $F = 96\,485\ \frac{\mathrm{C}}{\mathrm{mol}}$

$$\begin{aligned} m &= \frac{\eta \cdot M \cdot I \cdot t}{z \cdot F} \\ &= \frac{0.9 \cdot 196.97\ \frac{\mathrm{g}}{\mathrm{mol}} \cdot 0.05\ \mathrm{A} \cdot 600\ \mathrm{s}}{1 \cdot 96\,485\ \frac{\mathrm{C}}{\mathrm{mol}}} \\ &\approx 55.1\ \mathrm{mg} \end{aligned}$$

About 55 mg of gold is deposited on the case.

Coating thickness from the mass

(9.39) ADVANCED

$$s = \frac{m}{\varrho \cdot A}$$

The mass spreads over the surface as a coating. At edges and points the coating becomes thicker, in recesses thinner; the formula gives the mean value.

Valid: Uniform, dense coating.

s	Thickness (coating thickness)	$\mu\mathrm{m}$
m	Mass (deposited metal)	mg
ϱ	Density (of the coating metal)	$\frac{\mathrm{g}}{\mathrm{cm}^3}$
A	Area (coated surface)	cm^2

EXAMPLE The 55.1 mg of gold from the previous example is spread over 15 cm² of surface. Gold has a density of $19.3\ \frac{\mathrm{g}}{\mathrm{cm}^3}$.

Given: $m = 55.1\,\mathrm{mg}$; $\varrho = 19.3\ \frac{\mathrm{g}}{\mathrm{cm}^3}$; $A = 15\,\mathrm{cm}^2$

$$\begin{aligned} s &= \frac{m}{\varrho \cdot A} \\ &= \frac{55.1\,\mathrm{mg}}{19.3\ \frac{\mathrm{g}}{\mathrm{cm}^3} \cdot 15\,\mathrm{cm}^2} \\ &\approx 1{,}90 \cdot 10^{-4}\,\mathrm{cm} \\ &= 1.90\,\mu\mathrm{m} \end{aligned}$$

The gold coating is 1.9 µm thick on average.

Coating thickness from current density and time

(9.40) ADVANCED

$$s = \frac{\eta \cdot M \cdot j \cdot t}{z \cdot F \cdot \varrho}$$

Baths are operated at a specified current density. The coating thickness follows directly from it and the time, without knowing the mass.

Valid: As for formulas (9.38) and (9.39).

s	Thickness (coating thickness)	$\mu\mathrm{m}$
η	Current efficiency (fraction of the current that deposits metal)	1
M	Molar mass of the deposited metal	$\frac{\mathrm{g}}{\mathrm{mol}}$
j	Current density (current per unit area, $j = \frac{I}{A}$)	$\frac{\mathrm{A}}{\mathrm{dm}^2}$
t	Time (duration of deposition)	s
z	Valency (charge number of the metal ions in the bath)	1
F	Faraday constant ($96\,485\ \frac{\mathrm{C}}{\mathrm{mol}}$)	$\frac{\mathrm{C}}{\mathrm{mol}}$
ϱ	Density (of the coating metal)	$\frac{\mathrm{g}}{\mathrm{cm}^3}$

EXAMPLE The same gold bath: 0.05 A on 15 cm² is $0.333\ \frac{\mathrm{A}}{\mathrm{dm}^2}$. How thick is the coating after 10 minutes?

Given: $\eta = 0.9$; $M = 196.97\ \frac{\mathrm{g}}{\mathrm{mol}}$; $j = 0.333\ \frac{\mathrm{A}}{\mathrm{dm}^2}$; $t = 600\,\mathrm{s}$; $z = 1$; $F = 96\,485\ \frac{\mathrm{C}}{\mathrm{mol}}$; $\varrho = 19.3\ \frac{\mathrm{g}}{\mathrm{cm}^3}$

$$\begin{aligned} s &= \frac{\eta \cdot M \cdot j \cdot t}{z \cdot F \cdot \varrho} \\ &= \frac{0.9 \cdot 196.97\ \frac{\mathrm{g}}{\mathrm{mol}} \cdot 0.333\ \frac{\mathrm{A}}{\mathrm{dm}^2} \cdot 600\,\mathrm{s}}{1 \cdot 96\,485\ \frac{\mathrm{C}}{\mathrm{mol}} \cdot 19.3\ \frac{\mathrm{g}}{\mathrm{cm}^3}} \\ &\approx 1.90\,\mu\mathrm{m} \end{aligned}$$

Again 1.9 µm. For twice the coating thickness, plating takes twice as long, not twice the current: the current density is set by the bath.

Values for electroplating

Metal	Valency z in the bath	Molar mass M in $\frac{\mathrm{g}}{\mathrm{mol}}$	Density ϱ in $\frac{\mathrm{g}}{\mathrm{cm}^3}$
Gold (cyanide baths)	1	196.97	19.3
Silver	1	107.87	10.5
Copper (acid baths)	2	63.55	8.96
Nickel	2	58.69	8.90
Rhodium	3	102.91	12.4

The density applies to the pure metal; alloy coatings such as hard gold deviate slightly.

10 Electrical Engineering, Quartz and Battery

The quartz watch replaces mainspring, escapement and balance with a battery, a circuit and a quartz crystal. Testing and repairing it requires only a little electrical engineering: Ohm's law, power and charge, plus the behaviour of the battery, the capacitor and the coil.

The formulas of the watch itself build on this: battery life, frequency division, rate and temperature behaviour of the quartz crystal, current consumption of the stepper motor. In this chapter, U stands for voltage, A for the conductor cross-section, P for power and C for capacitance.

10.1 Basic quantities of the circuit

Voltage drives the current through the circuit; the resistance opposes it. With these three quantities and time, power, energy and charge can be calculated. In the quartz watch the voltages are small and the currents tiny: calculations are made in volts, microamperes and kiloohms.

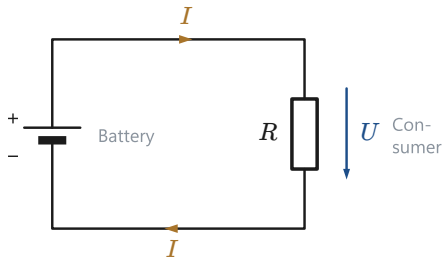


Fig. 10.1 Simple circuit: the voltage U drives the current I through the resistance R

Ohm's law

(10.1) JOURNEYMAN

$$U = R \cdot I$$

REARRANGED $\quad I = \dfrac{U}{R} \qquad R = \dfrac{U}{I}$

U	Voltage (across the resistor)	V
R	Resistance	Ω
I	Current	A

Across a resistor, voltage and current are proportional. Twice the voltage drives twice the current; twice the resistance lets only half the current flow.

Valid: Direct current and resistors whose value does not depend on voltage and current (wires, coils, film resistors).

EXAMPLE A motor coil of 1.8 kΩ is connected to 1.55 V. What current flows once the current has fully risen?

Given: $U = 1.55\,\mathrm{V}; \quad R = 1.8\,\mathrm{k\Omega}$

$$\begin{aligned} I &= \frac{U}{R} \\ &= \frac{1.55\,\mathrm{V}}{1.8\,\mathrm{k\Omega}} \\ &\approx 0.86\,\mathrm{mA} \end{aligned}$$

About 0.86 mA. The current in the motor pulse cannot rise any higher. On average the watch needs only about a thousandth of this, because the pulse lasts only a few milliseconds.

Electrical power

(10.2) JOURNEYMAN

$$P = U \cdot I$$

P	Power	W
U	Voltage	V
I	Current	A

Power states how much energy is converted per second. One watt is one volt times one ampere.

Valid: Direct current; with pulsed operation at constant voltage, use the mean value of the current.

EXAMPLE A quartz movement draws an average of 1.2 µA at 1.55 V. What is the power?

Given: $U = 1.55\,\mathrm{V}$; $I = 1.2\,\mu\mathrm{A}$

$$\begin{aligned} P &= U \cdot I \\ &= 1.55\,\mathrm{V} \cdot 1.2\,\mu\mathrm{A} \\ &= 1.86\,\mu\mathrm{W} \end{aligned}$$

Just under 2 µW. A mechanical wristwatch movement likewise converts only one to a few microwatts from the mainspring (Chapter 3).

Power in a resistor from the voltage

(10.3) JOURNEYMAN

$$P = \frac{U^2}{R}$$

P	Power	W
U	Voltage (across the resistor)	V
R	Resistance	Ω

Follows from power and Ohm's law. Power increases with the square of the voltage: twice the voltage, four times the power.

Valid: as for Ohm's law.

EXAMPLE A battery tester loads a cell with 2 kΩ. The voltage under load is 1.55 V. What power does the cell deliver?

Given: $U = 1.55\,\mathrm{V}$; $R = 2\,\mathrm{k\Omega}$

$$\begin{aligned} P &= \frac{U^2}{R} \\ &= \frac{(1.55\,\mathrm{V})^2}{2\,\mathrm{k\Omega}} \\ &\approx 1.20\,\mathrm{mW} \end{aligned}$$

About 1.2 mW, roughly 600 times the power the movement needs in operation. This is why the cell is loaded only briefly.

Power in a resistor from the current

(10.4) JOURNEYMAN

$$P = I^2 \cdot R$$

P	Power	W
I	Current	A
R	Resistance	Ω

This power is turned into heat in the resistor. In the motor coil it is a loss: only the rest of the pulse moves the rotor.

Valid: as for Ohm's law.

EXAMPLE A current of 0.5 mA flows through a coil of 1.8 kΩ during the pulse. How much power is turned into heat in the wire?

Given: $I = 0.5\,\mathrm{mA}$; $R = 1.8\,\mathrm{k\Omega}$

$$\begin{aligned} P &= I^2 \cdot R \\ &= (0.5\,\mathrm{mA})^2 \cdot 1.8\,\mathrm{k\Omega} \\ &= 0.45\,\mathrm{mW} \end{aligned}$$

0.45 mW, but only during the few milliseconds of the pulse.

Electrical energy

(10.5) JOURNEYMAN

$$W = U \cdot I \cdot t$$

Energy is power times time. One joule is one watt-second; one watt-hour is 3600 J.

Valid: Constant voltage; if the current fluctuates, its mean value is used.

W	Energy (electrical work, energy converted)	J
U	Voltage	V
I	Current	A
t	Time (duration of current flow)	s

EXAMPLE How much energy does a quartz movement convert in one year if it draws an average of 1.2 µA at 1.55 V?

Given: $U = 1.55\,\mathrm{V}$; $I = 1.2\,\mu\mathrm{A}$; $t = 365\,\mathrm{d}$

$$\begin{aligned} W &= U \cdot I \cdot t \\ &= 1.55\,\mathrm{V} \cdot 1.2\,\mu\mathrm{A} \cdot 365\,\mathrm{d} \\ &\approx 58.7\,\mathrm{J} \end{aligned}$$

Just under 59 J in a whole year. A 60 W light bulb converts the same energy in one second.

Charge

(10.6) JOURNEYMAN

$$Q = I \cdot t$$

A current of one ampere transports a charge of one coulomb in one second. For batteries, charge is given in milliampere-hours: 1 mAh is 3.6 C.

Valid: constant current or mean value of the current.

Q	Charge (quantity of electricity)	C
I	Current	A
t	Time (duration of current flow)	s

EXAMPLE What charge does a movement with a current consumption of 1.2 µA draw from the battery in one year (8760 h)?

Given: $I = 1.2\,\mu\mathrm{A}$; $t = 8760\,\mathrm{h}$

$$\begin{aligned} Q &= I \cdot t \\ &= 1.2\,\mu\mathrm{A} \cdot 8760\,\mathrm{h} \\ &\approx 10.5\,\mathrm{mAh} \end{aligned}$$

About 10.5 mAh per year. A cell of 28 mAh therefore lasts, by calculation, a good two and a half years.

Resistance of a wire

(10.7) JOURNEYMAN

$$R = \frac{\rho \cdot l}{A}$$

REARRANGED $l = \frac{R \cdot A}{\rho}$

The longer and thinner the wire, the greater its resistance. Motor coils are wound from enamelled copper wire only 0.015 mm to 0.02 mm in diameter, thinner than a hair.

Valid: uniform cross-section; ρ applies at 20 °C. Circular cross-section: $A = \frac{\pi}{4} d^2$ (Chapter 1).

R	Resistance	Ω
ρ	Resistivity of the material	$\frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}}$
l	Length (of the wire)	m
A	Area (cross-section of the wire)	mm^2

EXAMPLE A coil is wound from 26 m of copper wire 0.018 mm in diameter; the cross-section is 0.000254 mm². What is its resistance?

Given: $\rho = 0.0178\ \frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}}$; $l = 26\,\mathrm{m}$; $A = 0.000\,254\,\mathrm{mm}^2$

$$R = \frac{\rho \cdot l}{A} = \frac{0.0178\ \frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}} \cdot 26\,\mathrm{m}}{0.000\,254\,\mathrm{mm}^2} \approx 1.82\,\mathrm{k\Omega}$$

About 1.8 kΩ, a typical value for the coil of a three-hand movement.

Resistance and temperature

(10.8) APPROXIMATION ADVANCED

$$R_\vartheta \approx R_{20} \cdot (1 + \alpha_{20} \cdot \Delta\vartheta)$$

Metals conduct less well when warm. For copper the resistance rises by about 0.4 % per kelvin, i.e. 4 % per 10 K.

Valid: Metals in the range from about −50 °C to 150 °C; $\Delta\vartheta = \vartheta - 20\,^\circ\mathrm{C}$.

R_ϑ	Resistance at temperature ϑ	Ω
R_{20}	Resistance at 20 °C	Ω
α_{20}	Temperature coefficient of resistance (referred to 20 °C)	$\frac{1}{\mathrm{K}}$
$\Delta\vartheta$	Temperature change (relative to 20 °C; + means warmer)	K

EXAMPLE A coil has a resistance of 1.80 kΩ at 20 °C. What is it at 35 °C, as in a watch just taken off the wrist?

Given: $R_{20} = 1.80\,\mathrm{k\Omega}$; $\alpha_{20} = 0.0039\ \frac{1}{\mathrm{K}}$; $\Delta\vartheta = 15\,\mathrm{K}$

$$R_\vartheta \approx R_{20} \cdot (1 + \alpha_{20} \cdot \Delta\vartheta) = 1.80\,\mathrm{k\Omega} \cdot \left(1 + 0.0039\ \tfrac{1}{\mathrm{K}} \cdot 15\,\mathrm{K}\right) \approx 1.91\,\mathrm{k\Omega}$$

About 1.91 kΩ, i.e. 6 % more. Coil resistances are therefore compared with the manufacturer's specification at room temperature.

Conductor materials at 20 °C (guide values)

Material	Resistivity ρ in $\frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}}$	Temperature coefficient α_{20} in $\frac{1}{\mathrm{K}}$	Use
Silver	0.016	0.0038	Contacts
Copper	0.0178	0.0039	Coil wire, conductor tracks
Aluminium	0.028	0.0040	Cables
Brass (CuZn37)	0.07	0.0015	Contact springs, battery clamps
Constantan	0.49	almost 0	Precision (measuring) resistors

Values vary with the purity and condition of the material.

10.2 Resistor circuits, battery under load

Resistors can be connected one after another (in series) or side by side (in parallel). The battery itself also contains a resistance, the internal resistance. It decides whether a cell can still deliver the motor pulse.

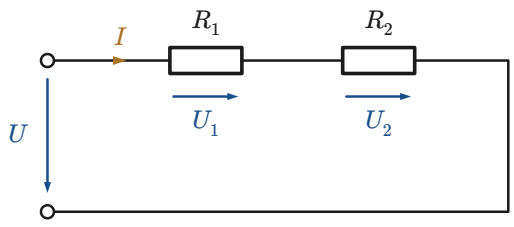


Fig. 10.2 Series connection: the same current I through both resistors; the partial voltages add up

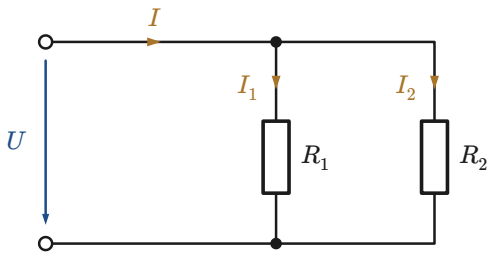


Fig. 10.3 Parallel connection: the same voltage U across both resistors; the partial currents add up

Series connection of resistors

(10.9) JOURNEYMAN

$$R = R_1 + R_2 + R_3 + \dots$$

The same current flows through all resistors. The partial voltages add up to the total voltage and are in the same ratio as the resistances. The formula applies to any number of resistors.

R	Resistance (total resistance of the circuit)	Ω
R_1	First partial resistance	Ω
R_2	Second partial resistance	Ω
R_3	Third partial resistance (further ones accordingly)	Ω

EXAMPLE The coil of a stepper motor has 1.8 kΩ. For measurement, a series resistor of 3.3 kΩ and a measuring resistor of 1.0 kΩ are connected in series with it. What is the total resistance?

Given: $R_1 = 1.8\,\mathrm{k\Omega}$; $R_2 = 3.3\,\mathrm{k\Omega}$; $R_3 = 1.0\,\mathrm{k\Omega}$

$$\begin{aligned} R &= R_1 + R_2 + R_3 \\ &= 1.8\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega} + 1.0\,\mathrm{k\Omega} \\ &= 6.1\,\mathrm{k\Omega} \end{aligned}$$

6.1 kΩ. The total resistance is always greater than the largest partial resistance.

Parallel connection of resistors

(10.10) JOURNEYMAN

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

REARRANGED $$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

R	Resistance (total resistance of the circuit)	Ω
R_1	First partial resistance	Ω
R_2	Second partial resistance	Ω
R_3	Third partial resistance (further ones accordingly)	Ω

The same voltage is applied across all resistors. The partial currents add up to the total current; the larger current flows through the smaller resistance. The formula applies to any number of resistors. For two resistors the shorter form is $R = \frac{R_1 \cdot R_2}{R_1 + R_2}$.

Caution: Moisture or dirt between the coil terminals acts like a resistor connected in parallel. The coil then measures too low, and the current consumption of the movement rises.

EXAMPLE Three resistors of 2 kΩ, 3 kΩ and 6 kΩ are connected in parallel. What is the total resistance?

Given: $R_1 = 2\,\mathrm{k\Omega}$; $R_2 = 3\,\mathrm{k\Omega}$; $R_3 = 6\,\mathrm{k\Omega}$

$$\begin{aligned} R &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \\ &= \frac{1}{\frac{1}{2\,\mathrm{k\Omega}} + \frac{1}{3\,\mathrm{k\Omega}} + \frac{1}{6\,\mathrm{k\Omega}}} \\ &= 1\,\mathrm{k\Omega} \end{aligned}$$

1 kΩ. The total resistance is always smaller than the smallest partial resistance.

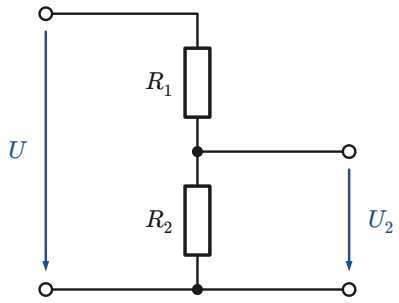


Fig. 10.4 Unloaded voltage divider: across resistor R_2 lies the partial voltage U_2

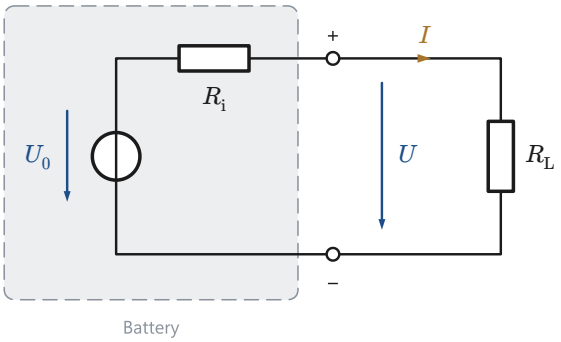


Fig. 10.5 Equivalent circuit of the battery: source voltage U_0 and internal resistance R_i; across the load R_L lies the terminal voltage U

Voltage divider (unloaded)

(10.11) JOURNEYMAN

$$U_2 = \frac{U \cdot R_2}{R_1 + R_2}$$

Two resistors in series divide the voltage in the ratio of their values. In this way a smaller voltage is obtained from a larger one.

Valid: unloaded: the current drawn at the tap is much smaller than the current through the divider. A load in parallel with R_2 lowers the partial voltage.

U_2	Partial voltage across resistor R_2	V
U	Voltage (total voltage)	V
R_1	First partial resistance	Ω
R_2	Second partial resistance (the voltage is tapped across it)	Ω

EXAMPLE A partial voltage is to be obtained from a 3.0 V lithium cell. The divider consists of 120 kΩ and 100 kΩ. What voltage is present across the 100 kΩ resistor?

Given: $U = 3.0\,\mathrm{V}$; $R_1 = 120\,\mathrm{k\Omega}$; $R_2 = 100\,\mathrm{k\Omega}$

$$\begin{aligned} U_2 &= \frac{U \cdot R_2}{R_1 + R_2} \\ &= \frac{3.0\,\mathrm{V} \cdot 100\,\mathrm{k\Omega}}{120\,\mathrm{k\Omega} + 100\,\mathrm{k\Omega}} \\ &\approx 1.36\,\mathrm{V} \end{aligned}$$

About 1.36 V, as long as no significant current is drawn at the tap.

Terminal voltage of the loaded battery

(10.12) JOURNEYMAN

$$U = U_0 - I \cdot R_\mathrm{i}$$

REARRANGED $R_\mathrm{i} = \dfrac{U_0 - U}{I}$

As soon as current flows, part of the voltage drops across the internal resistance of the battery. Less remains at the terminals, and the less, the larger the current and the older the cell.

Valid: Equivalent circuit of source voltage and a fixed internal resistance. In real cells R_i increases with discharge, with age and in the cold.

U	Voltage (terminal voltage under load)	V
U_0	Source voltage (open-circuit voltage, measured without load)	V
I	Current (load current)	A
R_i	Internal resistance of the battery	Ω

Caution: The voltage without load says little about the condition of a cell. Even an almost exhausted silver oxide cell still shows over 1.5 V on a high-impedance meter. Always test batteries under load.

EXAMPLE A silver oxide cell has 1.57 V without load and an internal resistance of 50 Ω. What voltage is present while the motor pulse draws 0.8 mA?

Given: $U_0 = 1.57\,\mathrm{V}$; $I = 0.8\,\mathrm{mA}$; $R_\mathrm{i} = 50\,\Omega$

$$\begin{aligned} U &= U_0 - I \cdot R_\mathrm{i} \\ &= 1.57\,\mathrm{V} - 0.8\,\mathrm{mA} \cdot 50\,\Omega \\ &\approx 1.53\,\mathrm{V} \end{aligned}$$

1.53 V. A fresh cell drops by only a few hundredths of a volt under the pulse.

Internal resistance from open-circuit and load measurement

(10.13) MASTER

$$R_\mathrm{i} = \frac{R_\mathrm{L} \cdot (U_0 - U)}{U}$$

Two measurements are sufficient: one without load and one with a known load resistance. Internal resistance and load form a voltage divider; the internal resistance follows from the voltage drop.

Valid: Meter impedance much higher than the load resistance; apply the load only briefly so that the cell is not discharged.

R_i	Internal resistance of the battery	Ω
R_L	Load resistance (of the tester)	Ω
U_0	Source voltage (open-circuit voltage, measured without load)	V
U	Voltage (terminal voltage under load)	V

EXAMPLE A used cell shows 1.57 V without load and only 1.49 V across a load resistance of 2 kΩ. What is its internal resistance?

Given: $R_\mathrm{L} = 2\,\mathrm{k\Omega}$; $U_0 = 1.57\,\mathrm{V}$; $U = 1.49\,\mathrm{V}$

$$\begin{aligned} R_\mathrm{i} &= \frac{R_\mathrm{L} \cdot (U_0 - U)}{U} \\ &= \frac{2\,\mathrm{k\Omega} \cdot (1.57\,\mathrm{V} - 1.49\,\mathrm{V})}{1.49\,\mathrm{V}} \\ &\approx 107\,\Omega \end{aligned}$$

About 107 Ω. The cell has aged considerably and should be replaced, even if it still looks good without load.

Key point: Batteries are tested under load. The voltage without load only shows which cell chemistry is involved.

10.3 Capacitor and coil

The capacitor stores charge; the coil opposes any change in current. In the watch we meet capacitors at the quartz oscillator (a few picofarads) and as the energy store of older solar and automatic quartz movements (a few tenths of a farad). The coil is the heart of the stepper motor.

Charge of the capacitor

(10.14) MASTER

$$Q = C \cdot U$$

Capacitance states how much charge the capacitor takes up per volt. One farad is one coulomb per volt.

Q	Charge (stored in the capacitor)	C
C	Capacitance	F
U	Voltage (across the capacitor)	V

EXAMPLE A storage capacitor of 0.33 F is charged to 1.8 V. What charge does it contain?

Given: $C = 0.33\,\mathrm{F}$; $U = 1.8\,\mathrm{V}$

$$\begin{aligned} Q &= C \cdot U \\ &= 0.33\,\mathrm{F} \cdot 1.8\,\mathrm{V} \\ &= 0.594\,\mathrm{C} \end{aligned}$$

0.594 C, which is only 0.165 mAh. A small silver oxide cell stores more than a hundred times as much.

Capacitance of the parallel-plate capacitor

(10.15) ADVANCED

$$C = \frac{\varepsilon_0 \cdot \varepsilon_\mathrm{r} \cdot A}{d}$$

Large plates, a small gap and a good insulator give a large capacitance. In a rotary trimmer the area over which the plates overlap is changed.

Valid: Plate spacing small compared with the dimensions of the plates.

C	Capacitance	F
ε_0	Electric constant, $8.854 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}$	$\frac{\mathrm{F}}{\mathrm{m}}$
ε_r	Relative permittivity of the insulator (air 1, plastic films 2 to 4)	1
A	Area (of one plate, where the plates face each other)	m^2
d	Distance (between the plates)	m

EXAMPLE Two plates of 100 mm² each face each other at a distance of 0.01 mm, with a film of relative permittivity 4 between them. What is the capacitance?

Given: $\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}$; $\varepsilon_\mathrm{r} = 4$; $A = 100\,\mathrm{mm}^2$; $d = 0.01\,\mathrm{mm}$

$$\begin{aligned} C &= \frac{\varepsilon_0 \cdot \varepsilon_\mathrm{r} \cdot A}{d} \\ &= \frac{8.854 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}} \cdot 4 \cdot 100\,\mathrm{mm}^2}{0.01\,\mathrm{mm}} \\ &\approx 354\,\mathrm{pF} \end{aligned}$$

About 354 pF. Capacitors of microfarads or even farads need wound films or porous electrodes with an enormous surface area.

Parallel connection of capacitors

(10.16) MASTER

$$C = C_1 + C_2 + C_3 + \ldots$$

Capacitors connected in parallel act like one capacitor with a larger plate area: the capacitances add up. This is exactly the reverse of resistors. The formula applies to any number of capacitors.

C	Capacitance (total capacitance of the circuit)	F
C_1	First partial capacitance	F
C_2	Second partial capacitance	F
C_3	Third partial capacitance (further ones accordingly)	F

EXAMPLE A fixed capacitor of 10 pF is connected to the quartz crystal, with a trimmer set to 12 pF in parallel. The conductor tracks add another 3 pF. What is the total capacitance?

Given: $C_1 = 10\,\mathrm{pF}$; $C_2 = 12\,\mathrm{pF}$; $C_3 = 3\,\mathrm{pF}$

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 10\,\mathrm{pF} + 12\,\mathrm{pF} + 3\,\mathrm{pF} \\ &= 25\,\mathrm{pF} \end{aligned}$$

25 pF. The trimmer allows the total capacitance, and thus the frequency of the quartz crystal, to be finely adjusted.

Series connection of capacitors

(10.17) MASTER

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

REARRANGED $$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$$

C	Capacitance (total capacitance of the circuit)	F
C_1	First partial capacitance	F
C_2	Second partial capacitance	F
C_3	Third partial capacitance (further ones accordingly)	F

In series, the reciprocals add up. The total capacitance is smaller than the smallest partial capacitance. The formula applies to any number of capacitors; for two the shorter form is $C = \frac{C_1 \cdot C_2}{C_1 + C_2}$.

EXAMPLE Three capacitors of 30 pF, 20 pF and 12 pF are connected in series. What capacitance results?

Given: $C_1 = 30\,\mathrm{pF}$; $C_2 = 20\,\mathrm{pF}$; $C_3 = 12\,\mathrm{pF}$

$$\begin{aligned} C &= \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \\ &= \frac{1}{\frac{1}{30\,\mathrm{pF}} + \frac{1}{20\,\mathrm{pF}} + \frac{1}{12\,\mathrm{pF}}} \\ &= 6\,\mathrm{pF} \end{aligned}$$

6 pF. In the quartz oscillator the two load capacitors of the crystal are in series: 22 pF and 15 pF give 8.9 pF. A few picofarads from the conductor tracks are added; the sum is the load capacitance for which the crystal must be calibrated.

Time constant for charging and discharging

(10.18) MASTER

$$\tau = R \cdot C$$

τ	Time constant	s
R	Resistance (in the charging or discharging circuit)	Ω
C	Capacitance	F

A capacitor charged through a resistor does not charge instantly. After one time constant 63 % of the charging voltage is reached, after five time constants over 99 %. On discharging, the voltage falls at the same pace.

EXAMPLE A storage capacitor of 0.33 F is charged through 1 kΩ. What is the time constant?

Given: $R = 1\,\mathrm{k\Omega}$; $C = 0.33\,\mathrm{F}$

$$\begin{aligned} \tau &= R \cdot C \\ &= 1\,\mathrm{k\Omega} \cdot 0.33\,\mathrm{F} \\ &= 330\,\mathrm{s} \end{aligned}$$

330 s. The capacitor is practically full after five time constants, i.e. after a good 27 minutes.

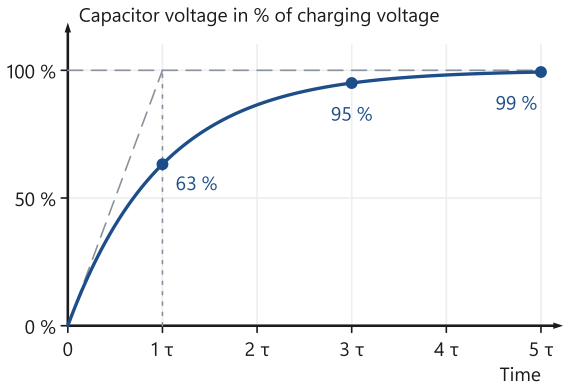


Fig. 10.6 Charging curve of the capacitor: 63 % of the charging voltage after one time constant τ, over 99 % after $5\,\tau$

Capacitor voltage during charging

(10.19) ADVANCED

$$u_\mathrm{C} = U \cdot \left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right)$$

The voltage rises quickly at first and then approaches the charging voltage ever more slowly. For discharging: $u_\mathrm{C} = U \cdot \mathrm{e}^{-\frac{t}{\tau}}$.

Valid: Capacitor initially empty; charging voltage and resistance constant.

u_C	Capacitor voltage at time t	V
U	Voltage (charging voltage of the source)	V
t	Time (since the start of charging)	s
τ	Time constant	s

EXAMPLE The capacitor from the previous example (time constant 330 s) is charged at 1.8 V. What is its voltage after 10 minutes?

Given: $U = 1.8\,\mathrm{V}$; $t = 600\,\mathrm{s}$; $\tau = 330\,\mathrm{s}$

$$\begin{aligned} u_\mathrm{C} &= U \cdot \left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right) \\ &= 1.8\,\mathrm{V} \cdot \left(1 - \mathrm{e}^{-\frac{600\,\mathrm{s}}{330\,\mathrm{s}}}\right) \\ &\approx 1.51\,\mathrm{V} \end{aligned}$$

About 1.51 V, which is 84 % of the charging voltage.

Energy in the capacitor

(10.20) MASTER

$$W = \frac{1}{2} \cdot C \cdot U^2$$

The stored energy increases with the square of the voltage. Only the part above the minimum operating voltage of the movement is usable.

W	Energy (stored in the capacitor)	J
C	Capacitance	F
U	Voltage (across the capacitor)	V

EXAMPLE How much energy does a 0.33 F capacitor store at 1.8 V?

Given: $C = 0.33\,\mathrm{F}$; $U = 1.8\,\mathrm{V}$

$$\begin{aligned} W &= \frac{1}{2} \cdot C \cdot U^2 \\ &= \frac{1}{2} \cdot 0.33\,\mathrm{F} \cdot (1.8\,\mathrm{V})^2 \\ &\approx 0.53\,\mathrm{J} \end{aligned}$$

About 0.53 J. At a power consumption of 2 µW this would be three days by calculation; less is usable because the voltage drops during discharge.

Coil. When the current in a coil changes, an induced voltage arises in it that opposes the change. This is why the motor current does not rise instantly, and why a voltage spike occurs on switch-off. The rotating rotor also induces a voltage in the coil; many circuits use this to detect whether the step has been executed.

In the workshop we test the coil by its resistance. Depending on the calibre it lies between about 0.8 kΩ and 2.7 kΩ. If the meter shows no continuity, the wire is broken; a value that is clearly too low indicates a short between turns.

10.4 Battery

The capacity of a battery is the charge that can be drawn from it, given in milliampere-hours (mAh). It has only the name in common with the capacitance of a capacitor. Together with the current consumption of the movement it gives the battery life.

Battery life from capacity and current consumption

(10.21) JOURNEYMAN

$$t = \frac{C_\mathrm{N}}{I}$$

REARRANGED $I = \dfrac{C_\mathrm{N}}{t}$

Capacity divided by current gives the battery life. Conversely, the formula shows the maximum current consumption a movement may have for the battery to last the promised time.

Valid: constant mean current consumption; self-discharge not taken into account. Use consistent units: mAh and mA give hours.

t	Time (battery life)	h
C_N	Nominal capacity of the battery (available charge)	mAh
I	Current (mean current consumption of the movement)	mA

EXAMPLE An SR626SW cell has 28 mAh. The movement draws an average of 1.2 µA. How long does the watch run?

Given: $C_\mathrm{N} = 28\,\mathrm{mAh}$; $I = 1.2\,\mu\mathrm{A}$

$$\begin{aligned} t &= \frac{C_\mathrm{N}}{I} \\ &= \frac{28\,\mathrm{mAh}}{1.2\,\mu\mathrm{A}} \\ &= 23\,333\,\mathrm{h} \\ &\approx 2.66\,\mathrm{a} \end{aligned}$$

About 23 300 hours, a good two and a half years. If the movement draws 3 µA because of contamination, the cell is exhausted after only one year.

Energy content of the battery

(10.22) APPROXIMATION MASTER

$$W \approx U_\mathrm{N} \cdot C_\mathrm{N}$$

Nominal voltage times capacity gives the stored energy, usually in milliwatt-hours. It allows cells of different voltages to be compared.

Valid: Voltage approximately constant during discharge, as with silver oxide and lithium cells.

W	Energy (stored in the battery)	J
U_N	Nominal voltage of the battery	V
C_N	Nominal capacity of the battery (available charge)	mAh

EXAMPLE How much energy does a silver oxide cell of 1.55 V and 28 mAh contain?

Given: $U_\mathrm{N} = 1.55\,\mathrm{V}$; $C_\mathrm{N} = 28\,\mathrm{mAh}$

$$\begin{aligned} W &\approx U_\mathrm{N} \cdot C_\mathrm{N} \\ &= 1.55\,\mathrm{V} \cdot 28\,\mathrm{mAh} \\ &= 43.4\,\mathrm{mWh} \end{aligned}$$

43.4 mWh, which is about 156 J. A fully wound wristwatch mainspring stores less than 1 J (Chapter 3).

Battery life with self-discharge

(10.23) APPROXIMATION ADVANCED

$$t \approx \frac{C_\mathrm{N}}{I + s_\mathrm{E} \cdot C_\mathrm{N}}$$

Every cell slowly loses charge even without a load. Self-discharge acts like a small additional current $s_\mathrm{E} \cdot C_\mathrm{N}$ alongside the current consumption of the movement.

Valid: Self-discharge calculated as a constant loss. Guide value for silver oxide cells at room temperature: a few per cent per year, considerably more in warm conditions.

t	Time (battery life)	h
C_N	Nominal capacity of the battery (available charge)	mAh
I	Current (mean current consumption of the movement)	mA
s_E	Self-discharge (fraction of nominal capacity lost per unit time)	$\frac{\%}{\mathrm{a}}$

EXAMPLE How long does the watch from the battery-life example (28 mAh, 1.2 µA) run if the cell also loses 4 % of its capacity per year through self-discharge?

Given: $C_\mathrm{N} = 28\,\mathrm{mAh}$; $I = 1.2\,\mu\mathrm{A}$; $s_\mathrm{E} = 4\,\frac{\%}{\mathrm{a}}$

$$t \approx \frac{C_\mathrm{N}}{I + s_\mathrm{E} \cdot C_\mathrm{N}} = \frac{28\,\mathrm{mAh}}{1.2\,\mu\mathrm{A} + 4\,\frac{\%}{\mathrm{a}} \cdot 28\,\mathrm{mAh}} \approx 2.41\,\mathrm{a}$$

About 2.4 years instead of 2.7 years. The lower the current consumption of the movement, the more self-discharge matters.

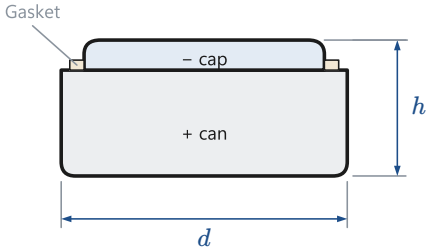


Fig. 10.7 Button cell: the can is the positive terminal, the cap the negative terminal; diameter d and height h are contained in the designation

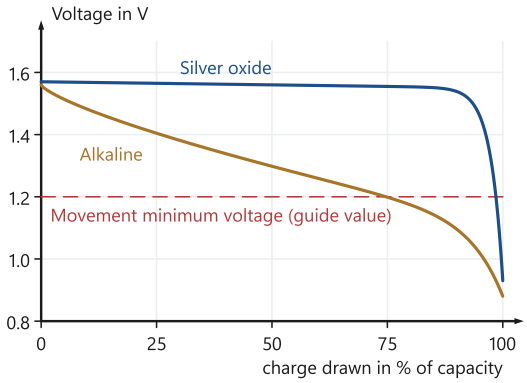


Fig. 10.8 Discharge curves in principle: the silver oxide cell holds its voltage until shortly before the end; the alkaline cell falls steadily

Battery systems for watches

System	Code letters	Nominal voltage	Discharge behaviour	Use
Silver oxide	SR	1.55 V	voltage remains almost constant until shortly before the end	standard for quartz wristwatches
Alkaline manganese	LR	1.5 V	voltage falls steadily from the start	low-cost replacement, clocks, alarm clocks
Lithium manganese dioxide	CR	3 V	voltage remains almost constant for a long time	watches with high current demand, long battery life

An alkaline cell of the same size fits mechanically but has less capacity and a falling voltage. A wristwatch takes a silver oxide cell.

IEC designation of button cells

Part of the designation	Meaning	Example SR626SW
first letter	electrochemical system: S silver oxide, L alkaline manganese, C lithium manganese dioxide	S: silver oxide
second letter	shape: R round	R: round cell
first digit or first two digits	diameter in mm, rounded down to whole millimetres	6: diameter 6.8 mm
last two digits	height in tenths of a millimetre	26: height 2.6 mm
manufacturer's suffix	SW: for small continuous currents (analogue watches); W: for high current peaks (light, alarm)	SW: for small currents

A CR2032 is thus a round lithium cell 20 mm in diameter and 3.2 mm high. Older designations such as SR44 are serial numbers without dimensional information.

Common silver oxide button cells (1.55 V)

Designation	Trade number	Diameter in mm	Height in mm	Capacity in mAh (typical)
SR521SW	379	5.8	2.15	14 to 16
SR616SW	321	6.8	1.65	14 to 16
SR621SW	364	6.8	2.15	19 to 23
SR626SW	377	6.8	2.6	24 to 28
SR721SW	362	7.9	2.1	21 to 25
SR920SW	371	9.5	2.05	35 to 40
SR927SW	395	9.5	2.7	50 to 55
SR1130SW	390	11.6	3.05	60 to 85
SR44W	357	11.6	5.4	150 to 180

Capacity depends on the manufacturer; the cell's data sheet is authoritative.

10.5 Quartz crystal and frequency divider

The oscillator of the quartz watch is a tiny quartz crystal in the shape of a tuning fork. It oscillates at 32 768 Hz, that is 2^{15} oscillations per second. A divider halves this frequency 15 times and so obtains exactly one pulse per second.

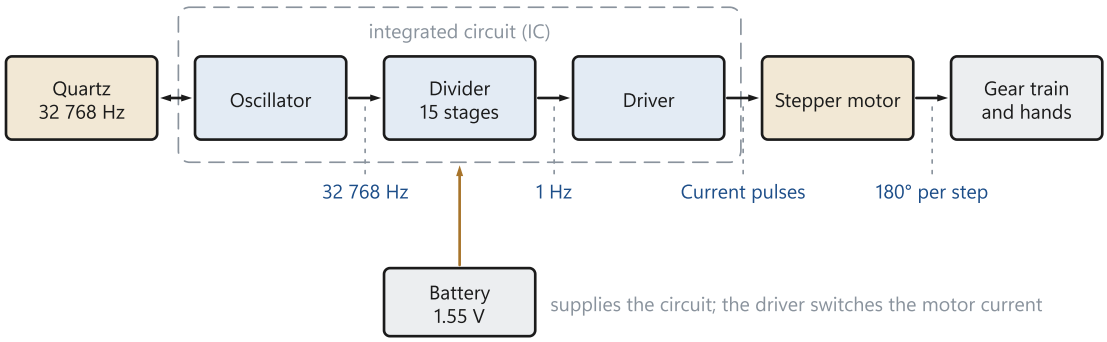


Fig. 10.9 Block diagram of the analogue quartz watch

Frequency divider

(10.24) JOURNEYMAN

$$f_{\text{aus}} = \frac{f}{2^n}$$

Each stage is a flip-flop that emits one output pulse for every second input pulse. This is why a power of two is chosen as the quartz frequency.

f_{aus}	Frequency at the divider output	Hz
f	Frequency (of the quartz crystal)	Hz
n	Number (of divider stages; each halves the frequency)	1

EXAMPLE What frequency does a 15-stage divider deliver from the quartz frequency of 32 768 Hz?

Given: $f = 32\,768\,\text{Hz}$; $n = 15$

$$\begin{aligned} f_{\text{aus}} &= \frac{f}{2^n} \\ &= \frac{32\,768\,\text{Hz}}{2^{15}} \\ &= 1\,\text{Hz} \end{aligned}$$

Exactly 1 Hz: one pulse per second for the stepper motor.

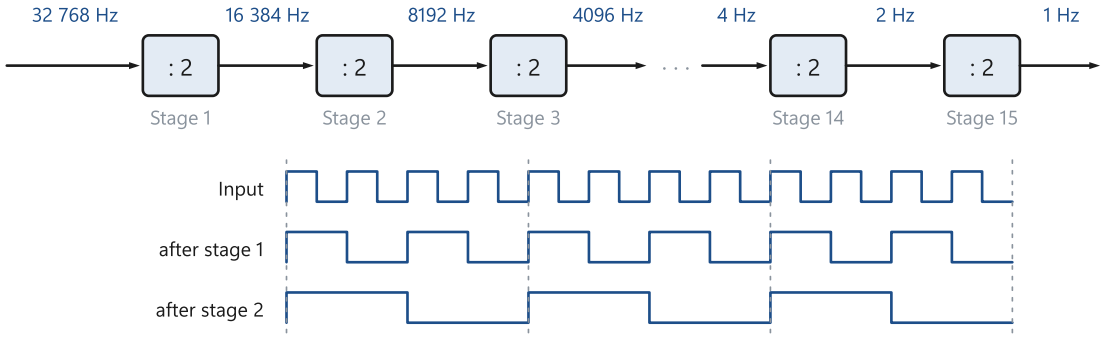


Fig. 10.10 Divider chain: each stage halves the frequency; after 15 stages 1 Hz remains

Rate of the quartz watch from the frequency deviation

(10.25) JOURNEYMAN

$$G = \frac{\Delta f}{f} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}}$$

If the crystal oscillates too fast, the watch gains in the same ratio. Relative deviations are given in millionths (ppm, Chapter 1). The same formula for the balance is given in Section 5.4.

Valid: the watch counts every quartz oscillation (no inhibition).

G	Rate (+ means: watch gains)	$\frac{\text{s}}{\text{d}}$
Δf	Frequency deviation (+ means: crystal oscillates too fast)	Hz
f	Frequency (nominal frequency of the crystal)	Hz

Good to know: 1 ppm corresponds to 0.0864 $\frac{\text{s}}{\text{d}}$, about 2.6 s per month and about 31.5 s per year.

EXAMPLE A crystal oscillates at 32 768.2 Hz instead of 32 768 Hz, i.e. 0.2 Hz too fast. What is the rate of the watch?

Given: $\Delta f = 0.2\,\text{Hz}$; $f = 32\,768\,\text{Hz}$

$$\begin{aligned} G &= \frac{\Delta f}{f} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &= \frac{0.2\,\text{Hz}}{32\,768\,\text{Hz}} \cdot 86\,400\ \tfrac{\text{s}}{\text{d}} \\ &\approx 0.53\ \tfrac{\text{s}}{\text{d}} \end{aligned}$$

The watch gains 0.53 s per day, about 16 s per month. The deviation is 6.1 ppm.

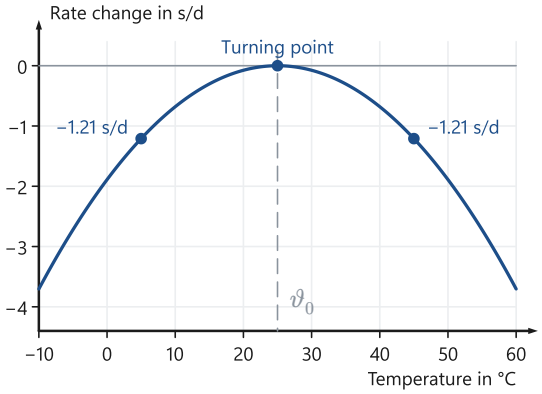


Fig. 10.11 Temperature behaviour of the tuning-fork crystal, calculated with $\beta = 0.035\ \frac{\text{ppm}}{\text{K}^2}$ and turnover point 25 °C

Temperature behaviour of the tuning-fork crystal

(10.26) APPROXIMATION MASTER

$$\Delta G \approx -\beta \cdot (\vartheta - \vartheta_0)^2 \cdot 86\,400\ \frac{\text{s}}{\text{d}}$$

The frequency of the tuning-fork crystal is highest at the turnover point and falls on both sides with the square of the temperature difference. The watch therefore runs slower than at the turnover point both in the cold and in the heat, never faster.

Valid: Tuning-fork crystals of 32 768 Hz; coefficient and turnover point vary from crystal to crystal, the data sheet is authoritative.

ΔG	Rate change (relative to the rate at the turnover point; − means: watch slows down)	$\frac{\text{s}}{\text{d}}$
β	Temperature coefficient of the crystal (guide value 0.03 to 0.04 ppm per kelvin squared)	$\frac{\text{ppm}}{\text{K}^2}$
ϑ	Temperature (of the crystal)	°C
ϑ_0	Turnover temperature (about 25 °C)	°C

Good to know: Ageing comes on top of this: in the first year the frequency of a crystal changes by a few ppm, thereafter ever more slowly. Thermocompensated movements measure the temperature and compensate for the parabola by calculation.

EXAMPLE In winter a quartz watch lies in a car at 5 °C. By how much does its rate change compared with 25 °C if the coefficient is 0.035 ppm per kelvin squared?

Given: $\beta = 0.035\ \frac{\text{ppm}}{\text{K}^2}$; $\vartheta = 5°\text{C}$; $\vartheta_0 = 25°\text{C}$

$$\begin{aligned}\Delta G &\approx -\beta \cdot (\vartheta - \vartheta_0)^2 \cdot 86\,400\ \frac{\text{s}}{\text{d}} \\ &= -0.035\ \frac{\text{ppm}}{\text{K}^2} \cdot (5°\text{C} - 25°\text{C})^2 \cdot 86\,400\ \frac{\text{s}}{\text{d}} \\ &\approx -1.21\ \frac{\text{s}}{\text{d}}\end{aligned}$$

The watch slows down by about 1.2 s per day. On the wrist the crystal stays close to the turnover point; a watch that is worn therefore runs more evenly than one that is laid aside.

Inhibition: rate correction by suppressed pulses

(10.27) MASTER

$$\Delta G = -\frac{N}{f \cdot t_\mathrm{I}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

Movements without a trimmer adjust the rate digitally. The crystal is deliberately slightly too fast; in each inhibition period the circuit omits a stored number of pulses and thus slows the watch down to the nominal rate.

Valid: Inhibition period usually 60 s depending on the circuit, 10 s in some movements, and several minutes in thermocompensated movements (ETA 252.511: 8 min); observe the manufacturer's specification.

ΔG	Rate change (due to inhibition; − means: watch slows down)	$\frac{\mathrm{s}}{\mathrm{d}}$
N	Number (of pulses suppressed per inhibition period)	1
f	Frequency (of the pulses at the divider stage where they are suppressed)	Hz
t_I	Inhibition period	s

Caution: Between corrections the watch runs at the uncorrected quartz frequency. The rate must therefore be measured over a whole inhibition period or a multiple of it, at the motor pulse and not at the quartz frequency. Otherwise the tester shows an apparent gain.

EXAMPLE A circuit suppresses one pulse of the 32 768 Hz quartz frequency every 60 s. How large is this smallest correction step?

Given: $N = 1;\quad f = 32\,768\,\mathrm{Hz};\quad t_\mathrm{I} = 60\,\mathrm{s}$

$$\begin{aligned}\Delta G &= -\frac{N}{f \cdot t_\mathrm{I}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= -\frac{1}{32\,768\,\mathrm{Hz} \cdot 60\,\mathrm{s}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx -0.044\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

Each suppressed pulse slows the watch down by 0.044 s per day, which is 0.51 ppm. A crystal that oscillates 5 ppm too fast therefore needs about 10 suppressed pulses per minute.

Key point: The watch crystal oscillates at 32 768 Hz. 15 divider stages turn this into the one-second pulse. 1 ppm is about 2.6 s per month.

10.6 Stepper motor

The Lavet stepper motor consists of a coil, a stator and a rotor with a two-pole permanent magnet. Each current pulse turns the rotor on by half a revolution. The next pulse must have the opposite polarity, otherwise the rotor stays still. Between pulses the positioning notches of the stator hold it in place without current.

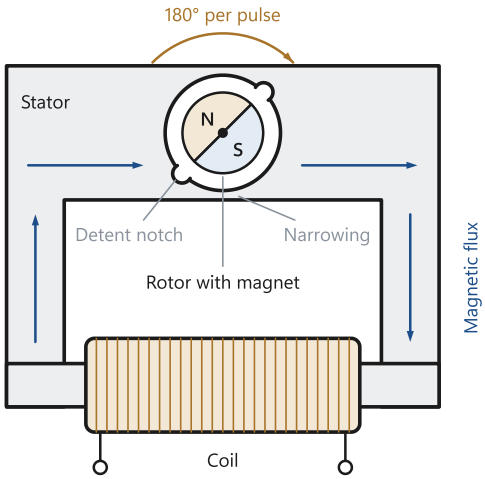


Fig. 10.12 Lavet stepper motor in principle: the narrow necks of the stator are magnetically saturated, so the flux of the coil passes through the rotor and turns it by 180° into the next detent position

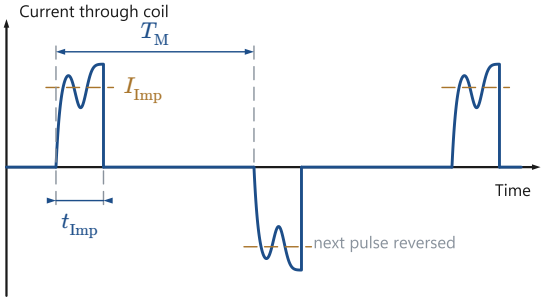


Fig. 10.13 Motor current: short pulses of duration t_{Imp} at intervals of T_M with alternating direction (pulse width greatly exaggerated)

Rotational speed of the rotor

(10.28) JOURNEYMAN

$$n_R = \frac{f_M \cdot \varphi_S}{360°}$$

With one pulse per second and half a revolution per step, the rotor turns once every two seconds.

n_R	Rotational speed (of the rotor)	$\frac{1}{\text{min}}$
f_M	Frequency of the motor pulses (steps per unit time)	Hz
φ_S	Step angle of the rotor (180° for the Lavet motor)	°

EXAMPLE How fast does the rotor of a movement with seconds hand turn if it makes one step of 180° every second?

Given: $f_M = 1\,\text{Hz}$; $\varphi_S = 180°$

$$\begin{aligned} n_R &= \frac{f_M \cdot \varphi_S}{360°} \\ &= \frac{1\,\text{Hz} \cdot 180°}{360°} \\ &= 0{,}5\,\frac{1}{\text{s}} \cdot 60\,\frac{\text{s}}{\text{min}} \\ &= 30\,\tfrac{1}{\text{min}} \end{aligned}$$

30 revolutions per minute.

Reduction ratio from rotor to fourth wheel

(10.29) JOURNEYMAN

$$i_{\mathrm{U}} = \frac{n_{\mathrm{R}}}{n_{\mathrm{S}}}$$

In the quartz movement the small rotor pinion drives the wheels, and the gear train gears down. The gear ratio i according to Chapter 4 is then less than 1. Its reciprocal, the reduction ratio, is more intuitive: it states how many times the rotor turns for one revolution of the fourth wheel. It is the product of the ratios of the number of teeth of the wheel to the number of leaves of the driving pinion at each stage.

i_{U}	Reduction ratio (rotational speed of the driving rotor divided by that of the fourth wheel; reciprocal of the gear ratio i from Chapter 4)	1
n_{R}	Rotational speed (of the rotor)	$\frac{1}{\mathrm{min}}$
n_{S}	Rotational speed of the fourth wheel (one revolution per minute)	$\frac{1}{\mathrm{min}}$

EXAMPLE The rotor makes 30 revolutions per minute, the fourth wheel one. What reduction ratio does the gear train between them need?

Given: $n_{\mathrm{R}} = 30\,\frac{1}{\mathrm{min}}$; $n_{\mathrm{S}} = 1\,\frac{1}{\mathrm{min}}$

$$\begin{aligned} i_{\mathrm{U}} &= \frac{n_{\mathrm{R}}}{n_{\mathrm{S}}} \\ &= \frac{30\,\frac{1}{\mathrm{min}}}{1\,\frac{1}{\mathrm{min}}} \\ &= 30 \end{aligned}$$

30, for example in two stages of 6 and 5: the seconds hand advances by 6° per step, one seconds division.

Mean current consumption of the movement

(10.30) MASTER

$$I_{\mathrm{m}} = \frac{I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}}{T_{\mathrm{M}}} + I_{\mathrm{IC}}$$

The motor draws current only during the short pulse. Averaged over the whole second, the fraction pulse duration divided by period remains; to this is added the small continuous current of the circuit.

Valid: regular sequence of identical pulses. A slow-responding meter or a measurement over several seconds shows this mean value.

I_{m}	Mean current consumption of the movement	$\mu\mathrm{A}$
I_{Imp}	Current through the coil during the pulse (mean value over the pulse duration)	mA
t_{Imp}	Pulse duration	ms
T_{M}	Period of the motor pulses (1 s for a seconds step)	s
I_{IC}	Current consumption of the circuit including oscillator	$\mu\mathrm{A}$

Good to know: In movements with a stop switch the motor stops when the crown is pulled out, and only the current of the circuit flows. In this way both components can be measured separately. If the gear train runs stiffly, many circuits lengthen the pulse, and the current consumption rises.

EXAMPLE During the pulse of 3.9 ms an average of 0.24 mA flows. The circuit needs 0.15 µA. What is the mean current consumption at one step per second?

Given: $I_{\mathrm{Imp}} = 0.24\,\mathrm{mA}$; $t_{\mathrm{Imp}} = 3.9\,\mathrm{ms}$; $T_{\mathrm{M}} = 1\,\mathrm{s}$; $I_{\mathrm{IC}} = 0.15\,\mu\mathrm{A}$

$$\begin{aligned} I_{\mathrm{m}} &= \frac{I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}}{T_{\mathrm{M}}} + I_{\mathrm{IC}} \\ &= \frac{0.24\,\mathrm{mA} \cdot 3.9\,\mathrm{ms}}{1\,\mathrm{s}} + 0.15\,\mu\mathrm{A} \\ &= 1.09\,\mu\mathrm{A} \end{aligned}$$

About 1.1 µA, a typical value for a three-hand movement. The motor accounts for a good 85 % of it.

Energy of a motor pulse

(10.31) MASTER

$$W = U \cdot I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}}$$

This is the energy one step draws from the battery. Only part of it moves the rotor and the gear train; the rest is turned into heat in the coil.

Valid: Voltage approximately constant during the pulse.

W	Energy (electrical energy per pulse)	J
U	Voltage (of the battery under load)	V
I_{Imp}	Current through the coil during the pulse (mean value over the pulse duration)	mA
t_{Imp}	Pulse duration	ms

EXAMPLE How much energy does a pulse of 3.9 ms duration with an average of 0.24 mA at 1.55 V cost?

Given: $U = 1.55\,\mathrm{V}$; $I_{\mathrm{Imp}} = 0.24\,\mathrm{mA}$; $t_{\mathrm{Imp}} = 3.9\,\mathrm{ms}$

$$\begin{aligned} W &= U \cdot I_{\mathrm{Imp}} \cdot t_{\mathrm{Imp}} \\ &= 1.55\,\mathrm{V} \cdot 0.24\,\mathrm{mA} \cdot 3.9\,\mathrm{ms} \\ &\approx 1.45\,\mu\mathrm{J} \end{aligned}$$

About 1.5 µJ per step. In one day with 86 400 steps this amounts to 0.13 J.

10.7 Measurements on the quartz watch

With a few measurements almost any fault of a quartz watch can be narrowed down: battery, current consumption, minimum operating voltage, coil and rate. The values in the table are guide values for three-hand movements. The specifications in the technical documentation of the calibre are binding.

Test values for the analogue quartz watch (guide values)

Test parameter	Guide value	Meaning
Battery voltage without load (silver oxide)	1.55 V to 1.6 V	shows only the cell chemistry; says little about the condition of the cell
Battery voltage under light load (measured in the watch)	at least 1.5 V	below this, replace the cell; across a heavy load resistance of the tester (a few hundred ohms) the voltage may drop lower; the scale of the instrument applies there
Current consumption of the movement	depending on the calibre, at most 0.7 µA to 1.3 µA	measured without battery at 1.55 V; too high: stiff gear train, contamination, moisture or short circuit; chronographs and movements with additional functions need more, about 3 µA to 6.5 µA
Current consumption with crown pulled out (movements with stop switch)	at most 0.4 µA to 0.5 µA	circuit only; too high: faulty circuit or circuit board
Minimum operating voltage	at most 1.3 V	lower the supply voltage from 1.55 V until the movement stops; down to 1.3 V it must still reliably execute every step. If it stops earlier, the gear train runs too stiffly or the motor or circuit is faulty
Coil resistance	depending on the calibre, about 0.8 kΩ to 2.7 kΩ	no continuity: wire broken; clearly too low: short between turns. Measure with a low test voltage, at most 0.4 V, preferably 0.2 V
Insulation resistance (coil to mainplate)	no continuity (reading infinite)	if the meter shows a resistance: moisture, damaged insulation
Rate	depending on the manufacturer, about −10 s to +20 s per month, i.e. −0.3 $\frac{\mathrm{s}}{\mathrm{d}}$ to +0.7 $\frac{\mathrm{s}}{\mathrm{d}}$	measure at the motor pulse over the inhibition period, usually 60 s (Section 10.5)

Guide values from manufacturers' specifications, measured at 20 °C to 25 °C. Examples of the maximum permissible current consumption: ETA 902.002 0.7 µA, Ronda 1005 0.95 µA, ETA F06.111 1.1 µA, ETA 255.411 1.2 µA, ETA 805.112 1.3 µA; chronograph ETA G10.211 3.5 µA, with stopwatch function running 6.5 µA. Examples of coil resistance: ETA 805.112 0.8 kΩ to 1.1 kΩ, ETA 255.411 1.3 kΩ to 1.75 kΩ, Ronda 1005 2.5 kΩ to 2.7 kΩ. The nominal values and test conditions in the technical documentation of the calibre are binding.

Key point: First the battery under load, then the current consumption, then the minimum operating voltage. If all three are correct, the fault rarely lies in the electronics.

11 Time and Calendar

Our clocks and watches show mean solar time: an imaginary, perfectly uniform solar day of 86 400 s. The true Sun, the stars and the Moon do not keep to this measure. For a sidereal clock, a perpetual calendar, an equation of time display or a moon phase we need the figures and formulas of this chapter.

The chapter ends with the chronograph scales: tachymeter, telemeter and pulsometer. In this chapter E is the equation of time, λ the geographical longitude, s a distance and c the speed of sound. The numbers of teeth for calendar and moon phase trains are given in Chapter 4.

11.1 Day and year

A day is one rotation of the Earth, but measured against what? Relative to the Sun it lasts 24 h on average, relative to the stars about 4 min less. The reason: in one day the Earth moves almost 1° further along its orbit and must turn through this extra angle before the Sun is due south again.

Astronomical units of time (mean values for the present epoch)

Unit of time	Duration	Written out	Reference
mean solar day	86 400 s	24 h	Sun due south to Sun due south, averaged over the year
sidereal day	86 164.09 s	23 h 56 min 4.09 s	one rotation of the Earth relative to the stars
tropical year	365.242 19 d	365 d 5 h 48 min 45 s	vernal equinox to vernal equinox; year of the seasons
sidereal year	365.256 36 d	365 d 6 h 9 min 10 s	one revolution of the Earth relative to the stars
synodic month	29.530 59 d	29 d 12 h 44 min 3 s	new moon to new moon; month of the lunar phases
sidereal month	27.321 66 d	27 d 7 h 43 min 11 s	one revolution of the Moon relative to the stars

Days here are always mean solar days. The calendar follows the tropical year, the moon phase the synodic month. An individual synodic month deviates from the mean by up to about 7 h.

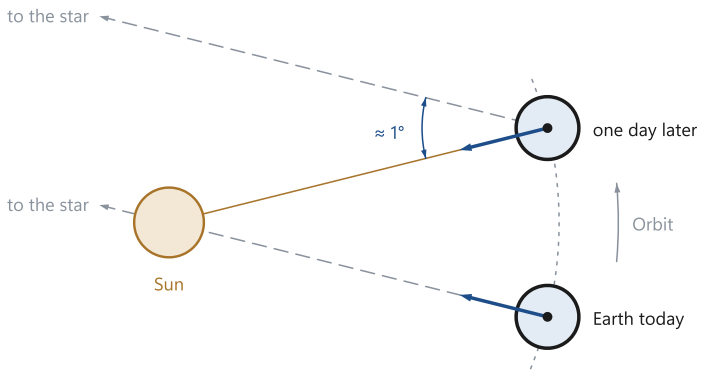


Fig. 11.1 Sidereal day and solar day: after one full rotation relative to the stars (dashed direction), the Earth must turn almost 1° further until the Sun is due south again (angle greatly exaggerated)

Sidereal day from solar day and year

(11.1) MASTER

$$T_{\mathrm{St}} = \frac{T_{\mathrm{So}} \cdot T_{\mathrm{J}}}{T_{\mathrm{J}} + T_{\mathrm{So}}}$$

T_{St}	Sidereal day	s
T_{So}	Mean solar day	s
T_{J}	Tropical year	d

In one year the Earth turns exactly once more relative to the stars than relative to the Sun: 366.2422 sidereal days correspond to 365.2422 solar days. This gives the length of the sidereal day.

Valid: Mean values for the present epoch.

EXAMPLE How long is the sidereal day if the tropical year has 365.2422 mean solar days?

Given: $T_{\mathrm{So}} = 1\,\mathrm{d}$; $T_{\mathrm{J}} = 365.2422\,\mathrm{d}$

$$\begin{aligned} T_{\mathrm{St}} &= \frac{T_{\mathrm{So}} \cdot T_{\mathrm{J}}}{T_{\mathrm{J}} + T_{\mathrm{So}}} \\ &= \frac{1\,\mathrm{d} \cdot 365.2422\,\mathrm{d}}{365.2422\,\mathrm{d} + 1\,\mathrm{d}} \\ &= 0{,}997\,269\,6\,\mathrm{d} \\ &\approx 86\,164.1\,\mathrm{s} \end{aligned}$$

The sidereal day lasts 23 h 56 min 4.1 s. It is 3 min 55.9 s shorter than the mean solar day.

Rate of a sidereal clock against mean time

(11.2) MASTER

$$G = \frac{T_{\mathrm{So}} - T_{\mathrm{St}}}{T_{\mathrm{St}}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}}$$

G	Rate (+ means: clock gains)	$\frac{\mathrm{s}}{\mathrm{d}}$
T_{So}	Mean solar day	s
T_{St}	Sidereal day	s

A sidereal clock counts sidereal days, so its day is shorter. Measured against a mean-time clock it gains almost 4 min every day.

Good to know: For the pendulum clock this means: the seconds pendulum for sidereal time is about 5.4 mm shorter than the seconds pendulum for mean time (Chapter 6).

EXAMPLE A sidereal clock is regulated against a radio-controlled clock showing mean time. What rate must it have relative to the radio-controlled clock?

Given: $T_{\mathrm{So}} = 86\,400\,\mathrm{s}$; $T_{\mathrm{St}} = 86\,164.09\,\mathrm{s}$

$$\begin{aligned} G &= \frac{T_{\mathrm{So}} - T_{\mathrm{St}}}{T_{\mathrm{St}}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= \frac{86\,400\,\mathrm{s} - 86\,164.09\,\mathrm{s}}{86\,164.09\,\mathrm{s}} \cdot 86\,400\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &\approx 236.6\ \tfrac{\mathrm{s}}{\mathrm{d}} \end{aligned}$$

The sidereal clock must gain 236.6 s per day, that is 3 min 56.6 s. In one year it gains exactly one day.

Synodic month from the Moon's revolution

(11.3) ADVANCED

$$T_{\mathrm{syn}} = \frac{T_{\mathrm{sid}} \cdot T_{\mathrm{E}}}{T_{\mathrm{E}} - T_{\mathrm{sid}}}$$

During one revolution of the Moon the Earth moves on along its orbit. The Moon therefore needs a good two days longer until it stands between Earth and Sun again. The same formula describes a fast hand catching up with a slow one.

Valid: Mean values; both revolutions in the same direction.

T_{syn}	Synodic month (new moon to new moon)	d
T_{sid}	Sidereal month (one lunar revolution relative to the stars)	d
T_{E}	Sidereal year (one revolution of the Earth relative to the stars)	d

EXAMPLE The Moon orbits the Earth once in 27.321 66 days, the Earth orbits the Sun once in 365.256 36 days. How long is it from new moon to new moon?

Given: $T_{\mathrm{sid}} = 27.321\,66\,\mathrm{d}$; $T_{\mathrm{E}} = 365.256\,36\,\mathrm{d}$

$$\begin{aligned} T_{\mathrm{syn}} &= \frac{T_{\mathrm{sid}} \cdot T_{\mathrm{E}}}{T_{\mathrm{E}} - T_{\mathrm{sid}}} \\ &= \frac{27.321\,66\,\mathrm{d} \cdot 365.256\,36\,\mathrm{d}}{365.256\,36\,\mathrm{d} - 27.321\,66\,\mathrm{d}} \\ &\approx 29.5306\,\mathrm{d} \end{aligned}$$

About 29.53 days. A moon phase train must reproduce this figure with its numbers of teeth (Chapter 4).

11.2 Calendar

The calendar year has whole days, the tropical year does not. Leap days make up the difference. How good a leap-year rule is shows in the mean length of its calendar year.

Leap-year rule

Calendar	Leap year (366 days, with 29 February)	Mean length of year
Julian	every year whose number is divisible by 4	$365.25\,\mathrm{d}$
Gregorian (since 1582)	divisible by 4; full centuries only if divisible by 400	$365.2425\,\mathrm{d}$

Under the Gregorian rule 1700, 1800 and 1900 were not leap years, 2000 was one, and 2100 again will not be.

Mean length of the calendar year

(11.4) JOURNEYMAN

$$T_{\mathrm{K}} = 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot n_{\mathrm{S}}}{n_{\mathrm{Z}}}$$

Each leap year adds one day. Spread over the whole cycle, this gives the mean length of the year.

T_{K}	Mean length of the calendar year	d
n_{S}	Number of leap years in the cycle	1
n_{Z}	Number of years in the cycle	1

EXAMPLE The Gregorian calendar has 97 leap years in 400 years. What is the mean length of its year?

Given: $n_{\mathrm{S}} = 97$; $n_{\mathrm{Z}} = 400$

$$\begin{aligned} T_{\mathrm{K}} &= 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot n_{\mathrm{S}}}{n_{\mathrm{Z}}} \\ &= 365\,\mathrm{d} + \frac{1\,\mathrm{d} \cdot 97}{400} \\ &= 365.2425\,\mathrm{d} \end{aligned}$$

365.2425 days. The Julian rule, with one leap year in four, gives 365.25 days.

Years until one day of error

(11.5) MASTER

$$n_{\mathrm{F}} = \frac{1\,\mathrm{d}}{T_{\mathrm{K}} - T_{\mathrm{J}}}$$

If the calendar year is longer than the tropical year, the calendar falls behind the seasons: the vernal equinox falls on an ever earlier date.

Valid: Present length of the tropical year; it changes by a few seconds over millennia.

n_{F}	Number of years until the calendar deviates by one day from the solar year	1
T_{K}	Mean length of the calendar year	d
T_{J}	Tropical year	d

EXAMPLE After how many years does the Julian calendar deviate by one day from the course of the seasons?

Given: $T_{\mathrm{K}} = 365.25\,\mathrm{d}$; $T_{\mathrm{J}} = 365.242\,19\,\mathrm{d}$

$$\begin{aligned} n_{\mathrm{F}} &= \frac{1\,\mathrm{d}}{T_{\mathrm{K}} - T_{\mathrm{J}}} \\ &= \frac{1\,\mathrm{d}}{365.25\,\mathrm{d} - 365.242\,19\,\mathrm{d}} \\ &\approx 128 \end{aligned}$$

After 128 years. From the Council of Nicaea in 325 to 1582, ten days had accumulated; they were dropped in the calendar reform. The Gregorian calendar, with 365.2425 days, takes about 3200 years to drift by one day.



Fig. 11.2 Four-year cycle of the perpetual calendar: $3 \cdot 365 + 366 = 1461$ days

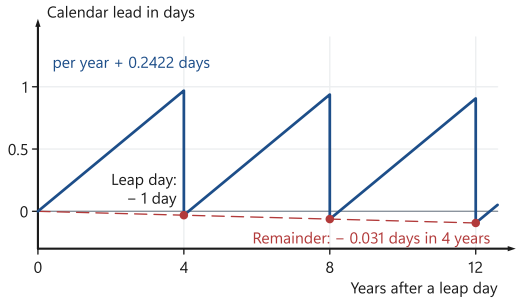


Fig. 11.3 Without a leap day the calendar year is 0.2422 days too short; every four years the leap day takes back a whole day

Days in the leap-year cycle

(11.6) JOURNEYMAN

$$N_{\mathrm{d}} = 365 \cdot n_{\mathrm{Z}} + n_{\mathrm{S}}$$

The perpetual calendar knows the lengths of the months and the leap year. Its program wheel turns once in four years.

N_{d}	Number of days in the cycle	1
n_{Z}	Number of years in the cycle	1
n_{S}	Number of leap years in the cycle	1

Caution: A perpetual calendar with a four-year cycle does not know the century rule. In 2100 it shows a 29 February and must be advanced by one day on 1 March 2100, likewise in 2200 and 2300. Only a secular calendar reproduces the 400-year cycle.

EXAMPLE A perpetual calendar reproduces four years with one leap year. How many days does this cycle have?

Given: $n_{\mathrm{Z}} = 4$; $n_{\mathrm{S}} = 1$

$$\begin{aligned} N_{\mathrm{d}} &= 365 \cdot n_{\mathrm{Z}} + n_{\mathrm{S}} \\ &= 365 \cdot 4 + 1 \\ &= 1461 \end{aligned}$$

1461 days or 48 months; this is how long the leap-year wheel takes for one turn. The Gregorian cycle of 400 years has 146 097 days. That is exactly 20 871 weeks, so after 400 years the days of the week repeat as well.

Key point: 365 days are 52 weeks and one day. A date therefore moves on by one weekday each year, and by two across a leap day.

11.3 Equation of time

The true Sun moves unevenly over the year: the Earth's orbit is an ellipse, and the Earth's axis is tilted to the orbit. A sundial therefore differs by up to a good quarter of an hour from a uniformly running clock. This difference is called the equation of time.

Equation of time

(11.7) MASTER

$$E = t_{\mathrm{W}} - t_{\mathrm{M}}$$

REARRANGED $t_{\mathrm{M}} = t_{\mathrm{W}} - E$

The equation of time is apparent solar time minus mean solar time. It depends only on the date, not on the location.

E	Equation of time (+ means: sundial is fast)	min
t_{W}	Apparent local time (sundial reading)	min
t_{M}	Mean local time	min

Caution: Older tables and some watches with an equation of time display use the opposite sign (mean minus apparent time). Before setting, check which direction is meant.

EXAMPLE On 11 February the equation of time is −14.2 min. A sundial shows 12:00, which is 720 min after midnight. What is the mean local time?

Given: $t_{\mathrm{W}} = 720\,\mathrm{min}$; $E = -14.2\,\mathrm{min}$

$$\begin{aligned} t_{\mathrm{M}} &= t_{\mathrm{W}} - E \\ &= 720\,\mathrm{min} - (-14.2\,\mathrm{min}) \\ &= 734.2\,\mathrm{min} \end{aligned}$$

734.2 min after midnight: it is 12:14 mean local time. The sundial is a good 14 min slow.

Annual angle for the equation of time

(11.8) ADVANCED

$$B = \frac{360^\circ \cdot (N - 81)}{364}$$

The annual angle divides the year into 360° and starts on 22 March, the 81st day. It is only an auxiliary quantity for the approximation formula.

B	Annual angle (auxiliary quantity; 0° on 22 March)	$^\circ$
N	Day number in the year (1 January = 1)	1

EXAMPLE In a common year, 3 November is the 307th day. What is the annual angle?

Given: $N = 307$

$$\begin{aligned} B &= \frac{360^\circ \cdot (N - 81)}{364} \\ &= \frac{360^\circ \cdot (307 - 81)}{364} \\ &\approx 223.5^\circ \end{aligned}$$

About 223.5°. The following formula works with this angle.

Equation of time from the date

(11.9) APPROXIMATION ADVANCED

$$E \approx 9.87\,\text{min} \cdot \sin(2 \cdot B) - 7.53\,\text{min} \cdot \cos B - 1.5\,\text{min} \cdot \sin B$$

The formula reproduces the two causes: the term with $2B$ comes from the tilted Earth axis and has two waves per year; the terms with B come from the elliptical orbit and have one wave per year.

E	Equation of time (+ means: sundial is fast)	min
B	Annual angle (auxiliary quantity; 0° on 22 March)	°

Valid: Approximation, accurate to about one minute (largest deviation, depending on the year, up to about 1.4 min, in early December). This is sufficient for sundials and for checking an equation of time display; exact daily values are given in astronomical almanacs.

Note: For 11 February (day 42, $B = -38.6^\circ$) the approximation gives −14.6 min, the table of key values −14.2 min. Differences of a few tenths of a minute lie within the accuracy of the approximation; for setting, the table values apply.

EXAMPLE What is the equation of time on 3 November, with the annual angle of 223.5° from the previous example?

Given: $B = 223.5^\circ$

$$\begin{aligned} E &\approx 9.87\,\text{min} \cdot \sin(2 \cdot B) - 7.53\,\text{min} \cdot \cos B - 1.5\,\text{min} \cdot \sin B \\ &= 9.87\,\text{min} \cdot \sin(2 \cdot 223.5^\circ) - 7.53\,\text{min} \cdot \cos(223.5^\circ) - 1.5\,\text{min} \cdot \sin(223.5^\circ) \\ &\approx 16.4\,\text{min} \end{aligned}$$

About +16.4 min: in early November the sundial is furthest ahead.

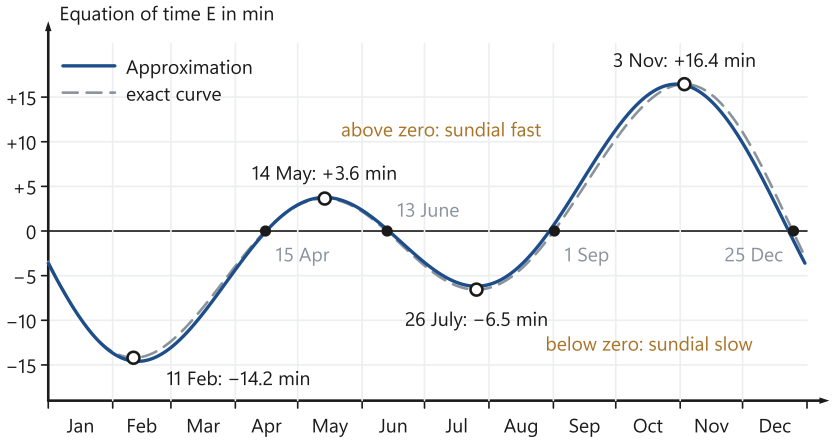


Fig. 11.4 Equation of time over the year: approximation formula and exact curve

Key values of the equation of time (rounded, for the present epoch)

Date	E	Sundial relative to mean local time
11 February	−14.2 min	furthest behind
15 April	0	agrees
14 May	+3.6 min	fast (small maximum)
13 June	0	agrees
26 July	−6.5 min	slow (small minimum)
1 September	0	agrees
3 November	+16.4 min	furthest ahead
25 December	0	agrees

Depending on the position in the leap-year cycle, the dates shift by up to one day.

11.4 Local time and zone time

Mean local time depends on the longitude: the Sun moves across 15 degrees of longitude in one hour and needs 4 min for one degree. So that not every place has its own time, the local time of the zone meridian applies throughout a time zone. For Central European Time (CET) this is the meridian 15° East.

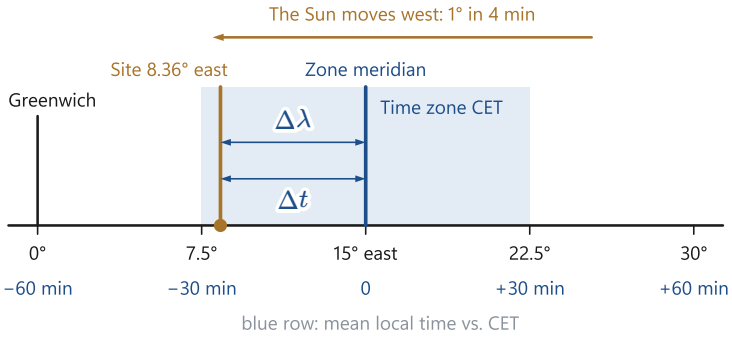


Fig. 11.5 Local time and zone time: a place west of the zone meridian has the Sun due south later

Local time from the longitude

(11.10) JOURNEYMAN

$$\Delta t = \frac{(\lambda - \lambda_Z) \cdot 4\,\text{min}}{1^\circ}$$

East of the zone meridian, local time is ahead of zone time; west of it, local time lags behind.

Valid: Longitudes east of Greenwich positive, west negative.

Δt	Local time difference (mean local time minus zone time; + means: local time is ahead)	min
λ	Geographical longitude of the place (east longitude positive)	$^\circ$
λ_Z	Geographical longitude of the zone meridian (CET: 15° East)	$^\circ$

EXAMPLE A workshop lies at 8.36° east longitude. By how much does its mean local time differ from CET?

Given: $\lambda = 8.36^\circ$; $\lambda_Z = 15^\circ$

$$\begin{aligned}\Delta t &= \frac{(\lambda - \lambda_Z) \cdot 4\,\text{min}}{1^\circ} \\ &= \frac{(8.36^\circ - 15^\circ) \cdot 4\,\text{min}}{1^\circ} \\ &\approx -26.6\,\text{min}\end{aligned}$$

Mean local time lags 26.6 min behind CET. Here the mean Sun is due south only at 12:26:34 CET.

Zone time from the sundial

(11.11) MASTER

$$t_Z = t_W - E - \Delta t$$

Two corrections lead from the sundial to the wristwatch: the equation of time for the date and the local time difference for the longitude.

Valid: Sundial showing apparent local time. Some sundials are already laid out for the longitude of their location; Δt is then omitted.

t_Z	Zone time (CET)	min
t_W	Apparent local time (sundial reading)	min
E	Equation of time (+ means: sundial is fast)	min
Δt	Local time difference (mean local time minus zone time; + means: local time is ahead)	min

EXAMPLE On 3 November the sundial at the workshop from the previous example shows 12:00, i.e. 720 min. The equation of time is +16.4 min. What is the time in CET?

Given: $t_W = 720\,\mathrm{min}$; $E = 16.4\,\mathrm{min}$; $\Delta t = -26.6\,\mathrm{min}$

$$\begin{aligned} t_Z &= t_W - E - \Delta t \\ &= 720\,\mathrm{min} - 16.4\,\mathrm{min} - (-26.6\,\mathrm{min}) \\ &= 730.2\,\mathrm{min} \end{aligned}$$

730.2 min after midnight, i.e. 12:10 CET. During summer time (daylight saving time) one hour is added.

11.5 Moon phase

The lunar phase repeats on average every 29.530 59 days. The age of the Moon counts the days since the last new moon; a moon phase display is set by it. How the 29.53 days are approximated with numbers of teeth, and how large the error of the display becomes, is covered in Chapter 4.

Age of the Moon from the date

(11.12) MASTER

$$a = t - k \cdot T_{syn}$$

All complete lunations are subtracted from the interval since a known new moon. What remains is the age of the Moon.

Valid: Mean lunation; the true phase can deviate from it by up to about one day. This is sufficient for setting a moon phase display.

a	Age of the Moon (days since the last new moon)	d
t	Time (since a known new moon)	d
k	Number (of lunations completed since then; the largest whole number for which a does not become negative)	1
T_{syn}	Synodic month (new moon to new moon)	d

EXAMPLE There was a new moon on 12 August 2026. How old is the Moon on 24 December 2026, i.e. 134 days later? In 134 days, 4 lunations are completed.

Given: $t = 134\,\mathrm{d}$; $k = 4$; $T_{syn} = 29.530\,59\,\mathrm{d}$

$$\begin{aligned} a &= t - k \cdot T_{syn} \\ &= 134\,\mathrm{d} - 4 \cdot 29.530\,59\,\mathrm{d} \\ &\approx 15.9\,\mathrm{d} \end{aligned}$$

The age of the Moon is about 15.9 days. Full moon is at 14.8 days, so the Moon has just been full.

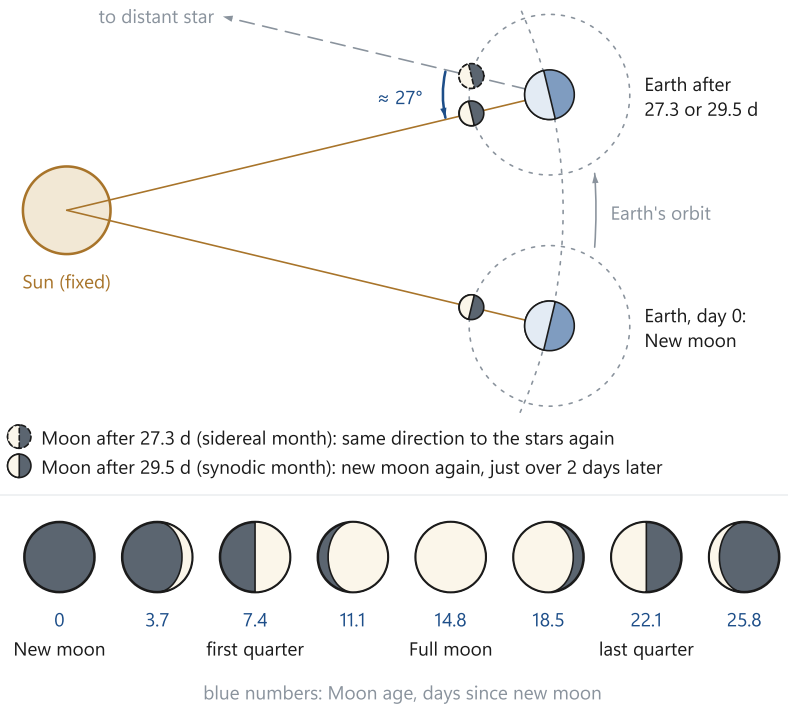


Fig. 11.6 Top: Sun fixed, Earth on its orbit, Moon around the Earth. After one sidereal month (27.3 d) the Moon is again in the same direction relative to the stars; it needs a good two days more until new moon (synodic month, 29.5 d; sizes not to scale). Bottom: moon phases as seen from the northern hemisphere, with the mean age of the Moon

11.6 Chronograph scales

Many chronographs carry a scale on the bezel or dial that immediately converts the measured time into another quantity. Behind every scale is a simple formula. With it a scale can be checked, or calculated for a new dial.

Speed from measured distance and stopped time

(11.13) JOURNEYMAN

$$v = \frac{s}{t}$$

v	Speed	$\frac{\mathrm{km}}{\mathrm{h}}$
s	Distance (measured distance)	km
t	Time (stopped)	s

The chronograph is started at the beginning of a known distance and stopped at its end.

Valid: Constant speed over the measured distance.

EXAMPLE A train needs 45 s for the distance between two kilometre posts. How fast is it travelling?

Given: $s = 1\,\mathrm{km}$; $t = 45\,\mathrm{s}$

$$\begin{aligned} v &= \frac{s}{t} \\ &= \frac{1\,\mathrm{km}}{45\,\mathrm{s}} \\ &= \frac{1\,\mathrm{km}}{45\,\mathrm{s}} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} \\ &= 80\,\tfrac{\mathrm{km}}{\mathrm{h}} \end{aligned}$$

The train travels at 80 km per hour.

Tachymeter scale

(11.14) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$v = \frac{3600}{t}$$

v	Speed (scale value in km per hour)	$\frac{\mathrm{km}}{\mathrm{h}}$
t	Time (stopped, in s)	s

The tachymeter scale is the preceding formula for a measured distance of 1 km: when stopped after t seconds, the hand points to the speed. In the same way the scale counts units per hour if the time for one unit is stopped.

Valid: Numerical values: t in s; measured distance 1 km (or 1 mile, then miles per hour). The scale only reaches 60 s. For slower events, time half the distance and halve the scale value.

EXAMPLE Which number is on a tachymeter scale at 24 s?

Given: $t = 24\,\mathrm{s}$

$$\begin{aligned} v &= \frac{3600}{t} \\ &= \frac{3600}{24} \\ &= 150\ \tfrac{\mathrm{km}}{\mathrm{h}} \end{aligned}$$

At 24 s the scale reads 150.

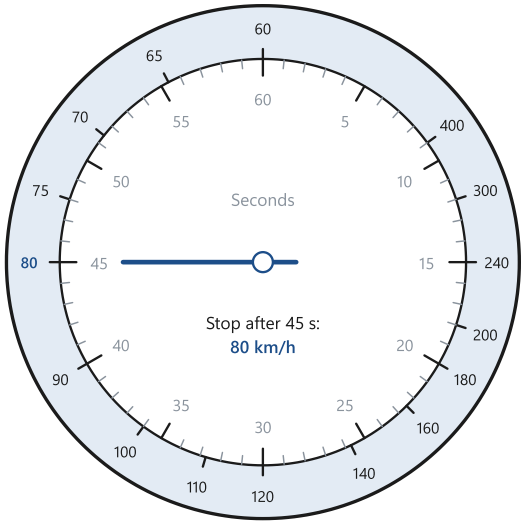


Fig. 11.7 Tachymeter scale: scale value equals 3600 divided by the seconds

Telemeter: distance from the travel time of sound

(11.15) JOURNEYMAN

$$s = c \cdot t$$

The lightning is seen practically at once; the thunder needs almost 3 s for every kilometre. The telemeter scale shows the distance in kilometres for the stopped travel time.

Valid: Speed of sound in air at 20 °C: $343\ \frac{\mathrm{m}}{\mathrm{s}}$; the travel time of light is neglected.

s	Distance	m
c	Speed of sound (in air)	$\frac{\mathrm{m}}{\mathrm{s}}$
t	Time (stopped, between light signal and sound)	s

EXAMPLE 6 s are timed between lightning and thunder. How far away is the thunderstorm?

Given: $c = 343\ \frac{\mathrm{m}}{\mathrm{s}}$; $t = 6\,\mathrm{s}$

$$\begin{aligned} s &= c \cdot t \\ &= 343\ \tfrac{\mathrm{m}}{\mathrm{s}} \cdot 6\,\mathrm{s} \\ &\approx 2.06\,\mathrm{km} \end{aligned}$$

The thunderstorm is about 2 km away.

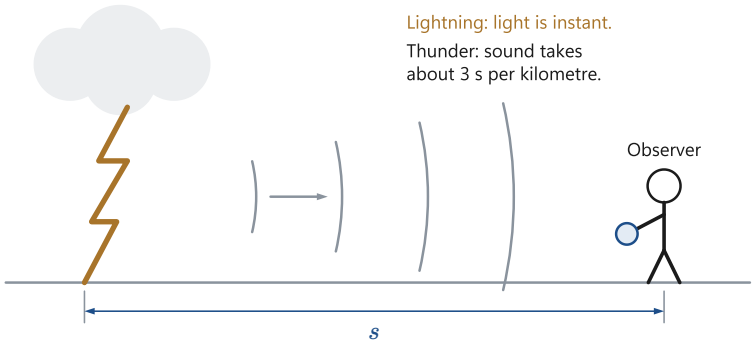


Fig. 11.8 Telemeter: the distance s follows from the time between lightning and thunder

Speed of sound and temperature

(11.16) NUMERICAL-VALUE EQUATION ADVANCED

$$c \approx 331.3 + 0.606 \cdot \vartheta$$

Sound travels faster in warm air. Telemeter scales are calculated for a fixed temperature, usually 20 °C.

Valid: Numerical values: ϑ in °C, c in $\frac{\mathrm{m}}{\mathrm{s}}$; dry air, approx. −20 °C to 40 °C.

c	Speed of sound (in air, numerical value in m per second)	$\frac{\mathrm{m}}{\mathrm{s}}$
ϑ	Temperature (of the air, in °C)	$^\circ\mathrm{C}$

EXAMPLE How fast is sound on a winter day at −10 °C?

Given: $\vartheta = -10^\circ\mathrm{C}$

$$\begin{aligned} c &\approx 331.3 + 0.606 \cdot \vartheta \\ &= 331.3 + 0.606 \cdot (-10) \\ &\approx 325.2\ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

About $325\ \frac{\mathrm{m}}{\mathrm{s}}$, which is 5 % less than at 20 °C. A telemeter scale then reads 5 % too high.

Pulsometer

(11.17) JOURNEYMAN

$$f_{\mathrm{P}} = \frac{N_{\mathrm{P}}}{t}$$

Start the chronograph, count the number of pulse beats stated on the scale, stop: the hand points to the beats per minute. The asthmometer scale counts breaths in the same way.

f_{P}	Pulse rate	$\frac{1}{\mathrm{min}}$
N_{P}	Number of pulse beats counted (graduation of the scale, usually 15 or 30)	1
t	Time (stopped)	s

EXAMPLE A pulsometer scale is graduated for 30 pulse beats. 20 s are timed for 30 beats. Which number is on the scale at that point?

Given: $N_{\mathrm{P}} = 30; \quad t = 20\,\mathrm{s}$

$$\begin{aligned} f_{\mathrm{P}} &= \frac{N_{\mathrm{P}}}{t} \\ &= \frac{30}{20\,\mathrm{s}} \\ &= 1{,}5\,\frac{1}{\mathrm{s}} \cdot 60\,\frac{\mathrm{s}}{\mathrm{min}} \\ &= 90\,\frac{1}{\mathrm{min}} \end{aligned}$$

The hand points to 90 beats per minute.

Decimal minutes

(11.18) NUMERICAL-VALUE EQUATION JOURNEYMAN

$$t_{\mathrm{dez}} = \frac{t}{60}$$

For time studies the decimal scale divides the minute into 100 parts. One hundredth of a minute lasts 0.6 s. With decimal minutes, times can be added and multiplied by quantities without conversion.

Valid: Numerical values: t in s, result in min.

t_{dez}	Time in minutes as a decimal number	min
t	Time (stopped, in s)	s

EXAMPLE An operation takes 39 s. What does the decimal scale show?

Given: $t = 39\,\mathrm{s}$

$$\begin{aligned} t_{\mathrm{dez}} &= \frac{t}{60} \\ &= \frac{39}{60} \\ &= 0.65\,\mathrm{min} \end{aligned}$$

0.65 min, i.e. 65 hundredths of a minute.

Key point: Three seconds between lightning and thunder are about one kilometre.

12 Quality and Statistics

A single measured value says little. Only a series of measurements shows where a rate, a dimension or a process lies on average and how widely it scatters. With the mean and the standard deviation one can judge whether a production process reliably holds its tolerance and which defects must be eliminated first.

This chapter is aimed mainly at the master craftsman examination and at series production. Here s is the standard deviation, not the blade thickness; in failure analysis B, A and E are rating numbers.

12.1 Series of measurements

From a series of measurements we draw two statements: the location (mean) and the spread (range, standard deviation). The example used throughout this section is a series of ten rate values in $\frac{\mathrm{s}}{\mathrm{d}}$: +4, +6, +3, +7, +5, +2, +6, +5, +8, +4.

Arithmetic mean

(12.1) JOURNEYMAN

$$\bar{x} = \frac{\sum x_i}{n}$$

All measured values are added and divided by their number. The mean has the same unit as the measured values. As a basic tool, the formula also appears in Section 1.5.

$\bar{x}$	Mean (arithmetic)	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum x_i$	Sum of all measured values (with sign)	$\frac{\mathrm{s}}{\mathrm{d}}$
n	Number (of measured values)	1

EXAMPLE The ten rate values of the series add up to 50 $\frac{\mathrm{s}}{\mathrm{d}}$. What is the mean rate?

Given: $\sum x_i = 50\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad n = 10$

$$\begin{aligned}\bar{x} &= \frac{\sum x_i}{n} \\ &= \frac{50\ \frac{\mathrm{s}}{\mathrm{d}}}{10} \\ &= 5.0\ \frac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

On average the watch gains 5 s per day.

Range

(12.2) JOURNEYMAN

$$R = x_{\max} - x_{\min}$$

The range is the quickest measure of spread. However, it depends only on the two extreme values; a single outlier distorts it.

R	Range	$\frac{\mathrm{s}}{\mathrm{d}}$
$x_{\max}$	Largest measured value	$\frac{\mathrm{s}}{\mathrm{d}}$
$x_{\min}$	Smallest measured value	$\frac{\mathrm{s}}{\mathrm{d}}$

EXAMPLE The largest rate value of the series is +8 $\frac{\mathrm{s}}{\mathrm{d}}$, the smallest +2 $\frac{\mathrm{s}}{\mathrm{d}}$.

Given: $x_{\max} = 8\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad x_{\min} = 2\ \frac{\mathrm{s}}{\mathrm{d}}$

$$\begin{aligned}R &= x_{\max} - x_{\min} \\ &= 8\ \tfrac{\mathrm{s}}{\mathrm{d}} - 2\ \tfrac{\mathrm{s}}{\mathrm{d}} \\ &= 6\ \tfrac{\mathrm{s}}{\mathrm{d}}\end{aligned}$$

The rate values lie within a band of 6 s per day.

Sample standard deviation

(12.3) MASTER

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

s	Standard deviation	$\frac{\mathrm{s}}{\mathrm{d}}$
$\sum (x_i - \bar{x})^2$	Sum of the squared deviations from the mean	$\frac{\mathrm{s}^2}{\mathrm{d}^2}$
n	Number (of measured values)	1

The standard deviation is the average deviation of the individual values from the mean, taken as a root mean square. The division is by $n - 1$ because the mean comes from the same sample.

Valid: Sample from a larger population; for a meaningful statement at least about 10 values, preferably 30 or more.

Good to know: Calculators show this quantity as s or σ_{n-1}. The σ_n key divides by n and applies only when every part has been measured.

EXAMPLE The deviations of the ten rate values from the mean of 5 are −1, +1, −2, +2, 0, −3, +1, 0, +3, −1. Their squares 1, 1, 4, 4, 0, 9, 1, 0, 9, 1 add up to 30.

Given: $\sum (x_i - \bar{x})^2 = 30\ \frac{\mathrm{s}^2}{\mathrm{d}^2}$; $\quad n = 10$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{30\ \frac{\mathrm{s}^2}{\mathrm{d}^2}}{10 - 1}}$$

$$\approx 1.83\ \frac{\mathrm{s}}{\mathrm{d}}$$

The rate values scatter about the mean with a standard deviation of about 1.8 s per day.

Standard deviation of the mean

(12.4) ADVANCED

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$s_{\bar{x}}$	Standard deviation of the mean	$\frac{\mathrm{s}}{\mathrm{d}}$
s	Standard deviation	$\frac{\mathrm{s}}{\mathrm{d}}$
n	Number (of measured values)	1

The mean of a series of measurements scatters less than a single measured value. Four times as many measurements halve its uncertainty.

Valid: Mutually independent measured values.

EXAMPLE How reliable is the mean of the ten rate values if the individual values scatter with 1.83 $\frac{\mathrm{s}}{\mathrm{d}}$?

Given: $s = 1.83\ \frac{\mathrm{s}}{\mathrm{d}}$; $\quad n = 10$

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$$= \frac{1.83\ \frac{\mathrm{s}}{\mathrm{d}}}{\sqrt{10}}$$

$$\approx 0.58\ \frac{\mathrm{s}}{\mathrm{d}}$$

The mean is reliable to about 0.6 s per day. With 40 measurements it would be 0.3 s per day.

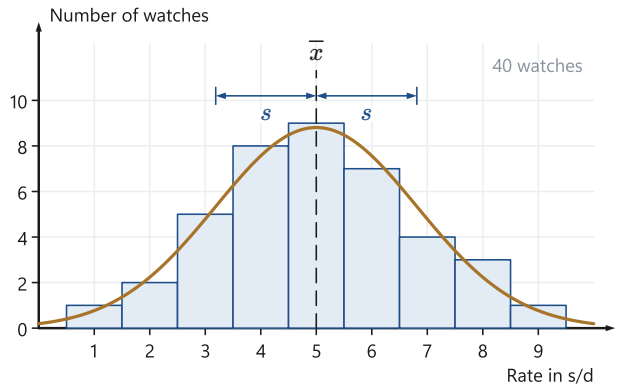


Fig. 12.1 Histogram of a rate measurement series of 40 watches with bell curve: $\bar{x} = 5.0\ \frac{\text{s}}{\text{d}}$, $s = 1.8\ \frac{\text{s}}{\text{d}}$

Many measured quantities scatter according to the normal distribution, the bell curve: values close to the mean are frequent, values far away are rare. The standard deviation then tells what proportion of the values lies within which range.

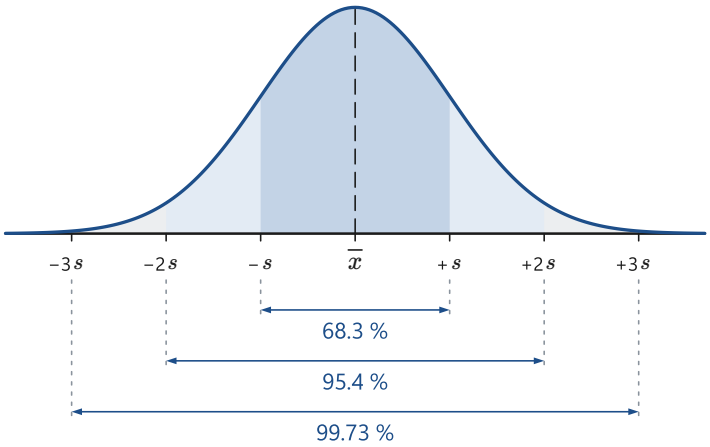


Fig. 12.2 Normal distribution: proportion of values within one, two and three standard deviations of the mean

Normal distribution: proportion of values about the mean

Range	Inside	Outside
$\bar{x} \pm 1\,s$	68.3 %	31.7 %
$\bar{x} \pm 2\,s$	95.4 %	4.6 %
$\bar{x} \pm 3\,s$	99.73 %	0.27 %
$\bar{x} \pm 4\,s$	99.994 %	63 ppm

Applies to normally distributed values. 1 ppm is one part per million.

Key point: Two thirds of all values lie within one standard deviation of the mean, almost all within three.

12.2 Machine and process capability

Capability compares the tolerance with the spread: does the bell curve fit safely between the lower and the upper specification limit? For machine capability (C_m, C_mk) a short series is measured under identical conditions, usually 50 consecutive parts. For process capability (C_p, C_pk) samples are measured over a longer period. The formulas are the same.

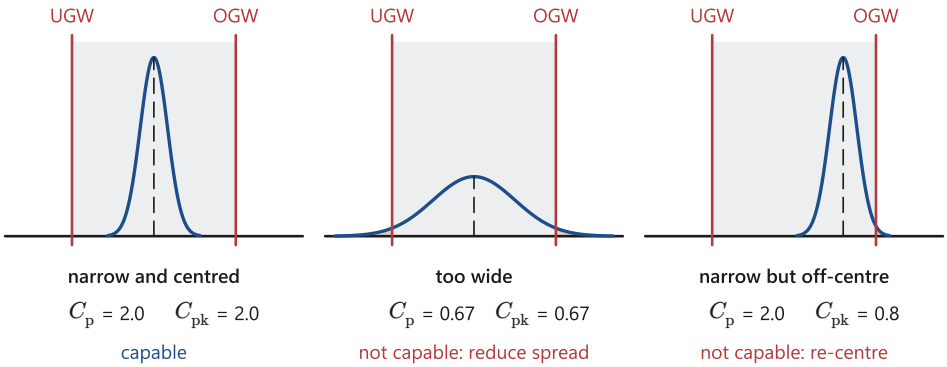


Fig. 12.3 Tolerance zone and distribution: a process is capable only if its spread is narrow enough and it is centred

Capability from tolerance and spread

(12.5) MASTER

$$C_\mathrm{p} = \frac{\mathrm{OGW} - \mathrm{UGW}}{6 \cdot s}$$

C_p	Process capability index (machine: C_m)	1
OGW	Upper specification limit	µm
UGW	Lower specification limit	µm
s	Standard deviation	µm

C_p states how many times the spread width of six standard deviations fits into the tolerance. Where the mean lies plays no part.

Valid: Normally distributed measured values, stable process.

EXAMPLE A pivot is to measure 0.100 mm ± 0.004 mm, so the specification limits are 96 µm and 104 µm. A series of measurements gives a standard deviation of 0.8 µm.

Given: $\mathrm{OGW} = 104\,\mu\mathrm{m}; \quad \mathrm{UGW} = 96\,\mu\mathrm{m}; \quad s = 0.8\,\mu\mathrm{m}$

$$\begin{aligned} C_\mathrm{p} &= \frac{\mathrm{OGW} - \mathrm{UGW}}{6 \cdot s} \\ &= \frac{104\,\mu\mathrm{m} - 96\,\mu\mathrm{m}}{6 \cdot 0.8\,\mu\mathrm{m}} \\ &\approx 1.67 \end{aligned}$$

The tolerance is 1.67 times as wide as the spread width: the spread is small enough.

Critical capability index (upper limit)

(12.6) MASTER

$$C_\mathrm{pu} = \frac{\mathrm{OGW} - \bar{x}}{3 \cdot s}$$

C_pu	Capability index to the upper limit	1
OGW	Upper specification limit	µm
$\bar{x}$	Mean (arithmetic)	µm
s	Standard deviation	µm

The critical capability index C_pk also takes the location into account. It is the smaller of the two values C_pu and C_pl: the distance from the mean to the nearer limit, measured in units of three standard deviations.

Valid: As above. For the machine the smaller value is called C_mk.

EXAMPLE The pivots from the previous example measure 101 µm on average.

Given: $\mathrm{OGW} = 104\,\mu\mathrm{m}; \quad \bar{x} = 101\,\mu\mathrm{m}; \quad s = 0.8\,\mu\mathrm{m}$

$$\begin{aligned} C_\mathrm{pu} &= \frac{\mathrm{OGW} - \bar{x}}{3 \cdot s} \\ &= \frac{104\,\mu\mathrm{m} - 101\,\mu\mathrm{m}}{3 \cdot 0.8\,\mu\mathrm{m}} \\ &= 1.25 \end{aligned}$$

Towards the upper limit only 1.25 times half the spread width remains.

Critical capability index (lower limit)

(12.7) MASTER

$$C_{\mathrm{pl}} = \frac{\bar{x} - \mathrm{UGW}}{3 \cdot s}$$

If the mean lies exactly in the centre of the tolerance, both values are equal and as large as C_{p}.

C_{pl}	Capability index to the lower limit	1
$\bar{x}$	Mean (arithmetic)	µm
UGW	Lower specification limit	µm
s	Standard deviation	µm

EXAMPLE The same pivots, now calculated towards the lower limit.

Given: $\bar{x} = 101\,\mu\mathrm{m}$; $\mathrm{UGW} = 96\,\mu\mathrm{m}$; $s = 0.8\,\mu\mathrm{m}$

$$C_{\mathrm{pl}} = \frac{\bar{x} - \mathrm{UGW}}{3 \cdot s} = \frac{101\,\mu\mathrm{m} - 96\,\mu\mathrm{m}}{3 \cdot 0.8\,\mu\mathrm{m}} \approx 2.08$$

The smaller value applies: $C_{\mathrm{pk}} = 1.25$. The spread would be small enough ($C_{\mathrm{p}} = 1.67$), but the process lies 1 µm too high and does not meet the requirement of 1.33. If the machine is readjusted by 1 µm, $C_{\mathrm{pk}} = C_{\mathrm{p}} = 1.67$.

Capability indices: usual requirements (guide values)

C_{pk}	Distance to the nearer limit	Defect rate	Assessment
1.00	$3\,s$	0.27 %	conditionally capable; usually not sufficient
1.33	$4\,s$	63 ppm	capable; usual minimum requirement for the process
1.67	$5\,s$	0.6 ppm	usual minimum requirement for the machine (C_{mk})
2.00	$6\,s$	0.002 ppm	very safe

Defect rate for a centred, normally distributed process, both sides together. What has been agreed with the customer is decisive.

Key point: C_{p} measures the spread, C_{pk} spread and location. C_{pk} is never greater than C_{p}.

The control chart keeps a capable process under observation: at fixed intervals a small sample is measured and its mean is plotted. As long as the points scatter randomly about the centre line and stay within the control limits, the process is running undisturbed. The upper and lower control limits (UCL, LCL) usually lie three standard deviations of the mean, according to formula (12.4), above and below the centre line.

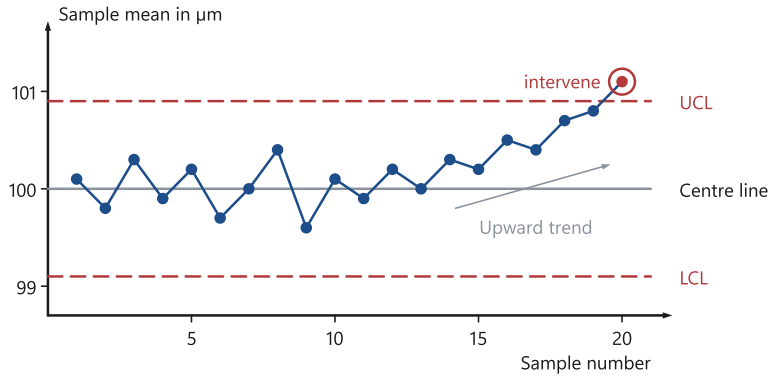


Fig. 12.4 Control chart (principle): sample means with centre line and control limits; a trend announces the disturbance

12.3 Defects and risk

Defect rates are given in per cent or, when they are small, in ppm: parts per million. The risk priority number of the FMEA (failure mode and effects analysis) ranks possible failures according to which must be eliminated first.

Defect rate

(12.8) JOURNEYMAN

$$q = \frac{n_\mathrm{F}}{n}$$

If only the unusable parts are counted, the proportion is called the scrap rate. If the parts that must be reworked are counted, it is called the rework rate.

q	Defect rate (proportion of defective parts)	ppm
n_F	Number of defective parts	1
n	Number (of inspected parts)	1

EXAMPLE Of 1200 spring bars delivered, 3 are too short. What is the defect rate?

Given: $n_\mathrm{F} = 3; \quad n = 1200$

$$\begin{aligned} q &= \frac{n_\mathrm{F}}{n} \\ &= \frac{3}{1200} \\ &= 0{,}0025 \\ &= 2500\,\mathrm{ppm} \end{aligned}$$

2500 ppm, that is 0.25 %.

First pass yield of an operation

(12.9) MASTER

$$\mathrm{FPY} = 1 - q_\mathrm{A} - q_\mathrm{N}$$

First pass yield counts only parts that are good without rework. Reworked parts are usable in the end, but they have cost time and money.

FPY	First pass yield (proportion of parts that are good on the first pass)	%
q_A	Scrap rate	%
q_N	Rework rate	%

EXAMPLE When turning arbors, 1.5 % are scrap and a further 2.5 % must be reworked.

Given: $q_\mathrm{A} = 1.5\,\%; \quad q_\mathrm{N} = 2.5\,\%$

$$\begin{aligned} \mathrm{FPY} &= 1 - q_\mathrm{A} - q_\mathrm{N} \\ &= 1 - 1.5\,\% - 2.5\,\% \\ &= 96\,\% \end{aligned}$$

96 % of the arbors are good first time.

First pass yield over several operations

(12.10) MASTER

$$\mathrm{FPY}_\mathrm{ges} = \mathrm{FPY}_1 \cdot \mathrm{FPY}_2 \cdot \mathrm{FPY}_3$$

A part is good first time only if it passes every operation without a defect. The first pass yields are therefore multiplied; with more operations, further factors are added.

Valid: Defects of the operations independent of one another.

$\mathrm{FPY}_\mathrm{ges}$	First pass yield over all operations	%
FPY_1	First pass yield of the first operation	%
FPY_2	First pass yield of the second operation	%
FPY_3	First pass yield of the third operation	%

EXAMPLE An arbor is turned (first pass yield 96 %), polished (98 %) and assembled (95 %).

Given: $\mathrm{FPY}_1 = 96\,\%; \quad \mathrm{FPY}_2 = 98\,\%; \quad \mathrm{FPY}_3 = 95\,\%$

$$\begin{aligned} \mathrm{FPY}_\mathrm{ges} &= \mathrm{FPY}_1 \cdot \mathrm{FPY}_2 \cdot \mathrm{FPY}_3 \\ &= 96\,\% \cdot 98\,\% \cdot 95\,\% \\ &\approx 89.4\,\% \end{aligned}$$

Only 89.4 % of the parts pass through without a defect, although each individual operation looks good. Every tenth part requires rework or is scrap.

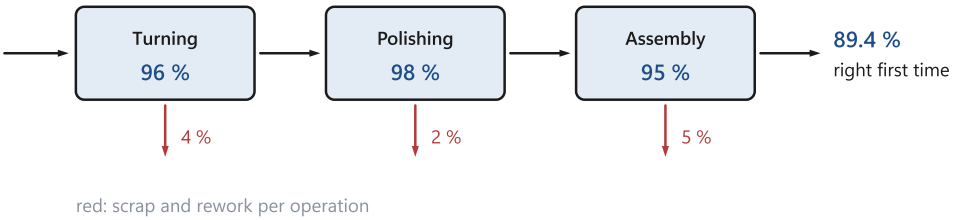


Fig. 12.5 First pass yield over three operations: $0.96 \cdot 0.98 \cdot 0.95 = 0.894$

Risk priority number (FMEA)

(12.11) MASTER

$$\mathrm{RPN} = B \cdot A \cdot E$$

RPN	Risk priority number (1 ... 1000)	1
B	Severity of the failure effect (1 ... 10)	1
A	Occurrence of the failure cause (1 ... 10)	1
E	Detection before delivery (1 ... 10)	1

Each possible failure receives three ratings from 1 (harmless, rare, certainly detected) to 10 (very serious, very frequent, undetectable). The larger the product, the more urgent the remedy.

Valid: Rating according to a fixed catalogue, always as a team and always by the same criteria; only then are the numbers comparable.

Caution: The RPN is a ranking, not a measured value. Failures with a very serious effect (B = 9 or 10) are always dealt with, even if the RPN is small. Newer FMEA standards therefore read the urgency from a table (action priority) instead of multiplying the three numbers.

EXAMPLE Possible failure: during a battery change the caseback gasket is pinched, and later water enters. Rating: severity 8 (movement destroyed), occurrence 3 (rare), detection 4 (visual inspection only).

Given: $B = 8; \quad A = 3; \quad E = 4$

$$\begin{aligned}\mathrm{RPN} &= B \cdot A \cdot E \\ &= 8 \cdot 3 \cdot 4 \\ &= 96\end{aligned}$$

RPN = 96. If every watch is tested for water resistance after closing, E falls to 1 and the RPN to 24.

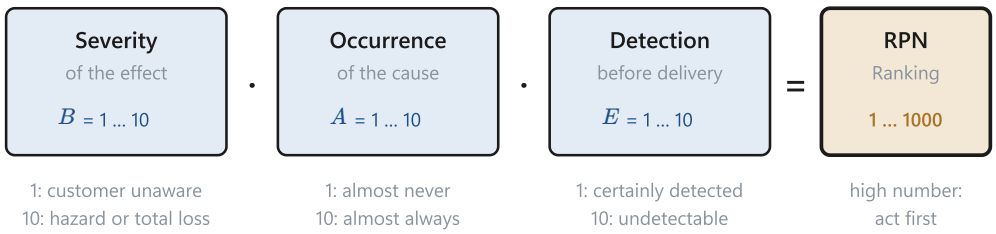


Fig. 12.6 Risk priority number of the FMEA: three ratings from 1 to 10 are multiplied

12.4 Measuring instruments and measuring conditions

A measured value is only as good as the measuring instrument and the conditions during measurement. Two questions come before every measurement: Is the measuring instrument fine enough for the tolerance? And are the workpiece and the measuring instrument at the reference temperature of 20 °C?

Golden rule of metrology

(12.12) JOURNEYMAN

$$U_{\max} = \frac{\mathrm{OGW} - \mathrm{UGW}}{10}$$

$U_{\max}$	Permissible measurement uncertainty of the measuring instrument	$\mu\mathrm{m}$
OGW	Upper specification limit	$\mu\mathrm{m}$
UGW	Lower specification limit	$\mu\mathrm{m}$

The measurement uncertainty should be at most one tenth of the tolerance. The tolerance then remains almost entirely available for production.

Valid: Rule of thumb; for very tight tolerances one fifth is allowed if necessary. The scale division or digit step must be at least as fine, because the measurement uncertainty is usually larger than the resolution.

EXAMPLE A pivot is to measure 0.100 mm ± 0.004 mm, so the specification limits are 96 µm and 104 µm. How accurate must the measuring instrument be?

Given: $\mathrm{OGW} = 104\,\mu\mathrm{m}$; $\mathrm{UGW} = 96\,\mu\mathrm{m}$

$$\begin{aligned} U_{\max} &= \frac{\mathrm{OGW} - \mathrm{UGW}}{10} \\ &= \frac{104\,\mu\mathrm{m} - 96\,\mu\mathrm{m}}{10} \\ &= 0.8\,\mu\mathrm{m} \end{aligned}$$

The measurement uncertainty may be at most 0.8 µm. An outside micrometer with 0.01 mm scale division is not sufficient; what is needed is an instrument that reliably measures to better than 1 µm.

Effect of temperature on measurement

(12.13) JOURNEYMAN

$$\Delta l = \alpha \cdot l \cdot \Delta\vartheta$$

Δl	Change in length	$\mu\mathrm{m}$
α	Coefficient of linear expansion	$\frac{1}{\mathrm{K}}$
l	Length (measured dimension)	mm
$\Delta\vartheta$	Temperature change (here: deviation from 20 °C)	K

Dimensions apply at 20 °C. A warmer workpiece measures too large, a colder one too small. This is linear expansion according to formula (2.55); the material values are given in Section 2.8.

Valid: Workpiece evenly warmed through, measuring instrument at 20 °C. If workpiece and measuring instrument have the same temperature and the same expansion coefficient, the error cancels out.

EXAMPLE A brass mainplate 30 mm in diameter ($\alpha = 19 \cdot 10^{-6}\,\frac{1}{\mathrm{K}}$) comes under the measuring instrument hand-warm at 30 °C, i.e. 10 K too warm.

Given: $\alpha = 19 \cdot 10^{-6}\,\frac{1}{\mathrm{K}}$; $l = 30\,\mathrm{mm}$; $\Delta\vartheta = 10\,\mathrm{K}$

$$\begin{aligned} \Delta l &= \alpha \cdot l \cdot \Delta\vartheta \\ &= 19 \cdot 10^{-6}\,\tfrac{1}{\mathrm{K}} \cdot 30\,\mathrm{mm} \cdot 10\,\mathrm{K} \\ &\approx 5.7\,\mu\mathrm{m} \end{aligned}$$

The mainplate measures 5.7 µm too large. That is more than the tolerance of some fits. Allow parts to reach room temperature before measuring and do not hold them in the hand longer than necessary.

Symbols

All symbols used in this formulary with their meaning, usual unit and chapter. The same symbol may mean different things in different chapters; the table of quantities given with each formula is always authoritative.

A	area	$\mathrm{mm^2}$	Ch. 1, 2
A	amplitude	$^\circ$, rad	Ch. 5
A	Amplitude	rad, $^\circ$	Ch. 6, 7, 8
A	Area	$\mathrm{cm^2}$, $\mathrm{mm^2}$, $\mathrm{m^2}$	Ch. 9, 10
A	Occurrence of the failure cause	1	Ch. 12
ΔA	Amplitude drop	$^\circ$	Ch. 8
a	side	mm	Ch. 1
a	semi-axis	mm	Ch. 1
a	leg	mm	Ch. 1
a	centre distance	mm	Ch. 1, 4
a	acceleration	$\frac{\mathrm{m}}{\mathrm{s^2}}$	Ch. 2
a	distance	mm	Ch. 2
a	Distance	m, mm	Ch. 6, 9
a	Centre distance	mm	Ch. 7
a	Proportion of time in horizontal positions	1	Ch. 8
a	Age of the Moon	d	Ch. 11
A_0	amplitude at the start	$^\circ$	Ch. 5
A_0	Amplitude fully wound	$^\circ$	Ch. 8
A_1	Amplitude before	rad	Ch. 6
A_2	Amplitude after	rad	Ch. 6
A_{24}	Amplitude after 24 hours of running	$^\circ$	Ch. 8
α	angle	$^\circ$	Ch. 1
α	central angle	$^\circ$	Ch. 1
α	angle of inclination	$^\circ$	Ch. 2
α	coefficient of linear expansion	$\frac{1}{\mathrm{K}}$	Ch. 2
α	pressure angle	$^\circ$	Ch. 4
α	Coefficient of linear expansion	$\frac{1}{\mathrm{K}}$	Ch. 6, 12
α	Taper angle	$^\circ$	Ch. 9
α	Bend angle	$^\circ$	Ch. 9
α_1	Coefficient of linear expansion of the parts acting downwards	$\frac{1}{\mathrm{K}}$	Ch. 6
α_2	Coefficient of linear expansion of the parts acting upwards	$\frac{1}{\mathrm{K}}$	Ch. 6
α_{20}	Temperature coefficient of resistance	$\frac{1}{\mathrm{K}}$	Ch. 10
α_S	coefficient of linear expansion of the balance spring	$\frac{1}{\mathrm{K}}$	Ch. 5
α_U	coefficient of linear expansion of the balance	$\frac{1}{\mathrm{K}}$	Ch. 5
a_max	Distance between the bearing faces	mm	Ch. 9
a_min	Distance between the bearing faces	mm	Ch. 9
A_N	amplitude after N_S oscillations	$^\circ$	Ch. 5
a_p	Depth of cut	mm	Ch. 9
a_U	Centre distance of pallet fork and balance	mm	Ch. 7
A_Z	Amplitude displayed by the timing machine	$^\circ$	Ch. 8
B	Magnetic flux density	$\frac{\mathrm{V \cdot s}}{\mathrm{m^2}}$	Ch. 8

B	Annual angle	$^\circ$	Ch. 11
B	Severity of the failure effect	1	Ch. 12
b	semi-axis	mm	Ch. 1
b	leg	mm	Ch. 1
b	side	mm	Ch. 1
b	blade height	mm	Ch. 2, 3, 5
b	blows at each half hour	1	Ch. 4
b_1	blade height of the original spring	mm	Ch. 3
b_2	blade height of the replacement spring	mm	Ch. 3
β	angle	$^\circ$	Ch. 1, 5
β	wedge angle	$^\circ$	Ch. 2
β	Temperature coefficient of the crystal	$\frac{\mathrm{ppm}}{\mathrm{K^2}}$	Ch. 10
β_B	Travel of the pallet fork from banking to banking	$^\circ$	Ch. 7
β_E	temperature coefficient of the modulus of elasticity of the balance spring	$\frac{1}{\mathrm{K}}$	Ch. 5
β_G	Angle of the pallet fork while the impulse pin is in the fork	$^\circ$	Ch. 7
β_H	Impulse at the pallet fork	$^\circ$	Ch. 7
β_H	Impulse at the anchor	$^\circ$	Ch. 7
β_R	Lock	$^\circ$	Ch. 7
β_V	Run	$^\circ$	Ch. 7
b_K	Width of the pallet, measured at the circumference of the wheel	mm	Ch. 7
C	temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d \cdot K}}$	Ch. 5
C	Temperature coefficient of the rate	$\frac{\mathrm{s}}{\mathrm{d \cdot K}}$	Ch. 8
C	Taper ratio	1	Ch. 9
C	Capacitance	F	Ch. 10
c	side	mm	Ch. 1
c	hypotenuse	mm	Ch. 1
c	spring rate	$\frac{\mathrm{N}}{\mathrm{mm}}$	Ch. 2
c	tip clearance between wheel tip and pinion root	mm	Ch. 4
c	Speed of sound	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 11
C_1	First partial capacitance	F	Ch. 10
C_2	Second partial capacitance	F	Ch. 10
C_3	Third partial capacitance	F	Ch. 10
C_N	Nominal capacity of the battery	mAh	Ch. 10
C_p	Process capability index	1	Ch. 12
C_pl	Capability index to the lower limit	1	Ch. 12
C_pu	Capability index to the upper limit	1	Ch. 12
D	diameter	mm	Ch. 1
D	restoring torque coefficient	$\frac{\mathrm{N \cdot m}}{\mathrm{rad}}$, $\frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$	Ch. 2, 5
D	Restoring torque coefficient	$\frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$	Ch. 7
D	Vertical–horizontal difference	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
D	Diameter	mm	Ch. 9
ΔD	change in restoring torque coefficient	$\frac{\mu\mathrm{N \cdot m}}{\mathrm{rad}}$	Ch. 5
d	diameter	mm	Ch. 1, 2, 3
d	pitch circle diameter	mm	Ch. 4
d	Diameter	mm	Ch. 8, 9
d	Distance	m	Ch. 10

d'_{a}	tip circle diameter of the pinion	mm	Ch. 4
d'_{f}	root circle diameter of the pinion	mm	Ch. 4
d_2	Pitch diameter	mm	Ch. 9
d_3	Minor diameter	mm	Ch. 9
d_{a}	tip circle diameter	mm	Ch. 4
d_{a}	diameter of the outermost coil	mm	Ch. 5
d_{a}	Outside diameter of the cylinder	mm	Ch. 7
d_{b}	base circle diameter	mm	Ch. 4
d_{f}	root circle diameter	mm	Ch. 4
d_{i}	diameter of the innermost coil	mm	Ch. 5
d_{i}	Diameter	mm	Ch. 9
d_{K}	Tapping drill diameter	mm	Ch. 9
d_{m}	measured dimension between the measuring faces	mm	Ch. 4
d_{S}	thickness of the cable or gut line	mm	Ch. 3
d_{w}	effective barrel diameter	mm	Ch. 3
E	modulus of elasticity	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2, 3, 5
E	Equation of time	min	Ch. 11
E	Detection before delivery	1	Ch. 12
e	across-corners dimension	mm	Ch. 1
e	terminal radius	mm	Ch. 5
e	Tailstock offset	mm	Ch. 9
E_{F}	energy of the tensioned spring	mJ, J	Ch. 2
E_{kin}	kinetic energy	J	Ch. 2
E_{pot}	potential energy	J	Ch. 2
E_{rot}	rotational energy	J	Ch. 2
η	efficiency	1	Ch. 2, 3, 4
η	Efficiency	1	Ch. 7
η	Current efficiency	1	Ch. 9
η_1	efficiency of the first stage	1	Ch. 2
η_2	efficiency of the second stage	1	Ch. 2
η_3	efficiency of the third stage	1	Ch. 2
η_{F}	efficiency of the spring in the barrel	1	Ch. 3
η_{ges}	overall efficiency	1	Ch. 2
η_{ges}	overall efficiency of the gear train	1	Ch. 4
F	force	N	Ch. 2
F	Force	N	Ch. 8
F	Faraday constant	$\frac{\mathrm{C}}{\mathrm{mol}}$	Ch. 9
Δf	Frequency deviation	Hz	Ch. 10
f	frequency	Hz	Ch. 1, 5
f	Frequency	Hz	Ch. 8, 10
f	Feed per revolution	mm	Ch. 9
F_1	first force	N	Ch. 2
F_2	second force	N	Ch. 2
f_{aus}	Frequency at the divider output	Hz	Ch. 10
F_{G}	weight force	N	Ch. 2
F_{H}	downhill force	N	Ch. 2

f_{ist}	measured frequency	Hz	Ch. 5
f_{M}	Frequency of the motor pulses	Hz	Ch. 10
F_{N}	normal force	N	Ch. 2
f_{P}	Pulse rate	$\frac{1}{\mathrm{min}}$	Ch. 11
FPY	First pass yield	$\%$	Ch. 12
FPY_{1}	First pass yield of the first operation	$\%$	Ch. 12
FPY_{2}	First pass yield of the second operation	$\%$	Ch. 12
FPY_{3}	First pass yield of the third operation	$\%$	Ch. 12
$\mathrm{FPY}_{\mathrm{ges}}$	First pass yield over all operations	$\%$	Ch. 12
F_{Q}	load perpendicular to the base	N	Ch. 2
F_{R}	resultant	N	Ch. 2
F_{R}	friction force	N	Ch. 2
F_{R0}	maximum static friction force	N	Ch. 2
f_{soll}	nominal frequency	Hz	Ch. 5
f_{z}	Feed per tooth	mm	Ch. 9
G	rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 1, 5
G	shear modulus	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
G	Rate of the clock in seconds per day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6
G	Rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8, 10, 11
ΔG	rate change	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 5
ΔG	Rate change	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 6, 10
ΔG	Isochronism error	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
Δg	Change in gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 6
$\bar{G}$	Mean rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
g	gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 2, 3, 5
g	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 6, 8
g	Gravitational acceleration at the old location	$\frac{\mathrm{m}}{\mathrm{s}^2}$	Ch. 6
G_{0}	Rate fully wound	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{1}	rate at the lower temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 5
G_{1}	Rate on the first test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{15}	Rate on the last test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{2}	rate at the higher temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 5
G_{2}	Rate on the second test day	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{24}	Rate after 24 hours of running	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{38}	Rate at 38 °C	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{8}	Rate at 8 °C	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
γ	angle	$^\circ$	Ch. 1, 2
γ	coefficient of volume expansion	$\frac{1}{\mathrm{K}}$	Ch. 2
γ	Half span angle	$^\circ$	Ch. 7
γ_{F}	Drop	$^\circ, \mathrm{rad}$	Ch. 7
γ_{H}	Impulse at the wheel	$^\circ$	Ch. 7
G_{f}	Mean rate of the horizontal positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{h}	Mean rate of the vertical positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
ΔG_{L}	Greatest positional variation	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{m}	rate at the middle temperature	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 5
G_{max}	Highest rate of all positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8

$G_{\min}$	Lowest rate of all positions	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{o}	Upper limit of size	mm	Ch. 9
G_{oB}	Upper limit of size	mm	Ch. 9
G_{oW}	Upper limit of size	mm	Ch. 9
G_{T}	Expected rate when worn	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
G_{uB}	Lower limit of size	mm	Ch. 9
G_{uW}	Lower limit of size	mm	Ch. 9
ΔG_{W}	Resumption of rate	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
H	Magnetic field strength	$\frac{\mathrm{A}}{\mathrm{m}}$	Ch. 8
h	height of arc	mm	Ch. 1
h	height	mm, m	Ch. 1, 2, 3
h	Altitude	m	Ch. 6
h	Height	m	Ch. 8
h_a	height	mm	Ch. 1
I	second moment of area	mm^4	Ch. 2
I	Current	A, mA	Ch. 9, 10
i	gear ratio	1	Ch. 4
i	Number	1	Ch. 9
i	Ratio	1	Ch. 9
i_{G}	Ratio in the central position	1	Ch. 7
i_{ges}	overall gear ratio	1	Ch. 4
I_{IC}	Current consumption of the circuit including oscillator	$\mu\mathrm{A}$	Ch. 10
I_{Imp}	Current through the coil during the pulse	mA	Ch. 10
I_{m}	Mean current consumption of the movement	$\mu\mathrm{A}$	Ch. 10
i_{R}	reduction ratio of automatic winding	1	Ch. 3
i_{U}	Reduction ratio	1	Ch. 10
i_{W}	gear ratio from pin wheel to fly	1	Ch. 4
J	moment of inertia	$\mathrm{kg{\cdot}m^2}$, $\mathrm{mg{\cdot}cm^2}$	Ch. 2, 5
ΔJ	change in moment of inertia	$\mathrm{mg{\cdot}cm^2}$	Ch. 5
j	backlash on the pitch circle	mm	Ch. 4
j	Current density	$\frac{\mathrm{A}}{\mathrm{dm^2}}$	Ch. 9
J_{A}	moment of inertia about the axis of rotation	$\mathrm{mg{\cdot}cm^2}$	Ch. 2
J_{A}	Moment of inertia about the pivot axis	$\mathrm{kg{\cdot}m^2}$	Ch. 6
J_{S}	moment of inertia about the parallel axis through the centre of gravity	$\mathrm{mg{\cdot}cm^2}$	Ch. 2
J_{S}	Moment of inertia about the parallel axis through the centre of gravity	$\mathrm{kg{\cdot}m^2}$	Ch. 6
J_{Z}	known moment of inertia of the attached disc	$\mathrm{mg{\cdot}cm^2}$	Ch. 5
K	CGS value of the balance spring	$\frac{\mathrm{N{\cdot}mm^3}}{\mathrm{rad}}$	Ch. 5
K	Carat number	1	Ch. 9
k	number	1	Ch. 3, 4
k	addendum coefficient	1	Ch. 4
k	last hour struck	1	Ch. 4
k	Span	1	Ch. 7
k	Correction factor	1	Ch. 9
k	Number	1	Ch. 11
k'	addendum coefficient of the pinion	1	Ch. 4

k_{A}	amplitude factor of the out-of-poise	1	Ch. 5
k_{F}	filling ratio: share of the free area covered by the spring	1	Ch. 3
L	Length	mm	Ch. 9
Δl	change	mm	Ch. 2, 5
Δl	Change	mm, $\mu\mathrm{m}$	Ch. 6, 12
Δl	Bob travel	mm	Ch. 6
l	length	mm	Ch. 2, 3, 5
l	Pendulum length	m, mm	Ch. 6
l	Length	mm, m	Ch. 9, 10, 12
l_0	original length	mm	Ch. 2
l_1	first lever arm	mm	Ch. 2
l_1	effective length in the trial	mm	Ch. 5
l_1	Total length of the parts acting downwards	m	Ch. 6
l_2	second lever arm	mm	Ch. 2
l_2	required effective length	mm	Ch. 5
l_2	Total length of the parts acting upwards	m	Ch. 6
λ	Lift angle	$^\circ$	Ch. 7, 8
λ	Geographical longitude of the place	$^\circ$	Ch. 11
λ_{Z}	Lift angle set on the timing machine	$^\circ$	Ch. 8
λ_{Z}	Geographical longitude of the zone meridian	$^\circ$	Ch. 11
l_{B}	arc length	mm	Ch. 1
l_{B}	Length	mm	Ch. 9
l_{G}	Fork length	mm	Ch. 7
$l_{\max}$	Length	mm	Ch. 9
$l_{\min}$	Length	mm	Ch. 9
l_{P}	Distance between the two contact points	mm	Ch. 7
l_{r}	Equivalent pendulum length	m	Ch. 6
L_{W}	Length	mm	Ch. 9
l_{Z}	Length of a tooth	mm	Ch. 7
M	torque	$\mathrm{N{\cdot}mm}$, $\mathrm{N{\cdot}m}$, $\mu\mathrm{N{\cdot}m}$	Ch. 2, 3, 4, 5
M	Molar mass of the deposited metal	$\frac{\mathrm{g}}{\mathrm{mol}}$	Ch. 9
Δm	Mass of the timing weight	g	Ch. 6
m	mass	g, kg, mg	Ch. 1, 2, 3, 5
m	module	mm	Ch. 4
m	Mass	kg, g, mg	Ch. 6, 9
M'	torque at the arbor of the driven pinion	$\mathrm{N{\cdot}mm}$	Ch. 4
M_1	torque of the original spring	$\mathrm{N{\cdot}mm}$	Ch. 3
m_1	module of the cannon pinion and minute wheel mesh	mm	Ch. 4
m_1	Mass	g	Ch. 9
M_2	torque of the replacement spring	$\mathrm{N{\cdot}mm}$	Ch. 3
m_2	module of the minute pinion and hour wheel mesh	mm	Ch. 4
m_2	Mass	g	Ch. 9
ei	Lower deviation	mm	Ch. 9
es	Upper deviation	mm	Ch. 9
M_{b}	bending moment	$\mathrm{N{\cdot}mm}$	Ch. 2

M_{F}	torque of the barrel at the same moment	$\mathrm{N{\cdot}mm}$	Ch. 3
M_{F}	torque at the barrel	$\mathrm{N{\cdot}mm}$	Ch. 4
m_{f}	Fine weight	g	Ch. 9
M_{H}	torque at the escape wheel	$\mathrm{\mu N{\cdot}m}$	Ch. 4
M_{H}	Torque at the escape wheel	$\mathrm{\mu N{\cdot}m}$	Ch. 7
m_{L}	mass	g	Ch. 1
M_{m}	mean torque over these turns	$\mathrm{N{\cdot}mm}$	Ch. 3
M_{max}	maximum torque transmissible elastically	$\mathrm{N{\cdot}mm}$	Ch. 3
M_{R}	friction torque	$\mathrm{N{\cdot}mm}$	Ch. 2
M_{S}	torque at the fusee	$\mathrm{N{\cdot}mm}$	Ch. 3
m_{S}	mass of one screw	mg	Ch. 5
M_{t}	torsional moment	$\mathrm{N{\cdot}mm}$	Ch. 2
μ	coefficient of friction	1	Ch. 2
μ	Coefficient of friction	1	Ch. 7
m_{u}	excess weight	$\mathrm{\mu g}$	Ch. 5
μ_0	coefficient of static friction	1	Ch. 2
μ_0	Magnetic constant	$\frac{\mathrm{V{\cdot}s}}{\mathrm{A{\cdot}m}}$	Ch. 8
m_{W}	reading of the balance when the part hangs fully immersed	g	Ch. 1
m_{zu}	Mass	g	Ch. 9
N	turns	1	Ch. 3
N	number	1	Ch. 4, 5
N	Turns	1	Ch. 6
N	Nominal size	mm	Ch. 9
N	Number	1	Ch. 10
N	Day number in the year	1	Ch. 11
n	number	1	Ch. 1
n	rotational speed	$\frac{1}{\mathrm{min}}, \frac{1}{\mathrm{h}}$	Ch. 2, 4
n	Number	1	Ch. 8, 10, 12
n	Spindle speed	$\frac{1}{\mathrm{min}}$	Ch. 9
n'	rotational speed of the driven pinion	$\frac{1}{\mathrm{h}}$	Ch. 4
n_1	beat rate in the trial	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 5
N_{12}	hour blows in 12 hours	1	Ch. 4
n_2	beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 5
N_{A}	blows per revolution of the driving wheel	1	Ch. 4
N_{ab}	coils of the run-down spring	1	Ch. 3
N_{auf}	coils of the wound spring	1	Ch. 3
N_{d}	blows in 24 hours	1	Ch. 4
N_{d}	Number of days in the cycle	1	Ch. 11
n_{d}	blows per day	$\frac{1}{\mathrm{d}}$	Ch. 4
N_{E}	terminal curve number	1	Ch. 5
n_{F}	Number of years until the calendar deviates by one day from the solar year	1	Ch. 11
n_{F}	Number of defective parts	1	Ch. 12
n_{G}	Number of threads	$\frac{1}{\mathrm{in}}$	Ch. 9
n_{H}	rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$	Ch. 4
n_{H}	Rotational speed of the escape wheel	$\frac{1}{\mathrm{min}}$	Ch. 7
n_{h}	beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 1, 4, 5

n_h	Beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 6, 7
n_h	Beat rate in beats per minute	$\frac{1}{\mathrm{min}}$	Ch. 6
n_ist	Measured beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 8
N_K	revolutions of the crown	1	Ch. 3
n_K	Crank turns per indexing step	1	Ch. 9
N_L	number of lignes	1	Ch. 1
N_L	Number of holes in the hole circle	1	Ch. 9
n_L	Hole spaces per indexing step	1	Ch. 9
N_M	revolutions of the minute hand	1	Ch. 3
n_M	rotational speed of the centre wheel	$\frac{1}{\mathrm{h}}$	Ch. 4
N_max	maximum possible turns of the barrel	1	Ch. 3
N_P	Number of pulse beats counted	1	Ch. 11
N_R	revolutions of the rotor	1	Ch. 3
n_R	Rotational speed	$\frac{1}{\mathrm{min}}$	Ch. 10
N_S	number of full oscillations	1	Ch. 5
n_S	rotational speed of the fourth wheel	$\frac{1}{\mathrm{min}}$	Ch. 4
n_S	Rotational speed of the fourth wheel	$\frac{1}{\mathrm{min}}$	Ch. 10
n_S	Number of leap years in the cycle	1	Ch. 11
N_Sch	blows per revolution of the count wheel	1	Ch. 4
n_soll	Nominal beat rate	$\frac{\mathrm{A}}{\mathrm{h}}$	Ch. 8
ν	safety factor	1	Ch. 2
N_W	revolutions of the barrel	1	Ch. 3
n_Z	Number of years in the cycle	1	Ch. 11
OGW	Upper specification limit	$\mu\mathrm{m}$	Ch. 12
ω	angular velocity	$\frac{\mathrm{rad}}{\mathrm{s}}$	Ch. 2
ω	angular frequency	$\frac{\mathrm{rad}}{\mathrm{s}}$	Ch. 5
ω_max	angular velocity of the balance when passing through the rest position	$\frac{\mathrm{rad}}{\mathrm{s}}$	Ch. 5
$\overline{\Delta G}$	Mean variation	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
P	power	W, $\mu\mathrm{W}$	Ch. 2, 3, 5
P	Thread pitch	mm	Ch. 6, 9
P	Power	W	Ch. 10
p	relative deviation	$\%$	Ch. 1
p	pressure	bar	Ch. 2
p	bearing pressure	$\frac{\mathrm{N}}{\mathrm{mm^2}}$	Ch. 2
p	circular pitch	mm	Ch. 4
p	coil pitch	mm	Ch. 5
p	Pressure	bar	Ch. 8
P_ab	output power	W	Ch. 2
p_dyn	Dynamic pressure	bar	Ch. 8
P_Imax	Maximum interference	mm	Ch. 9
P_Imin	Minimum interference	mm	Ch. 9
P_SH	Maximum clearance	mm	Ch. 9
P_SM	Minimum clearance	mm	Ch. 9
P_U	power reaching the balance or pendulum	$\mu\mathrm{W}$	Ch. 3
P_zu	input power	W	Ch. 2
Q	quality factor	1	Ch. 5

Q	Quality factor	1	Ch. 7
Q	Metal removal rate	$\frac{\mathrm{mm}^3}{\mathrm{min}}$	Ch. 9
Q	Charge	C	Ch. 10
q	Defect rate	ppm	Ch. 12
q_A	Scrap rate	$\%$	Ch. 12
q_N	Rework rate	$\%$	Ch. 12
R	inner radius	mm	Ch. 3
R	Radius	mm	Ch. 7
R	Resistance	Ω	Ch. 10
R	Range	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 12
Δr	shift of the centre of gravity	mm	Ch. 5
r	radius	mm, m	Ch. 1, 2, 3, 5
r	lever arm	mm	Ch. 2
r	distance	mm	Ch. 2
r	Bend radius	mm	Ch. 9
R_1	First partial resistance	Ω	Ch. 10
R_2	Second partial resistance	Ω	Ch. 10
R_{20}	Resistance at 20 °C	Ω	Ch. 10
R_3	Third partial resistance	Ω	Ch. 10
r_A	Pallet arm	mm	Ch. 7
r_a	outer radius	mm	Ch. 2
r_a	radius of the outermost coil	mm	Ch. 5
R_e	yield strength	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
r_E	Radius of the impulse pin about the balance arbor	mm	Ch. 7
r_F	radius of the barrel	mm	Ch. 3
ρ	angle of friction	$^\circ$	Ch. 2
ρ	radius	mm	Ch. 3
ρ	Angle of friction	$^\circ$	Ch. 7
ρ	Resistivity of the material	$\frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}}$	Ch. 10
R_i	Internal resistance of the battery	Ω	Ch. 10
r_i	inner radius	mm	Ch. 2
r_i	inner radius of the run-down spring pack	mm	Ch. 3
r_i	radius of the innermost coil	mm	Ch. 5
R_L	Load resistance	Ω	Ch. 10
RPN	Risk priority number	1	Ch. 12
r_S	effective radius of the fusee at the point where the chain runs off	mm	Ch. 3
r_S	distance of its centre of gravity from the balance axis	mm	Ch. 5
r_u	distance of the excess weight from the balance axis	mm	Ch. 5
R_ϑ	Resistance at temperature ϑ	Ω	Ch. 10
S	middle temperature error	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 5
S	Time error	s	Ch. 8
ΔS	Change in time error	s	Ch. 8
Δs	change in blade thickness	$\mu\mathrm{m}$	Ch. 5
s	chord	mm	Ch. 1
s	width across flats	mm	Ch. 1
s	distance	m	Ch. 2

s	spring deflection	mm	Ch. 2
s	blade thickness	mm, $\mu\mathrm{m}$	Ch. 2, 3, 5
s	tooth thickness	mm	Ch. 4
s	Thickness	mm, $\mu\mathrm{m}$	Ch. 9
s	Distance	km, m	Ch. 11
s	Standard deviation	$\frac{\mathrm{s}}{\mathrm{d}}$, $\mu\mathrm{m}$	Ch. 12
s'	leaf thickness of the pinion on the pitch circle	mm	Ch. 4
S_1	Time error at the first reading	s	Ch. 8
s_1	blade thickness of the original spring	mm	Ch. 3
S_2	Time error at the second reading	s	Ch. 8
s_2	blade thickness of the replacement spring	mm	Ch. 3
s_E	Self-discharge	$\frac{\%}{\mathrm{a}}$	Ch. 10
s_F	Drop as travel	mm	Ch. 7
S_h	End shake	mm	Ch. 9
$S_\mathrm{h,max}$	End shake	mm	Ch. 9
$S_\mathrm{h,min}$	End shake	mm	Ch. 9
σ	stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
σ_b	maximum bending stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
σ_zul	permissible stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
σ_zul	permissible bending stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 3
$\sum(x_i - \bar{x})^2$	Sum of the squared deviations from the mean	$\frac{\mathrm{s}^2}{\mathrm{d}^2}$	Ch. 12
$\sum G_i$	Sum of the individual rates, with sign	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
$\sum \lvert G_{2k} - G_{2k-1} \rvert$	Sum of the five absolute values: difference between the two daily rates in each position	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
$\sum x_i$	sum of all measured values	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 1
$\sum x_i$	Sum of all measured values	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 12
$s_{\bar{x}}$	Standard deviation of the mean	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 12
T	period	s	Ch. 5
T	Period	s	Ch. 6
T	Tolerance	mm	Ch. 9
Δt	time interval	s	Ch. 2
Δt	deviation of the display from the true age of the moon	d	Ch. 4
Δt	Time shift of the beats relative to the reference frequency during the measuring time	ms	Ch. 8
Δt	Local time difference	min	Ch. 11
t	time	h, s	Ch. 1, 2, 5
t	Time	d, s, h	Ch. 8, 9, 10, 11
t'_U	rotation period of the pinion	s	Ch. 4
T_0	Period at very small amplitude)	s	Ch. 6
T_1	period without disc	s	Ch. 5
t_1	Time of the first reading	d	Ch. 8
t_1	Duration of one vibration	ms	Ch. 8
T_2	period with disc	s	Ch. 5
t_2	Time of the second reading	d	Ch. 8
t_2	Duration of the following vibration	ms	Ch. 8
T_a	Tolerance	mm	Ch. 9

t_{A}	rotation period of the driving arbor	d	Ch. 4
t_{A}	Beat error	ms	Ch. 8
τ	shear stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
τ	Angular pitch	$^\circ$	Ch. 7
τ	Indexing angle	$^\circ$	Ch. 9
τ	Time constant	s	Ch. 10
τ_{t}	maximum torsional stress	$\frac{\mathrm{N}}{\mathrm{mm}^2}$	Ch. 2
t_{dez}	Time in minutes as a decimal number	min	Ch. 11
T_{E}	Sidereal year	d	Ch. 11
t_{F}	rotation period of the barrel	h	Ch. 3
t_{F}	running time until the display deviates by Δt	d	Ch. 4
t_{G}	running time	h, d	Ch. 3, 4
t_{H}	Lift time	ms	Ch. 8
t_{h}	whole hours	h	Ch. 1
t_{h}	Machining time	min	Ch. 9
t_{I}	Inhibition period	s	Ch. 10
t_{Imp}	Pulse duration	ms	Ch. 10
T_{J}	Tropical year	d	Ch. 11
T_{K}	Mean length of the calendar year	d	Ch. 11
T_l	Tolerance	mm	Ch. 9
t_{L}	displayed duration of one lunation	d	Ch. 4
T_{Luft}	Tolerance	mm	Ch. 9
T_{M}	Period of the motor pulses	s	Ch. 10
t_{M}	rotation period of the centre wheel	h	Ch. 3
t_{M}	Measuring time	s	Ch. 8
t_{M}	Mean local time	min	Ch. 11
t_{min}	minutes	min	Ch. 1
t_{N}	Reference time	s	Ch. 8
t_{R}	power reserve	h	Ch. 3
T_{S}	beat duration	s	Ch. 1
T_{S}	Beat duration	s	Ch. 6
t_{s}	seconds	s	Ch. 1
t_{s}	time between two advances	d	Ch. 4
T_{sid}	Sidereal month	d	Ch. 11
T_{So}	Mean solar day	s	Ch. 11
T_{St}	Sidereal day	s	Ch. 11
T_{syn}	synodic month	d	Ch. 4
T_{syn}	Synodic month	d	Ch. 11
t_{U}	rotation period of the driving arbor	h	Ch. 3
t_{U}	rotation period of the driving wheel	s	Ch. 4
t_{U}	Reading of the watch	s	Ch. 8
t_{W}	rotation period of the barrel	h	Ch. 3
t_{W}	Apparent local time	min	Ch. 11
t_{Z}	Zone time	min	Ch. 11
U	circumference	mm	Ch. 1
U	required revolutions of the driving wheel	1	Ch. 4

U	Voltage	V	Ch. 10
U_0	Source voltage	V	Ch. 10
U_2	Partial voltage across resistor R_2	V	Ch. 10
u_C	Capacitor voltage at time t	V	Ch. 10
u_G	Maximum error of the rate due to reading	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 8
UGW	Lower specification limit	$\mu\mathrm{m}$	Ch. 12
U_max	Permissible measurement uncertainty of the measuring instrument	$\mu\mathrm{m}$	Ch. 12
U_N	Nominal voltage of the battery	V	Ch. 10
u_S	Maximum reading error of one time error	s	Ch. 8
U_W	revolutions of the fly per blow	1	Ch. 4
V	volume	$\mathrm{mm^3}$, $\mathrm{cm^3}$	Ch. 1
ΔV	change	$\mathrm{cm^3}$	Ch. 2
Δv	change	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 2
v	speed	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 2
v	peripheral speed	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 2
v	Velocity	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 8
v	Speed	$\frac{\mathrm{km}}{\mathrm{h}}$	Ch. 11
V_0	original volume	$\mathrm{cm^3}$	Ch. 2
v_0	initial speed	$\frac{\mathrm{m}}{\mathrm{s}}$	Ch. 2
ε	strain	1	Ch. 2
ε	Angle	$^\circ$	Ch. 8
ε_0	Electric constant, $8.854 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}$	$\frac{\mathrm{F}}{\mathrm{m}}$	Ch. 10
ε_r	Relative permittivity of the insulator	1	Ch. 10
φ	angle	$^\circ$, rad	Ch. 1, 5
φ	angle of rotation	rad	Ch. 2
φ	angle of twist	rad	Ch. 2, 3
φ	Geographic latitude	$^\circ$	Ch. 6
φ	Lead angle	$^\circ$	Ch. 9
φ_E	Supplementary arc	$^\circ$	Ch. 7
φ_S	Step angle of the rotor	$^\circ$	Ch. 10
ϱ	density	$\frac{\mathrm{g}}{\mathrm{cm^3}}$	Ch. 1
ϱ	Density	$\frac{\mathrm{kg}}{\mathrm{m^3}}$, $\frac{\mathrm{g}}{\mathrm{cm^3}}$	Ch. 8, 9
ϱ_W	density of water	$\frac{\mathrm{g}}{\mathrm{cm^3}}$	Ch. 1
$\Delta\vartheta$	change	K	Ch. 2
$\Delta\vartheta$	Temperature change	K	Ch. 6, 10, 12
ϑ	Temperature	$^\circ\mathrm{C}$	Ch. 10, 11
ϑ_0	Turnover temperature	$^\circ\mathrm{C}$	Ch. 10
ϑ_1	lower test temperature	$^\circ\mathrm{C}$	Ch. 5
ϑ_2	higher test temperature	$^\circ\mathrm{C}$	Ch. 5
v_c	Cutting speed	$\frac{\mathrm{m}}{\mathrm{min}}$	Ch. 9
v_F	Proportion of the wheel's work lost in the drop	$\%$	Ch. 7
v_f	Feed rate	$\frac{\mathrm{mm}}{\mathrm{min}}$	Ch. 9
W	work	J, mJ	Ch. 2, 3
W	energy	$\mu\mathrm{J}$	Ch. 5
W	Energy	J	Ch. 10
ΔW	energy loss per full oscillation	$\mu\mathrm{J}$	Ch. 5

w	deflection at the free end	mm	Ch. 2
w	Fineness	‰	Ch. 9
w_1	Fineness	‰	Ch. 9
w_2	Fineness	‰	Ch. 9
W_{ab}	work delivered while running down	mJ	Ch. 3
W_{auf}	work expended	mJ	Ch. 3
W_{b}	section modulus	mm^3	Ch. 2
$\widehat{\alpha}$	angle	rad	Ch. 1
W_{max}	maximum elastically storable energy	mJ	Ch. 3
W_{S}	Work of the escape wheel per beat	$\mu\mathrm{J}$	Ch. 7
W_{U}	Work delivered to the balance per beat	$\mu\mathrm{J}$	Ch. 7
$\bar{x}$	mean value	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 1
$\bar{x}$	Mean	$\frac{\mathrm{s}}{\mathrm{d}}$, $\mu\mathrm{m}$	Ch. 12
x	relative deviation	ppm	Ch. 1
x	coordinate	mm	Ch. 1
x	Distance	mm	Ch. 6
x_1	coordinate	mm	Ch. 1
x_2	coordinate	mm	Ch. 1
x_{ist}	actual value	mm	Ch. 1
x_{max}	Largest measured value	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 12
x_{min}	Smallest measured value	$\frac{\mathrm{s}}{\mathrm{d}}$	Ch. 12
x_{soll}	nominal value	mm	Ch. 1
y	coordinate	mm	Ch. 1
y_1	coordinate	mm	Ch. 1
y_2	coordinate	mm	Ch. 1
z	number of teeth	1	Ch. 4
z	Number of teeth	1	Ch. 7, 9
z	Number of divisions	1	Ch. 9
z	Valency	1	Ch. 9
z'	number of leaves of the pinion	1	Ch. 4
z'_1	number of teeth of the pinion on the first reduction wheel	1	Ch. 3
z'_1	number of leaves of the first driven pinion	1	Ch. 4
z'_2	number of teeth of the pinion on the second reduction wheel	1	Ch. 3
z'_2	number of leaves of the second driven pinion	1	Ch. 4
z'_3	number of leaves of the third driven pinion	1	Ch. 4
z'_{A}	number of teeth of the winding pinion	1	Ch. 3
z'_{B}	number of teeth of the intermediate pinion	1	Ch. 3
z'_{H}	number of leaves of the escape pinion	1	Ch. 4
z'_{K}	number of teeth of the sliding pinion	1	Ch. 3
z'_{K}	number of leaves of the third pinion	1	Ch. 4
z'_{M}	number of teeth of the centre pinion	1	Ch. 3
z'_{N}	number of leaves of the pinion on the pin-wheel arbor	1	Ch. 4
z'_{R}	number of teeth of the rotor pinion	1	Ch. 3
z'_{S}	number of leaves of the fourth pinion	1	Ch. 4
z'_{Sch}	number of leaves of the pinion on the pin-wheel arbor that drives the count wheel	1	Ch. 4
z'_{W}	number of leaves of the minute pinion	1	Ch. 4

z_1	number of teeth of the first reduction wheel	1	Ch. 3
z_1	number of teeth of the first driving wheel	1	Ch. 4
z_2	number of teeth of the second reduction wheel	1	Ch. 3
z_2	number of teeth of the second driving wheel	1	Ch. 4
z_{24}	number of teeth of the 24-hour wheel	1	Ch. 4
z_3	number of teeth of the third driving wheel	1	Ch. 4
z_{A}	number of teeth of the driving wheel on the hour-wheel pipe	1	Ch. 4
z_{A}	number of teeth of the driving wheel	1	Ch. 4
z_{B}	number of teeth of the intermediate wheel	1	Ch. 3
z_{F}	number of teeth of the barrel	1	Ch. 3
z_{H}	number of teeth of the escape wheel	1	Ch. 4
z_{K}	number of teeth of the third wheel	1	Ch. 4
z_{M}	number of teeth of the centre wheel	1	Ch. 4
z_{N}	number of lifting pins	1	Ch. 4
z_{S}	number of teeth of the fourth wheel	1	Ch. 4
z_{Sch}	number of teeth of the count-wheel drive wheel	1	Ch. 4
z_{Sp}	number of teeth of the ratchet wheel	1	Ch. 3
z_{St}	number of teeth of the hour wheel	1	Ch. 4
z_{V}	number of teeth of the cannon pinion	1	Ch. 3
z_{V}	number of leaves of the cannon pinion	1	Ch. 4
z_{W}	number of teeth of the minute wheel	1	Ch. 4

Index

2

24-Hour Display 96
24-hour display 96

A

A/h 8
Acceleration 31
acceleration 31
across-corners dimension 17
addendum 100
adjacent side 24
Adjustment, digital 231
Age of the Moon 243
Age of the Moon from the date 243
Ageing, quartz crystal 230
Alligation cross 209
Alloy 207
Alloying 208
Alloying down 210
Alloying up 209
alternation 117
Ampere-hour 217
Amplitude drop 180
Amplitude drop in 24 hours 180
Amplitude from lift angle and lift time 175
Amplitude from the energy balance 164
Amplitude with incorrectly set lift angle 176
amplitude, balance 118
Amplitude, energy balance 164
Amplitude, incorrect lift angle 176
Amplitude, pendulum 147
Amplitude, timing machine 175
angle from three sides 25
Angle of Friction 48
angle of friction 48
Angle of friction 158
Angle of the Balance over Time 118
angle of twist 60
angular frequency 34, 117
Angular Frequency 117
Angular indexing 204
Angular indexing with the dividing head 204
Angular pitch of the escape wheel 152
Angular pitch, escape wheel 152
Angular Velocity 34
angular velocity 34
Angular Velocity from Rotational Speed 34
angular velocity, balance 118
Annual angle for the equation of time 240
Annulus 11
annulus 11
Antimagnetic 185
Apparent local time 243
Apparent solar time 240
Arc Length 12
arc length 12
Archimedean spiral 121
Archimedes 22
area moment 58
Area of a Circle 10
area of a circle 10
area, circle 10
Arithmetic Mean 28
Arithmetic mean 248
Asthmometer 247
automatic winding 84
Auxiliary weight 143
average 28
Average 248
Axial play 201

B

Backlash 105
backlash 105
balance rim 51, 125
balance rim, volume 19
balance screws 125
balance spring, length 121
balance spring, restoring torque coefficient 119
balance spring, thickness and rate 120
balance spring, vibrating 123
balance, period 116
balance-spring number 122
Banking 159
bar 36
barrel 64
barrel arbor 64
barrel diameter, effective 77
barrel turns 65
barrel, gear ratio 75
Base Circle 107
base circle 107
Battery capacity 226
Battery life 226
Battery life from capacity and current consumption 226
Battery life with self-discharge 227
Battery test 221
Bearing clearance 201
bearing load 37
bearing pressure 37
Bearing Pressure in a Pivot Bearing 37
Beat Duration 9
beat duration 9
Beat duration (vibration) 138
Beat error 174
Beat error as an angle 174
Beat error, angle 174
beat rate 8
Beat Rate and Frequency 8
Beat rate and rotational speed of the escape wheel 152
beat rate from numbers of teeth 91
Beat Rate from the Frequency 117
Beat Rate from the Gear Train 91
Beat Rate from the Last Mesh 94
Beat rate from the period 139
Beat Rate of a Watch from the Numbers of Teeth 92
beat rate, balance 117

Beat rate, deviation 171
Beat rate, escape wheel 152
Beat rate, pendulum 139
beat rate, watch 92
Beat, setting in 174
bell-crank lever 41
Bending 205
bending moment 56
Bending Stress 56
bending stress 56
bending stress, mainspring 71
Bezel 206
blade height 119
blade thickness 58, 71, 119
blade thickness from turns 68
Blank length 205
Blank length for a ring 206
blows in 12 hours 108
Blows per Day 108
Blows per Revolution of the Driving Wheel 109
Bob adjustment from rate and beat rate (workshop formula) 141
Bob, adjusting 141
braking distance 32
breakaway force 48
Breguet overcoil 123
broken pivot 57
buoyancy 22

C

Calendar error 239
Calendar year 238
cannon pinion 95
cantilever beam 59
Capability from tolerance and spread 251
Capacitance (capacitor) 222
Capacitance of the parallel-plate capacitor 223
Capacitor voltage during charging 225
Capacitor, charge 222
Capacitor, charging 225
Capacitor, energy 225
Carat 207
centre distance 26, 98
Centre Distance 98
Centre Distance from Coordinates 26
Centre distance of wheel and pallet fork 156
Centre distance, wheel and pallet fork 156
Centre of oscillation 145
centre-of-gravity error 127
CET 242
CGS number 122
CGS Number of the Balance Spring 122
CGS value 122
chain and fusee 80
Change in time error 170
Change in time error over a running period 170
Charge 217
Charge of the capacitor 222
Charging curve 225
Chord 13
chord 13
chord length 13
Chronometer testing 182
chronometer testing, temperature 132
circular arc 12
circular cross-section 57, 58
Circular error 147
circular pitch 97
Circular Sector 12
Circular Segment 14
Circumference of a Circle 10
circumference of a circle 10
circumference, circle 10
Clearance fit 199
Coating thickness 213
Coating thickness from current density and time 213
Coating thickness from the mass 213
coefficient of expansion 61
coefficient of friction 47
Coil Pitch 121
coil pitch 121
Coil wire 218
Coils of the Run-Down Spring 65
Coils of the Wound Spring 64
combining forces 37
Combining Two Forces 37
compensation balance 133
Compensation pendulum 149
Compound pendulum 145
compressive stress 53
Condition for the Fourth Wheel 93
Conductor resistance 218
Cone 20
cone 20
Contact points, distance 157
coordinates 26
Coordinates on the Pitch Circle: x 27
Coordinates on the Pitch Circle: y 28
core diameter, pinion 103
Correction factor for tight bend radii 206
Correction factor, bending 206
Cosine 24
cosine 24
Costing, turned part 191
Coulomb 217
Count Wheel 109
count wheel 109
count-wheel striking 109
counting teeth 92
Cpk 251, 252
Critical capability index (lower limit) 252
Critical capability index (upper limit) 251
cross-section, round 10
Crown Revolutions for Winding 83
crown wheel 83
crown winding 83
cuboid 18
Current 215
Current consumption 226
Current consumption, mean 233
Current density 213
Current efficiency 212
curve template 123
Cutting speed 189
cycloidal toothing 100
Cylinder 18

cylinder 18
Cylinder escapement 168
Cylinder escapement: pitch, tooth length and cylinder 168
Cylinder wheel 168
Cylinder, diameter 168
cylinder, moment of inertia 52

D

daily requirement, automatic 85
damping, balance 135
Day number 240
Days in the leap-year cycle 239
decay test 135
deceleration 31
decimal hours 6
Decimal minutes 247
dedendum 103
Defect rate 253
deflection 59
Deflection of a Cantilever 59
degree 8
Degrees to Radians 8
degrees, minutes, seconds 6
Delta, positions 178
density 21
Density by Weighing in Air and in Water 22
density determination 22
Deposited mass (Faraday's law) 212
Depth of cut 192
depthing distance 26
Detached escapement 154
determining a missing wheel 92, 102
Developed length 205
Developed length of a bend 205
Deviation (fits) 198
Deviation in Percent 29
deviation, percentage 29
diagonal 23
Difference between vertical and horizontal positions 179
Difference, vertical–horizontal 179
Dimension chain 201
Direct indexing 203
disc, moment of inertia 52
Distance between the contact points 157
Distance under Uniform Acceleration 32
distance–time law 32
Divider chain 229
dividing circle 67
Dividing Circle at Optimum Length 67
Dividing head 204
division 27
downhill force 38
Draw 158
Draw angle 158
Draw: angle of friction as the limit 158
driving weight 35
Drop 153
Drop angle 153
Drop as travel of the tooth tip 154
Drop loss 163
Drop, as travel 154
Drop, loss 163
Dynamic pressure 185
Dynamic pressure during movement in water 185

E

Effect of temperature on measurement 255
Effective Barrel Diameter 77
effective length 124
Effective Length for the Nominal Beat Rate 124
Efficiency 46
efficiency 46
Efficiency from Measured Torques 114
Efficiency of the escapement 164
Efficiency of the Mainspring 74
Efficiency, escapement 164
efficiency, gear train 113
efficiency, mainspring 74
efficiency, movement 82
eight-day movement 76
Electrical energy 217
Electrical power 216
Electrical work 217
Electroplating 212
Electroplating, coating thickness 213
ellipse 16
ellipse, circumference 16
Ellipse: Area 16
Ellipse: Circumference 16
End shake 201
End shake of an arbor 201
End shake, adjusting 202
End shake, limit values 201
Energy balance, balance 164
Energy content of the battery 226
energy flow 46
Energy in the capacitor 225
Energy of a motor pulse 234
Energy of a Tensioned Spring 44
Energy of a Torsion Spring 45
energy of motion 43
energy of the balance 43
energy of the balance spring 45
Energy of the Oscillating Balance 133
Energy of the Weight 79
energy reserve, mainspring 42
energy, balance 133
energy, barrel 74
Energy, electrical 217
energy, mainspring 73
energy, maximum 73
energy, weight drive 79
Equal Centre Distance in the Motion Work 96
Equation of time 240
Equation of time display 240
Equation of time from the date 241
Equation of time, approximation formula 241
equilibrium of moments 41
Equivalent pendulum length 145
Equivalent resistance 220
Error of the rate due to reading 187
Escape wheel, angular pitch 152
escape wheel, rotational speed 93
Escape wheel, rotational speed 152
Escape wheel, work per beat 163
Escapement, efficiency 164

F

falling speed 32
Farad 222
Faraday's law 212
Feed 190
Feed per tooth 191
Feed rate 190
Feed rate in milling 191
Feed rate in turning and drilling 190
filling factor 67
filling of the barrel 66
filling ratio 67
Filling Ratio of the Barrel 67
fine regulation 126
Fine Regulation with Timing Screws 126
Fine regulation, pendulum 143
Fine weight 207
Fineness 207
Fineness in carats 207
Fineness of a mixture 208
Fineness, lowering 210
Fineness, raising 209
First pass yield 253
First pass yield of an operation 253
First pass yield over several operations 253
First pass yield, overall 253
Fit 199
flat (on an arbor) 13
Flux density and field strength in air 185
fly 110
FMEA 254
force 35
Force and Acceleration 35
Fork 160
Fork and impulse pin: lengths and angles 160
Fork length 162
Fork length from the centre distance 162
Fork, ratio 162
fourth wheel 93
fourth wheel, missing 94
Fourth wheel, quartz movement 233
FPY 253
Free Decay without Drive 135
free fall 31, 43
free-sprung balance 126
frequency 8
frequency deviation 128
Frequency deviation 229
Frequency divider 229
Frequency of the Balance 116
frequency, balance 116
friction force 47
friction torque 50
Friction Torque in a Pivot Bearing 50
full winding 83
fusee 80

G

Gauss 185
gear ratio 87
Gear Ratio of One Mesh 87
gear train 89
going train 91
Gold alloy 207
Gold plating 212
Golden rule of metrology 255
gradient 24
Graham escapement 166
Graham escapement, pallet width 166
Gravitational acceleration from latitude and altitude 150
Gravitational acceleration, location 150
Gravity formula 150
Greatest positional variation 178
Gregorian calendar 238
Gridiron pendulum 149

H

half-hour striking 108
hand setting 85
Hand Setting: Travel of the Minute Hand 85
hand, moment of inertia 52
Height of Arc 13
height of arc 13
height of fall 32, 78
Height of Fall of the Weight 78
helix angle 49
hexagon 17
Hexagon: Across Corners 17
hole circle 27
Hole circle 203
hole circle, coordinates 27
Hole Circle: Distance between Adjacent Holes 27
Hollow Cylinder 19
hollow cylinder 19
hollow cylinder, moment of inertia 51
Hooke's Law 54
Hooke's law 54
horological toothing 100
hour wheel 95
Hundredth of a minute 247
hypotenuse 23

I

idler 89
impact speed 32
Impact velocity in a fall 186
Impulse and drop at the wheel 153
Impulse pin 160
Impulse pin radius from the centre distance 161
Impulse pin, radius 161
Impulse roller 160
Impulse, at the pallet fork 159
Impulse, at the wheel 153
In beat 174
Inch thread 195
inclined plane 38
Inclined Plane, Wedge and Thread with Friction 49
Inclined Plane: Downhill Force 38
Inclined Plane: Normal Force 38
Index crank 204
Index plate 203
Indexing 202
Indexing angle 202
Indexing, direct 203
Indexing, indirect 204
Indirect indexing with the dividing head 204

Influence of Blade Thickness 71
Inhibition 231
Inhibition: rate correction by suppressed pulses 231
Interference 200
Interference fit 200
intermediate wheel 76
Internal resistance 221
Internal resistance from open-circuit and load measurement 222
Internal resistance, measuring 222
involute toothing 107
ISO 1413 186
ISO 3159 182
ISO 764 185
Isochronism error from 0 h and 24 h 180
Isochronism, 0 h and 24 h 180
Isochronism, pendulum 147

J

Jewel, pressing in 200
joule 41
Joule heating 216
Julian calendar 238

K

Kinetic Energy 43
kinetic energy 43

L

Lavet motor 232
Law of Cosines 25
law of cosines 25
Law of Sines 25
law of sines 25
Law of the Lever 41
law of the lever 41
Lead angle 197
leaf spring 56, 59
Leap year 238
Leap-year cycle 239
leg (cathetus) 23
Length of the Flat Balance Spring 121
Length of the pallet arm 156
lever 41
lever arm 40
Lift angle, fork 160
Lift angle, timing machine 176
Lift time 175
ligne 6
Lignes to Millimetres 6
Limit of size 198
line of action 107
Linear Expansion 61
linear expansion 61
Load capacitance 224
Load resistance 222
Local time 242
Local time from the longitude 242
Lock 159
Longitude 242
Loss due to the drop 163
Lower limit of size 198
Lowering the fineness with alloying metal 210
Lunar revolution 238

M

Machine capability 251
Machining time 191
Machining time in turning and drilling 191
Magnetic field 185
mainspring, blade thickness 71
mainspring, coils 64, 65
mainspring, length 66
mainspring, replacement 72
mainspring, torque 70
manual winding 83
mass 21
Mass from Volume and Density 21
mass moment of inertia 51
masselottes 125
matching balance and balance spring 129
Matching: Restoring Torque Coefficient for Balance and Beat Rate 129
Maximum Angular Velocity 118
Maximum clearance 199
Maximum end shake 201
Maximum interference 200
Maximum Number of Turns 68
Maximum Storable Energy 73
Maximum Torque from the Permissible Bending Stress 71
Mean 248
Mean current consumption of the movement 233
Mean daily rate 171
Mean diameter 206
Mean length of the calendar year 238
Mean Power 81
mean rate 28
Mean rate 171
Mean solar time 240
mean value 28
Mean variation in rates 182
Mean, uncertainty 249
Measurement error, rate 187
Measurement error, temperature 255
Measurement uncertainty 255
measuring efficiency 114
Measuring instruments, selection 255
measuring the moment of inertia 130
Measuring the Moment of Inertia with an Additional Disc 130
measuring the tip circle 106
Measuring the Tip Circle with an Odd Number of Teeth 106
Measuring time 172
mercury pendulum 62
Metal removal rate 192
Metal removal rate in turning 192
Middle Temperature Error 133
middle temperature error 133
Milliampere-hour 226
Milling, feed 191
Minimum clearance 199
Minimum end shake 201
Minimum interference 200
Minimum swing 167
Minor diameter 196
Minor diameter of the external thread 196
minute of arc 6

minute wheel 95
Minutes and Seconds as Decimal Value 6
Mixture calculation 208
module 97
module from centre distance 98
Module from the Measured Tip Circle 102
module from tip circle 102
modulus of elasticity 54
modulus of elasticity, balance spring 119
moment of inertia 51
Moment of inertia about the suspension (parallel axis theorem) 146
Moment of Inertia of Screws and Masselottes 125
Moment of Inertia of the Rim (Thin Ring) 125
moment of inertia, balance 125
moon disc 111
moon phase 111
Moon phase 243
moon phase, accuracy 112
moon phase, precise 112
Moon-Phase Disc 111
motion work 95
motion work, centre distance 96
Motor pulse 233
movement size 6

N

neutral axis 56
Neutral axis 205
New moon 243
newton 35
Nominal size 198
nominal value and actual value 29
normal force 38
Normal gravity 150
number of coils 121

O

odd number of teeth 106
Ohm's law 215
Open-circuit voltage 221
opposite side 23
Optimum Spring Length 66
ordering a balance spring 122
oscillation energy 133
out-of-poise 127
oval 16
Overall Efficiency 46
overall efficiency 46
Overall Efficiency of the Gear Train 113
Overall Gear Ratio 89
overall gear ratio 89

P

Pallet arm 156
Pallet fork travel 159
Pallet fork, centre distance 156
Pallet fork, span 155
Pallet span 155
Pallet stones, distance 157
Pallet width 166
Pallet width of the Graham escapement 166
Parallel Axis Theorem (Steiner) 53
parallel axis theorem (Steiner) 53
Parallel axis theorem (Steiner) 146
Parallel connection of capacitors 223
Parallel connection of resistors 220
Parallel-plate capacitor 223
parallelogram of forces 37
Paris ligne 6
Pendulum length 139
Pendulum length from beats per minute (workshop formula) 140
Pendulum length from the beat rate 139
Pendulum, period 138
Pendulum, supplementary arc 167
percent 29
percent and rate 7
Period 138
period and beat duration 9
Period at larger amplitude 147
Period of the Balance 116
Period of the compound pendulum 145
period, balance 116
Peripheral Speed 33
peripheral speed 33
permissible stress 55
Permissible Stress and Safety Factor 55
Permittivity 223
Perpetual calendar 239
pin 55
pin wheel 109
pin wheel, gear ratio 109
pinion 87
pinion leaf 102
pinion, missing 102
Pitch and Module 97
pitch circle 27, 97
Pitch Circle Diameter 97
Pitch Circle from the Centre Distance 99
Pitch diameter 195
Pitch from threads per inch (inch threads) 195
pitch point 98
pivot bearing 37
pivot diameter 50
pivot friction 50
Point Mass and Thin Ring 51
poising 127
pond (gram-force) 40
positional error 127
Positional error 178
Positional variation 178
Positions, horizontal and vertical 179
Potential Energy 42
potential energy 42, 79
potential energy (gravitational) 42
Power 45
power 45
Power at the Oscillator 82
power from torque 81
Power from Torque and Rotation Period 81
Power in a resistor 216
Power in a resistor from the current 216
Power in a resistor from the voltage 216
Power in Rotation 46
Power loss 216

power of the barrel 46
power requirement 135
Power Requirement of the Balance 135
power reserve 76
power reserve, automatic 85
power, balance 82, 135
power, drive 81
Power, electrical 216
Power, escape wheel 163
power, mean 81
power, rotation 46
ppm 7, 253
ppm and rate 229
Precise Moon Phase with Gearing 112
Press fit 200
Pressure 36
pressure 36
pressure angle 107
Pressure force 184
Pressure force on crystal or caseback 184
Pressure of a water column 184
Prism: Solid with Constant Cross-Section 18
Process capability 251
Process capability, critical 251
Production counter 245
proof stress 55
pulley, loose 78
Pulse duration 233
Pulse energy 234
Pulsometer 247
pyramid 20
Pythagoras 23
Pythagoras' Theorem 23

Q

Q factor, balance 134
Quality Factor 134
quality factor 134
Quality factor 164
Quartz crystal, temperature behaviour 230
Quartz frequency 32 768 Hz 229

R

radian 8
radian measure 8, 34
Radius from Chord and Height of Arc 14
radius of curvature 14
Radius of the Barrel Arbor (Guide Value) 64
Raising the fineness with fine metal 209
Range 248
ratchet wheel 83
Rate 170
Rate as Relative Deviation 7
Rate change due to a different location 150
Rate Change due to Blade Thickness 120
Rate Change due to Out-of-Poise (Centre-of-Gravity Error) 127
Rate change due to temperature 148
Rate Change due to the Moment of Inertia 126
Rate change from a change in length 140
Rate Change from Restoring Torque Coefficient and Moment of Inertia 129
Rate change when the amplitude changes 147
Rate change with the rating nut 141
Rate Change with the Regulator 124
Rate from the deviation of the beat rate 171
Rate from the Frequency Deviation 128
Rate from the time shift of the beats 172
Rate from two time errors 170
Rate of a sidereal clock against mean time 237
Rate of the quartz watch from the frequency deviation 229
Rate when worn from two groups of positions 179
Rate, daily 170
rate, frequency 128
Rate, from the beat rate 171
Rate, mean 171
rate, moment of inertia 126
Rate, pendulum length 140
Rate, per month 170
Rate, quartz watch 229
Rate, resumption of 183
Rate, sidereal time 237
Rate, when worn 179
Rating nut 141
Ratio between fork and balance 162
Ratio, fork and balance 162
RC circuit 224
Re-alloying 208
Reading error 187
rectangle 15
rectangular cross-section 57
reduction gear train 84
Reduction ratio from rotor to fourth wheel 233
Reduction Ratio of Automatic Winding 84
Reduction Ratio of the Motion Work 95
Reduction ratio, quartz movement 233
Reference temperature 255
Reference time 169
Regulating weight 144
regulating, balance 126
regulating, basic formula 129
Regulating, pendulum clock 140, 141
regulating, regulator 124
regulation 129
regulator 124
relative deviation 7
replacement spring 72
Replacement Spring with the Same Effect 72
Resistance and temperature 218
Resistance of a wire 218
Resistance, electrical 215
Resistivity 218
resolving forces 38
restoring torque 119
restoring torque coefficient 45, 119
Restoring Torque Coefficient from the Dimensions of the Balance Spring 119
Restoring Torque of the Balance Spring 119
resultant 37
Resumption of rate 183
reverser 84
Revolutions of the Fly per Blow 110
Revolutions per Winding 110
Rework rate 253
Ring and Hollow Cylinder 51
ring area 11

Ring, blank length 206
Risk priority number 254
Risk priority number (FMEA) 254
rod, moment of inertia 52
Roller jewel (impulse pin) 160
root circle 103
Root Circle of Involute Toothing 107
Root Circle of the Pinion 104
Root Circle of the Pinion (Guide Value) 103
Root Circle of the Wheel 103
root circle, involute 107
root circle, pinion 103, 104
Rotation Period 89
rotation period 89
Rotation Period of the Barrel 75
Rotation Period of the Barrel with Intermediate Wheel 76
rotation period, barrel 75
Rotational Energy 43
rotational energy 43
rotational speed 33, 34, 88
Rotational Speed of the Escape Wheel 93
Rotational Speed of the Pinion 88
Rotational speed of the rotor 232
Rotational speed, escape wheel 152
rotor 84
Rotor (stepper motor) 232
Rotor Revolutions for a Power Reserve 85
RPN 254
Run 159
running time 76
Running Time from the Height of Fall 79
Running Time from the Turns of the Barrel 76
Running Time until the Moon Phase Deviates 112
running time, weight drive 79

S

safety factor 55
sagitta 13
Sample standard deviation 249
Scrap rate 253
second moment of area 58
Second Moment of Area: Circle 58
Second Moment of Area: Rectangle 58
second of arc 6
Seconds pendulum 138
section modulus 57
Section Modulus: Circle 57
Section Modulus: Rectangle 57
sector 12
Secular calendar 239
segment 14
selecting a balance spring 129
Self-discharge 227
self-locking 48, 49
Self-locking, thread 197
Series connection of capacitors 224
Series connection of resistors 219
setting angle 24
Setting angle 193
setting wheel 85
Shear 55
shear 55
shear modulus 60
shear stress 55
shock 31
shock protection 35
Shock protection, testing 186
shock test 43
Shock test 186
shortening the balance spring 124
shrink fitting 61
Sidereal clock 237
Sidereal day 237
Sidereal day from solar day and year 237
Sidereal month 238
Sidereal time 237
Sine 23
sine 23
sine bar 23
sinusoidal oscillation 118
Sliding Friction 47
sliding friction 47
sliding pinion 85
Slope 193
Solar day, mean 237
Solid Disc and Cylinder 52
Span 155
Span of the pallet fork 155
Speed 30
speed 30
Speed and Distance without Time 32
Speed from measured distance and stopped time 244
Speed of sound 246
Speed of sound and temperature 246
Speed of sound, temperature 246
Speed under Uniform Acceleration 31
Speed, chronograph 244
Sphere 19
sphere 19
Spindle speed from cutting speed (workshop formula) 189
Spindle speed, turning and milling 189
Spread 248, 249
spring constant 44
spring energy 44
Spring Force 44
spring force 44
spring length 66
spring length from coils 69
Spring Length from the Coils of the Run-Down Spring 69
spring pack 64
spring pack, width 67
spring rate 44
Spring Rate of a Leaf Spring 59
spring rate, leaf spring 59
spring torque 70
spring volume 73
square 17
Square: Across Corners 17
Standard deviation 249
Standard deviation of the mean 249
State of wind 180
Static Friction 48
static friction 48
Steady state 164
Step angle 232
Step per beat 152

Stepper motor 232
stopping distance 32
Storage capacitor 225
Stored Energy of the Ideal Spring 73
Strain 54
strain 54
strain energy 44
stress 53
striking work 108
striking work, running time 110
Sum of the Hour Blows 108
Sundial 243
Supplementary arc 154
Supplementary arc of the balance 154
Supplementary arc of the pendulum 167
Supplementary arc, pendulum 167
synodic month 111
Synodic month 238
Synodic month from the Moon's revolution 238

T

Tachymeter 245
Tachymeter scale 245
Tailstock offset 194
Tangent 24
tangent 24
Tangent, lever escapement 156
Taper 193
Taper angle 193
Taper angle and setting angle 193
Taper pin 193
Taper ratio 193
Taper ratio (conicity) 193
Taper turning 194
Tapping 196
Tapping drill diameter 196
Tapping hole 196
Telemeter 246
Telemeter: distance from the travel time of sound 246
Temperature behaviour of the tuning-fork crystal 230
temperature coefficient 131
Temperature Coefficient from Material Data 131
Temperature Coefficient from Measurement 132
Temperature coefficient in chronometer testing (rate variation per °C) 182
Temperature coefficient of resistance 218
Temperature coefficient, chronometer testing 182
temperature coefficient, measurement 132
Temperature compensation with two materials 149
temperature error, balance 131
Temperature error, pendulum 148
Tensile and Compressive Stress 53
tensile strength 54
tensile stress 53
tensioning work 73
terminal curve 123
Terminal Curve Number 123
terminal radius 123
Terminal voltage 221
Terminal voltage of the loaded battery 221
Test pressure 184
thermal expansion 61
thermoelastic coefficient 131
Thin Rod about Its Centre 52
third wheel 93
Thread depth 196
Thread pitch 195
thread, friction 49
Thread, metric 195
Threads per inch 195
Time constant 224
Time constant for charging and discharging 224
Time error 169
Timing machine, amplitude 175
Timing machine, rate 172
timing screw 51
timing screws 125, 126
Timing weight 143
Timing weight at any height 144
Timing weight at half the pendulum length 143
Timing weight, height 144
tip circle 100
Tip Circle of Involute Toothing 107
Tip Circle of the Pinion 102
Tip Circle of the Wheel 100
tip circle, involute 107
tip circle, pinion 102
tip clearance 104
Tolerance 198, 251
Tolerance chain 201, 202
Tolerance of end shake (tolerance chain) 202
Tolerance zone 198
tooth clearance 105
tooth depth 103
tooth space 104
tooth thickness 104
Tooth Thickness of the Wheel 104
Top slide, swivelling 193
Torque 40
torque 40
Torque at the Escape Wheel 114
Torque at the Fusee 80
Torque at the Pinion 90
Torque at the Weight Barrel 78
Torque of the Mainspring 70
torque, escape wheel 114
torque, gear train 90
torque, mainspring 70
torque, maximum 71
torque, weight drive 78
torsion 60
torsion constant 119
torsion spring 45, 60
Torsion: Angle of Twist 60
Torsion: Stress in a Round Shaft 60
torsional stress 60
Total capacitance 223
Total resistance 219
transmission ratio 87
Trapezium 15
trapezium 15
Travel of the pallet fork between the bankings 159
Triangle 15
triangle, area 15
triangle, general 25
Trimmer capacitor 223, 231

Tropical year 239
Truncated Cone 20
truncated cone 20
tube 19
Tuning-fork crystal 230
Turnover point 230
Turns of the Barrel 65
turns, barrel 65
turns, maximum number 68

U

uniformly accelerated motion 31
Upper and lower limit of size 198
Upper limit of size 198

V

Variation in rates, mean 182
vibrating the balance spring 123
Vibrating: Oscillations in the Counting Time 123
vibration (half-oscillation) 8, 117
Vibration (half-oscillation) 138
vibrations per hour 91
Voltage 215
Voltage divider 221
Voltage divider (unloaded) 221
Volume Expansion 62
volume expansion 62
volume, bar 18
volume, cone 20
volume, cylinder 18
volume, sphere 19

W

watch winder 85
Water column 184
Water pressure 184
water resistance 36
Water resistance 184
watt 45
Watt-hour 226
Watt-second 217
weakening a spring 59
Wearing rate 179
Wedge 39
wedge 39
wedge angle 39
wedge, friction 49
weighing in water 22
weight barrel 77
weight drive 42, 78
weight drive, height of fall 78
Weight Force 35
weight force 35
weight of a part 21
Weight tray 143
width across flats 17
winding pinion 83
winding stem 60
Work 41
work 41
Work Delivered while Running Down 74
Work in Rotation 42
Work of the escape wheel per beat 163
work, barrel 74
Work, escape wheel 163
work, rotation 42
Workshop formula, bob adjustment 141
Workshop formula, pendulum length 140
Workshop formula, spindle speed 189

Y

Years until one day of error 239
yield strength 54
Young's modulus 54

Z

Zone time 242
Zone time from the sundial 243